IMS Graduate Summer School in Logic 2026
Abstracts

29 Jun 2026–17 Jul 2026

June 22, 2026

Abstracts

Tutorial

Dp-minimal Theories

William Johnson

Fudan University, China

The class of *dp-minimal theories* contains many of the important theories in model theory, such as o-minimal theories, strongly minimal theories, Presburger arithmetic, and the theory of the p-adic numbers. Dp-minimal theories can be seen as the "1-dimensional" NIP theories. Although the class of dp-minimal theories is big, admitting many examples, it is small enough that we can prove many non-trivial things about dp-minimal theories. In a dp-minimal theory, *dp-rank* yields a good notion of dimension for definable sets. Under some mild topological assumptions, dp-minimal theories have topological tameness for definable sets, generalizing what happens in o-minimal theories. For example, definable functions are generically continuous, there is a weak form of cell decomposition, and definable groups have a natural manifold structure. Dp-minimal theories of fields always satisfy these topological assumptions. In the summer school, we will work through much of the above. We will also discuss some related results without proof, such as the known results on dp-minimal fields, rings, groups, and ordered groups, and other examples of dp-minimal theories.

References:

- Alfred Dolich, John Goodrick, David Lippel. "Dp-minimality: basic facts and examples"
- Will Johnson. "The canonical topology on dp-minimal fields"
- Itay Kaplan, Alf Onshuus, and Alexander Usvyatsov. "Additivity of the dp-rank"
- Pierre Simon. *A Guide to NIP Theories*, Chapters 1-4
- Pierre Simon. "Dp-minimality: invariant types and dp-rank"

- Pierre Simon and Erik Walsberg. “Tame topology over dp-minimal structures”

Further reading:

- Hans Adler. “Strong theories, burden, and weight”
- Hans Adler. “Theories controlled by formulas of Vapnik-Chervonenkis codimension 1”
- Sylvy Anscombe. “Shelah’s Conjecture and Johnson’s Theorem”
- Christian d’Elbée and Yatir Halevi. “Dp-minimal integral domains”
- Christian d’Elbée, Yatir Halevi, and Will Johnson. “The classification of dp-minimal integral domains”
- Alfred Dolich and John Goodrick. “Tame topology over definable uniform structures”
- Vince Guingona. “On VC-minimal fields and dp-smallness”
- Franziska Jahnke, Pierre Simon, Erik Walsberg. “Dp-minimal valued fields”
- Will Johnson. “The classification of dp-minimal and dp-small fields”
- Will Johnson. “Visceral theories without assumptions”
- Alf Onshuus and Alexander Usvyatsov. “On dp-minimality, strong dependence, and weight”
- Bruno Poizat. *Stable Groups*
- Atticus Stonestrom. “On non-abelian dp-minimal groups”
- Alexander Usvyatsov. “On generically stable types in dependent theories”
- Frank Wagner. “Dp-minimal groups”

Computable Structure Theory

Theodore A Slaman

University of California at Berkeley, USA

This course will be an overview of computability-theoretic and definability-theoretic perspectives on countable first-order structures. A central theme will be the distinction between internal properties, determined by the isomorphism type of a structure, and external properties, revealed through the computability-theoretic analysis of its presentations. We will employ computability-theoretic constructions, arithmetic forcing, the understanding of the types realized in countable models, and tools from effective descriptive set theory.

Nairian Models

W Hugh Woodin

Harvard University, USA

Nairian models introduced by Blue, Larson, and Sargsyan are ZF models generalizing the Chang Model and which are obtained from LSA models of the Axiom of Determinacy (more precisely from AD+ models). These models have many interesting properties and provide a new source of ground models for obtaining independence results in ZFC. We will survey the background material with the goal of proving an upper bound on the large cardinals which can exist in (full) Nairian models. This is relevant to the Ultimate L Program.

The subject of Nairian models combines descriptive set theory in the context of AD, fine-structure theory from the Inner Model Program, and Cohen's method of forcing.

Students' Talks

Computable Presentations of Linear Orders

Aknur Askarbekkyzy

Kazakh-British Technical University, Kazakhstan

"Computable reducibility has become a central tool for comparing the complexity of mathematical structures and classification problems. In this talk, we investigate computable reducibility among computable copies of linear orders. For a fixed computable linear order L, we consider the degree structure P(L) formed by the c-degrees of its computable copies. We study how properties of L are reflected in the structure of P(L).

Our main result shows that the structures P(L) associated with several natural and widely studied linear orders are pairwise non-isomorphic, demonstrating that these degree structures capture essential features of the underlying orders."

Formalizing Abstract Simplicial Complexes & Stellar Subdivisions in Lean

Garett Cunningham

University of Connecticut, USA

The theory of simplicial complexes is a cornerstone of topology, offering a sophisticated tool for computing invariants. We will discuss a formalization of abstract simplicial complexes and stellar subdivisions in the Lean proof assistant. We adopt a purely combinatorial framework, providing formalizations of morphisms between abstract simplicial complexes; several constructions and operations on complexes, such as links and joins; and perform a study of how stellar subdivisions interact with these operations. We state and prove a number of identities commonly used in the study of triangulated manifolds, such as deriving equivalences between links in an abstract simplicial complex K and in a stellar subdivision $\sigma_s K$, including new results with no references in the standard literature. Throughout the journey, we will note some of the complications that arise when trying to translate these classical ideas into Lean's type theoretic foundations. This is joint work with Daniel Zach and Stefan Friedl.

Controlling the Skips of Arithmetic Sets

Taeyoung Em

University of Wisconsin-Madison, USA

The notion of cototality has proven important in various areas of mathematics. The skip, a fundamental feature of the enumeration degree structure, exhibits a strong connection with cototality. Andrews, Ganchev, Kuyper, Lempp, Miller, A. Soskova, and M. Soskova asked whether skip-cototality coincides with weak cototality. We show that the answer is negative by constructing a Π_2^0 set which has its skip non-cototal. We further adapt this construction to obtain additional results on iterated skips of a set.

Reverse Mathematics and Well-quasi-orders

Luca Facchinetti

University of Udine, Italy

We revisit the reverse-mathematical analysis of equivalent definitions of well-quasi-orders (wqos), refining classical results by introducing new characterizations and studying their logical strength. Our results provide a more detailed picture of the reverse mathematics of wqos and leave several interesting questions open.

A Survey of Higher Proof Theory

Koshiro Ichikawa

Nagoya University, Japan

Traditional proof theory has mainly been concerned with analyzing which Γ-sentences are provable in a theory, where Γ is a pointclass such as Π_1^0, corresponding to consistency strength; Π_2^0, corresponding to provably recursive functions; or Π_1^1, corresponding to proof-theoretic ordinals. Girard's Π_2^1-logic was the first systematic attempt to extend proof theory to a higher pointclass, namely Π_2^1. In this talk, we introduce several concepts from higher proof theory and survey both classical and recent results concerning proof theory for higher pointclasses.

Rogers Semilattices of Computable Structures

Alibek Iskakov

Kazakh-British Technical University, Kazakhstan

The talk is about applying classical numbering theory tools to the classification problem for computable algebraic structures. We compare various structures by acomparing their Rogers semilattices. The talk will be supported by a wide array of examples.

Escaping Tennenbaum's Theorem

Duarte Maia

University of Chicago, USA

Tennenbaum's theorem states that PA does not admit any nonstandard computable model. In 2022, Fedor Pakhomov proved that this theorem is fragile in regards to how PA is expressed, by constructing a theory that is definitionally equivalent to PA (roughly: "it's PA but with a different choice of signature") for which there is a computable nonstandard model. I will introduce the audience to this result and, time allowing, present the way in which we have been able to improve on Pakhomov's original construction and some remaining open questions.

The Effective Convergence of Correlation Dimension Estimators

Svetlana Pack

The Pennsylvania State University, USA

The correlation dimension of a probability measure is a popular notion of fractal dimension among physicists, as correlation dimension is easy to estimate numerically from finite data. Indeed, Birkhoff's ergodic theorem guarantees that almost every orbit of an ergodic map can be used to approximate the correlation dimension of the underlying invariant measure.

We effectivize this result to show that if we are working with a computable ergodic map over cantor space with a computable invariant measure, then our correlation integral estimator converges for every Martin-Löf random orbit. Furthermore, the convergence is effective in the sense that the rate of convergence is computable.

Relative-strength Lattices of First-order Theories

Álvaro Díaz Ramos

Carnegie Mellon University, USA

In this talk, I will present novel extensions of Friedman's isomorphism theorem between the consistency degrees of binumerations of first-order theories and Pi-0-1 elements of the Lindenbaum-Tarski algebra of a particular base theory. Using general definitions of the Pi-0-1 conservation and interpretability degrees, the isomorphism is extended for suborders of these partial orders, leading to the translation of order-theoretic properties of the Lindenbaum-Tarski algebra to these settings.

Extended Complexity Theory and Arithmetic on First-order Structure

Shogo Saito

Tohoku University, Japan

Classical computability theory and complexity theory concerns computation over finite symbols. In practical applications, however, one often wishes to work directly with structures that are not necessarily finite. I introduce an extension of Turing machines in which a first-order structure M is treated as primitive instead of a finite alphabet. I then define extended complexity classes corresponding to standard classes such as P and NP. Using methods from bounded arithmetic, I also investigate arithmetic-like 2-sorted theories whose provably total functions are precisely the extended polynomial-time functions.

A Note on Busy Beaver Bounds

Tomas Schitter

University of Buenos Aires, Argentina

"The Busy Beaver functions are a family of non computable functions that have the property of growing faster than any computable function. here are several interesting research areas related to these functions, such as their relationship with any given axiomatic theory. For example, it was proven that there exists a relatively small Turing Machine such that the ZFC set theory cannot prove whether the machine will halt or not. There also exists a relationship between finding certain values for some functions of this family and refuting well-known open mathematical conjectures, such as Goldbach's Conjecture.

We present some relationship between different functions of the family, generalizing previously used techniques to Turing Machines defined on non-binary alphabets, we introduce a new function to this family and present some properties for it, as well as the relationship it has with the previously known functions. Finally, we prove a relationship between two open conjectures for these functions."

Ordinal Game Values over Infinite Games

Isobel Rae Shaw

University of Leeds, UK

Many combinatorial games (such as chess and go), can be adapted to be played over infinite space, giving us a much larger variety of possible positions. *Ordinal game values* describe the complexity of positions in infinite games, in which one of the players has a winning strategy. In this talk we will see a brief overview of known results about these games and game values. We then will look at a description of infinite go and positions of ordinal value $\omega + n$. This is joint work with Ethan Saunders.

Two-Cardinal Combinatorics

Martha Catalina Torres Pachon

University of Barcelona, Spain

Many classical combinatorial notions for regular cardinals have natural analogues on $P_\kappa(\lambda) = \{x \subseteq \lambda : |x| < \kappa\}$. In this talk, I will focus on higher forms of stationarity in this two-cardinal setting.

The motivation comes from Bagaria's work on derived topologies on ordinals and stationary reflection [1]. I will explain how this picture can be lifted to $P_\kappa(\lambda)$, leading to a hierarchy of n-stationarity notions and associated derived topologies. This is based on my work on higher stationarity and derived topologies on $P_\kappa(\lambda)$ [2].

I will then discuss some questions about the large cardinal strength of these principles. In particular, I will mention recent joint work with Hiroshi Sakai studying what happens at the first non-trivial level, namely 2-stationarity on $P_\kappa(\kappa^+)$, in relation to strong compactness [3].

References

1. J. Bagaria, *Derived topologies on ordinals and stationary reflection.*
2. M. C. Torres, *Higher stationarity and derived topologies on* $P_\kappa(\lambda)$.
3. H. Sakai and M. C. Torres, *Strong compactness and two-stationarity on* $P_\kappa(\kappa^+)$, in preparation.

Knaster-like Properties at Singular Cardinals

Andrés F. Uribe-Zapata

Vienna University of Technology, Austria

Recently, Diego A. Mejía and the speaker proved that it is possible to force Cichoń's Maximum in such a way that the covering number of the null ideal, together with all cardinal characteristics on the right-hand side of Cichoń's diagram, are singular cardinals. This required the development of new preservation results for forcing constructions at singular cardinals. As part of this work, we introduced a new linkedness notion, called Sequentially Γ-Knaster, which extends the classical notion of Γ-Knaster to the singular setting. In this talk, we will study this property and the associated Γ-Knaster numbers of a forcing notion $\mathbb{P}$, namely, the least cardinal θ such that $\mathbb{P}$ is θ-Γ-Knaster. We will focus on their regularity properties and on the connections between sequentially Γ-Knaster and other linkedness notions. This is ongoing joint work with Diego A. Mejía.

Identity Crisis on Tightly C^2-Compact Cardinal

Fengju Zhang

Sun Yat-Sen University, China

We show that the least tightly C^2-compact cardinal may be the least strongly compact cardinal. This negatively answers Question 4.7 of Bagaria and Goldberg in their paper Reflecting measures for the case n=2.

Some Results in Classical Recursion Theory

Xuanheng Zhao

Nanjing University, China

We give a survey of some recent results in classical recursion theory. Of particular interest for us will be the r.e. Degrees, 1-genericity, domination, aed degrees, low2 degrees, and superhighness.

On the Bounded Axiomatizability of Theories and Types

Hongyu Zhu

Sun Yat-Sen University, China

In this talk, we discuss the implications of a theory or a type having an axiomatization (or none) of bounded quantifier alternation complexity. While a syntactical notion to begin with, it turns out to capture descriptive and structrual complexities as well. Starting with arithmetic as a prototypical unbounded theory, we present several results around bounded axiomatizability.