Workshop on
Statistical Mechanics and Singular SPDEs
Abstracts

11 May 2026 –15 May 2026

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Anomalous Superdiffusion of Brownian Motion in Random Incompressible Flows

Scott Armstrong
Sorbonne Université, France

We consider Brownian motion advected by a stationary, divergence-free random drift whose spatial correlations decay slowly. Such long-range dependence is expected to produce *superdiffusion*: for a typical realization of the drift, the variance of the displacement of the particle grows faster than linearly in time, with a precise exponent determined by the correlation structure of the drift. This behavior was predicted in the physics literature around 1990 via perturbative renormalization group heuristic arguments.

We recast the problem in PDE terms via the infinitesimal generator of the stochastic process, which is a divergence-form drift-diffusion operator, whose random coefficients exhibit an approximate self-similarity across scales. Our main tool is a scale-by-scale coarse-graining scheme. At each scale we compare the operator to an effective Laplacian with a diffusivity that depends on the scale, and we obtain quantitative control of the approximation error. Because of the self-similar structure, this procedure must be iterated across all scales: the operator does not converge to a single homogenized limit; instead, the analysis yields a scale-dependent effective diffusivity. This may be viewed as a rigorous counterpart of the earlier renormalization group predictions.

A key analytic input is an *anomalous regularization* phenomenon: we prove near-Lipschitz regularity for solutions that are uniform in the microscopic (molecular) diffusivity parameter, allowing quantitative control even in the regime of very weak underlying diffusion. In other words, the chaotic behavior of the random drift produces regularity in the solutions of the equation.

The talk is based on joint work with Ahmed Bou-Rabee (UPenn) and Tuomo Kuusi (Helsinki) which is available here: https://arxiv.org/abs/2601.22142

An Introduction to the Stochastic Heat Flow

Quentin Berger

Université Sorbonne Paris Nord, France

The Stochastic Heat Equation (SHE) with multiplicative white noise is a stochastic PDE which is ill-defined in its critical dimension d=2. In that case, very recent results show that a subtle normalisation procedure is needed to make sense of it. For instance, the "probabilistic" approach consists in studying the directed polymer model (which can be thought as a discretised SHE) in some critical window of parameters. This is the approach followed by Caravenna, Sun and Zygouras, that allowed them to prove the existence of the so-called critical 2D Stochastic Heat Flow (SHF). Another approach by Tsai also give an axiomatic characterization of the SHF.

I will review some of the properties of this object, highlighting that contributions come from a wide range of ideas, from stochastic analysis to probability and statistical mechanics.

Construction of the 2D Yang-Mills-Higgs Measure I

Bjoern Bringmann

Princeton University, USA

In this series of two talks, we discuss the construction of the 2D Yang-Mills-Higgs (YMH) measure via stochastic quantization, or i.e. by Markov Chain Monte Carlo. To do so, we show global well-posedness and uniform-in-time bounds for the associated Langevin dynamics, given by the 2D stochastic YMH equations. A key component of our approach is the further development of techniques in stochastic geometric analysis, combining ideas from geometric analysis and stochastic analysis. These methods yield a manifestly gauge-covariant local existence theory, refined estimates for covariant stochastic objects, and a decay mechanism driven by unstable Yang-Mills connections.

Based on joint work with M. Hairer and W. Zhao.

A General Paracontrolled Ansatz for Singular SPDEs

Yvain Bruned

Univesity of Lorraine, France

In this talk, we will present a general paracontrolled ansatz for singular SPDEs built out of elementary differentials, polynomial weighted paraproducts and the driving noises. The main ingredient for writing this ansatz is to change the combinatorial perspective from decorated trees to words that are more suitable for encoding iterated paraproducts.

This is a joint work with Nicolas Moench.

A Stochastic Homogeneization Approach to Supercritical SPDEs

Giuseppe Cannizzaro

University of Warwick, UK

In the present talk, we will present a novel approach to study the large-scale fluctuations of a class of stationary quasi-linear SPDEs at super-critical dimensions. In particular, we show that this class of SPDEs has Gaussian fluctuations and that the limit is given by a linear additive Stochastic Heat equation with renormalised coefficients. Our approach is based on a suitable two-scale expansion for the generator of our dynamics and extends classical stochastic homogeneization to the infinite-dimensional setting. Byproducts of our methods are that we obtain quantitative rates for the convergence of the microscopic generator the limiting one and a more transparent characterisation of the macroscopic diffusivity in terms of microscopic observables.

Based on joint ongoing work with H. Giles (TU Vienna) and L. Gräfner (Warwick).

Construction of the 2D Yang-Mills-Higgs Measure II

Sky Cao

Massachusetts Institute of Technology, USA

In this series of two talks, we discuss the construction of the 2D Yang-Mills-Higgs (YMH) measure via stochastic quantization, or i.e. by Markov Chain Monte Carlo. To do so, we show global well-posedness and uniform-in-time bounds for the associated Langevin dynamics, given by the 2D stochastic YMH equations. A key component of our approach is the further development of techniques in stochastic geometric analysis, combining ideas from geometric analysis and stochastic analysis. These methods yield a manifestly gauge-covariant local existence theory, refined estimates for covariant stochastic objects, and a decay mechanism driven by unstable Yang-Mills connections.

Based on joint work with M. Hairer and W. Zhao.

Non-commutative Singular SPDE

Ajay Chandra

Purdue University, USA

In this talk, I will describe some recent progress on rough stochastic dynamics in the setting of non-commutative probability theory - examples will include the stochastic quantization of Fermionic quantum field theories and the setting of free probability.

This is based on joint work with Martin Hairer and Martin Peev and also work in preparation with Ilya Chevyrev, Martin Peev, and Harprit Singh.

Martingale Description of the Two-dimensional Stochastic Heat Equation

Yu-Ting Chen

University of Victoria, Canada

This talk will introduce the two-dimensional stochastic heat equation at criticality from a martingale-theoretic perspective. An interesting feature of the description thus obtained is the emergence of coefficients reflecting both pathwise extremality and quantum solvability at the level of expectations.

Gaussian Multiplicative Chaos Structure within the Polymer Measures Corresponding to the Critical 2d Stochastic Heat Flow

Jeremy Clark

University of Mississippi, USA

The critical two-dimensional stochastic heat flow (2d SHF) is a two-parameter process of random Borel measures on R^4 that was first derived in an article by Caravenna, Sun, and Zygouras as a universal distributional limit of point-to-point partition functions for (1+2)-dimensional models of a directed polymer in a random environment within a critical weak-coupling scaling regime. Later, Li-Cheng Tsai provided a list of properties uniquely characterizing the law of the 2d SHF, and he proved that it arises as a distributional limit of solutions to a mollified version of the two-dimensional stochastic heat equation. I will discuss continuum polymer measures associated with the 2d SHF. My emphasis will be on the conditional Gaussian multiplicative chaos distributional interrelationship between polymer measures having different disorder strengths.

The Maximum of 2d Directed Polymers

Clement Cosco

Université Paris Dauphine, France

Directed polymers can be described as a tilting of the simple random walk, where some local random noise can attract or repel the trajectory of the walk. It can also be seen as a discrete approximation of the stochastic heat equation.

In two dimensions and for high temperature, the partition function of the model is known to be asymptotically approximated by a Gaussian log-correlated field. In a work in collaboration with Shuta Nakajima and Ofer Zeitouni, we could refine this result by proving that the maximum of the partition function field converges to that of a (particular) branching Brownian motion.

Stationary Measure for the Open KPZ Equation: an Analytic Viewpoint

Alexander Dunlap

Duke University, USA

I will describe an analytic approach to proving the invariance of the stationary measure for the inhomogeneous open KPZ equation, given the knowledge of the (Brownian) stationary measure for the problem with homogeneous boundary conditions. In place of tools from integrable probability, we use tools from stochastic analysis to reduce the problem to a statement about the behavior of the KPZ nonlinearity at the boundary. We then use the theory of regularity structures to prove this statement.

Joint work with Yu Gu and Tommaso Rosati.

Fluctuations in the Weakly Coupled 4D Anderson Hamiltonian

Simon Gabriel

University of California, Berkeley, USA

We study the effect of a time-independent white noise disorder on a Brownian particle in four dimensions. This problem can be reformulated as a scaling-critical, singular SPDE. By tuning the noise strength appropriately, we obtain an exact analysis of the perturbative terms arising in the corresponding Green's function. This yields the first systematic approach towards renormalising scaling-critical SPDEs. To place our results in context, we will also compare with the case of space-time disorder, highlighting both parallels and key differences.

This is joint work with Tommaso Rosati.

Fernique-type Bounds for BPHZ Models and their Applications

Masato Hoshino

Institute of Science Tokyo, Japan

For general locally subcritical parabolic singular SPDEs without variance blow-up, we prove that the BPHZ model satisfies a Fernique-type theorem, namely exponential square integrability, whenever the driving noise is stationary and satisfies a suitable transportation cost inequality. As applications, we obtain two consequences. First, if the SPDE admits appropriate a priori estimates, then its solution satisfies a corresponding concentration inequality. Second, we show that the BPHZ model satisfies a Schilder-type large deviation principle, and that the solution to the SPDE satisfies a Freidlin-Wentzell-type large deviation principle, extending the result of Hairer and Weber (2015).

This talk is based on a joint work with Ismael Bailleul (Université de Bretagne Occidentale) and Ryoji Takano (Tokyo Metropolitan University).

Estimates for the Free Energy of Directed Polymers in d>=3

Stefan Junk

Gakushuin University, Japan

We consider the directed polymer model, which describes a random walk affected by an i.i.d. space-time random environment. This talk is about the regularity of the phase transition in transient dimensions.

First, we present bounds for the decay of the free energy $\mathfrak{f}(\beta)$, defined by $W_n^\beta = e^{-n\mathfrak{f}(\beta)+o(n)}$ as $n \to \infty$, at the critical point β_c for the transition from weak to strong disorder. Secondly, we also discuss bounds for the L^p-free energy $\mathrm{F}(\beta, p)$, defined by $\mathbb{E}[(W_n^\beta)^p] = e^{n\mathrm{F}(\beta,p)+o(n)}$ as $n \to \infty$, at the critical point β_p for L^p-boundedness of the partition function $(W_n^\beta)_{n\in\mathbb{N}}$. This talk is joint work with Hubert Lacoin.

Two Classical Diffusivity Results for Directed Polymers in the Weak Disorder Phase

Hubert Lacoin

IMPA Rio de Janeiro, Brazil

The directed polymer in a random environment is a statistical mechanics model which is defined attributing i.i.d. random multiplicative weights (of the form $e^{\beta\omega_{n,x}}$) to the sites visited by the graph of a simple random walk on $\mathbb{Z}^d$. When $d \geq 3$ it has long been established that the system undergoes at a critical value β_c a phase transition from a diffusive phase to a localized phase. In particular when $\beta \in [0, \beta_c]$:
1- The distribution of the rescaled polymer trajectories converge to Brownian motion.
2- The end-point distribution of the polymer satisfies a local central limit theorem with random corrections.

This talk is aimed at sketching new simple proofs of these two results. Based on collaborative work with S. Junk

Equivalence of Fluctuations between SHE and KPZ in Subcritical Weak Disorder Regime

Shuta Nakajima

Keio University, Japan

The Kardar-Parisi-Zhang (KPZ) equation is a mathematical model that describes the random evolution of interfaces. The equation has become a fundamental model in non-equilibrium statistical physics. Constructing a solution to the KPZ equation in any dimension presents a significant challenge due to its inherent non-linearity. This challenge has resulted in an enduring open problem, particularly in finding solutions in two and higher dimensions. This talk will explore the intriguing connection between the stochastic heat equation (SHE) and the KPZ equation. It offers a rigorous demonstration of the equivalence of fluctuations in these systems in the weak disorder regime for three and higher dimensions.

This talk is based on joint work with Stefan Junk (Gakushuin University).

Regularity and Positivity of the Critical $2d$ Stochastic Heat Flow

Makoto Nakashima
Nagoya University, Japan

In this talk, we focus on the critical $2d$ stochastic heat flow(critical $2d$ SHF). The critical $2d$ SHF is the measure-valued process, which can be regarded as a solution to a singular stochastic heat equation on $\mathbb{R}^2$.

It is known that critical $2d$ SHF is singular with respect to the Lebesgue measure and belongs to the Besov-Holder spaces of any negative regularity. Moreover, its average density converges to zero almost everywhere, almost surely.

We investigate its regularity and positivity.

Moments of Critical 2D Stochastic Heat Flow

Kyeongsik Nam
Korea Advanced Institute of Science & Technology, Korea

The stochastic heat equation (SHE) describes the evolution of a field under a random source. It arises as the universal scaling limit for a wide class of microscopic systems, including directed polymers and interacting particle systems, and is a cornerstone of the Kardar-Parisi-Zhang (KPZ) universality class in 1+1 dimensions. In 2+1 dimensions, SHE undergoes a weak to strong disorder phase transition depending on the strength of noise. Its scaling limit in the "critical" regime, called Critical 2D Stochastic Heat Flow, was conjectured to display extreme intermittency, with h-th moments growing like exp(exp(h)). In this talk, we establish a lower bound of this conjecture.

Joint work with Shirshendu Ganguly.

A Universality Property of the One-sided Ballistic Deposition Model near the Time Axis

Alejandro Ramirez
NYU Shanghai, China

Ballistic deposition is a model of interface growth introduced by Vold in 1959, which has remained largely mathematically intractable. In dimension d=2, it is conjectured to belong to the KPZ universality class. It is defined as a process of vertically falling blocks that stick to the top, the right or the left of growing columns. Here we introduce a variant, the one sided ballistic deposition model, in which vertically falling blocks can only stick to the top or the upper right corner of growing columns. We establish that in dimension d=2, strong KPZ universality holds near the time axis, proving that the fluctuations of the height function there are given by the Tracy-Widom GUE distribution. The proof is based on a graphical construction of the process in terms of a last passage percolation model.

This talk is based on a joint work with Pablo Groisman, Santiago Saglietti and Sebastián Zaninovich.

Global well posdeness for Exp(Phi)

Tommaso Rosati
University of Warwick, UK

We prove that the massless Exp(Phi) parabolic stochastic PDE is globally well-posed under a small drift assumption. The proof is based on nonlinear damping and a low mode decoupling.

Joint work with G. Cannizzaro and T. Galanis.

A New Approach to Loop Equations and Gauge-string Duality for Abelian Lattice Gauge Theories

Hao Shen

University of Wisconsin-Madison, USA

The proposal of Gauge-string duality suggests a possible connection between Yang-Mills (gauge theory) and the geometry of random surfaces. Important progress has been made recently by Chatterjee and Cao-Park-Sheffield. They studied the lattice gauge theories, which are well-defined statistical mechanics models, and derived a type of surface-sum formulas. These surface-sum formulas are closely related with so-called loop equations, which are Dyson-Schwinger equations in the context of gauge theories.

A notable feature of these existing results is that the surface-sum is defined with respect to a signed measure, not probability measure. It is an interesting question whether the duality can be established with positive measures on surfaces. In this talk we present a new approach of deriving loop equations. We show that for certain abelian groups and in low dimensions, the weights are positive; in many other cases, the leading terms exhibit positivity in a suitable sense.

Based on an ongoing joint work with Jack Piazza.

Disorder Relevance and Infinite-order Phase Transition for a Dimer Model with Random Weights

Fabio Toninelli

TU Wien, Austria

This talk will address the question of relevance/irrelevance of quenched randomness on phase transitions, in the context of the planar dimer model.

We study the dimer model on the square grid in a layered random environment: the edge weights are constant along each row but are i.i.d. sampled between rows. This disorder structure is inspired by the celebrated two-dimensional McCoy–Wu disordered Ising model. The disorder produces dramatic effects on the dimer model: the free energy exhibits an essential singularity (which has no analogue for the pure dimer model), where dimer-dimer correlations decay as $\exp(-\sqrt{distance})$. Moreover, at the so-called liquid-gas phase transition, where the pure model has a critical exponent 3/2, the disordered exponent has a continuously varying exponent, ranging from 3/2 to infinity.

This is based on a joint work with Quentin Moulard (arXiv:2507.11964).

Strong-disorder Asymptotics for Two-dimensional Directed Polymers and Stochastic Heat Flow

Nicola Turchi

Università degli Studi di Milano-Bicocca, Italy

We study two-dimensional directed polymers in random environment and stochastic heat flow in the supercritical disorder regime. This includes fixed inverse temperature and, more generally, disorder strengths tending to zero more slowly than the critical scale. For directed polymers, we derive sharp estimates on truncated and fractional moments of partition functions averaged over balls, which yield corresponding sharp bounds on the free energy. For stochastic heat flow, we prove a sharp form of local extinction and identify the spatial scale governing the crossover from extinction to averaged behavior, and between vanishing and diverging mass. The proof combines refined changes of measure with coarse graining in a unified framework for both models. As a consequence, in every supercritical regime, including fixed ¿0, the space-time discretized two-dimensional stochastic heat equation exhibits fluctuations on a superdiffusive scale.

Based on a joint work with Quentin Berger and Francesco Caravenna.

Moments of a Long-range Correlated 2d Stochastic Heat Equation at Criticality

Te-Chun Wang
École Polytechnique Fédérale de Lausanne, Switzerland

We study regularized versions of a long-range correlated stochastic heat equation (SHE) in two dimensions and analyze their scaling limit as the regularization vanishes. The driving noise in these regularized equations approximates, at large scales, Riesz-type spatial correlations of the form $|x|^{-2}$. In this work, we establish a universal limit. Under critical scaling, the higher moments converge to non-Gaussian limits, which coincide, up to a reparameterization and a multiplicative constant, with the higher moments of the critical two-dimensional stochastic heat flow. The latter arises both as a normalized limit of the two-dimensional (short-range) SHE and as a scaling limit of the two-dimensional directed polymer model, both under critical scaling.

Scaling Limit of a Weakly Asymmetric Simple Exclusion Process in the Framework of Regularity Structures

Hendrik Weber

University of Münster, Germany

We prove that a parabolically rescaled and suitably renormalised height function of a weakly asymmetric simple exclusion process on a circle converges to the Cole-Hopf solution of the KPZ equation. This is an analogue of the celebrated result by Bertini and Giacomin from 1997 for the exclusion process on a circle with any particles density. The main goal of this article is to analyse the interacting particle system using the framework of regularity structures without applying the Gärtner transformation, a discrete version of the Cole-Hopf transformation which linearises the KPZ equation.

Our analysis relies on discretisation framework for regularity structures developed by Erhard and Hairer [AIHP 2019] as well as estimates for iterated integrals with respect to jump martingales derived by Grazieschi, Matetski and Weber [PTRF 2025]. The main technical challenge addressed in this work is the renormalisation procedure which requires a subtle analysis of regularity preserving discrete convolution operators.

Joint work with R. Huang (Münster) and K. Matetski (Michigan State)

Periodic Homogenisation for Singular Stochastic PDEs

Weijun Xu

Westlake University, China

We consider periodic homogenisation problems for singular stochastic PDEs. These are natural models that combine two singular limiting procedures – homogenisation and renormalisation. We hope to understand how they interact each other when present in the same problem. We report on some of our recent understanding towards this direction.

Based on joint works with Yilin Chen and Ben Fehrman.

From Skew to Reflected Stochastic PDEs via Stochastic Sewing

Lorenzo Zambotti

Sorbonne Université, France

This talk concerns some stochastic PDEs with non-trivial behavior at one point, say zero, that are being investigated again recently with the new tools of stochastic sewing. In particular the so called ”skew stochastic heat equation” has been proved to satisfy pathwise uniqueness by Athreya-Butkovsky-Le-Mytnik using a critical version of the stochastic sewing lemma.

In two recent works, in collaboration with Butkovsky-Mattingly and, respectively, Labbé-Le Guerch, I have studied the ergodic properties of skew SPDEs and the convergenge of the latter to a reflected SPDE in the limit of infinite skewness.

Derivation of the Full Focusing Φ_1^6 Measure from Many-body Quantum Gibbs States

Rongchan Zhu

Beijing Institute of Technology, China

We derive the full focusing Φ_1^6 measure on the torus $\mathbb{T}$ as the high-temperature/mean-field limit of many-body quantum Gibbs states with a nonlocal three-body interaction. Our result reaches the optimal mass cutoff threshold $\|Q\|_{L^2()}$ identified in [**?**] and allows the interaction kernel in the quantum model to converge to the Dirac delta distribution. We also prove that the relative free energy diverges above this threshold, yielding a quantum phase transition consistent with the classical focusing theory. The main difficulty is to relate the classical mass cutoff to the quantum particle-number cutoff in the focusing regime. Our proof combines the variational framework of Lewin, Nam, and Rougerie with a regularized trial-state argument.

Defocusing Gibbs Measure from Quantum Gibbs States

Xiangchan Zhu

Chinese Academy of Sciences, China

We derive the Φ^4_3 measure on the torus as a rigorous limit of the quantum Gibbs state of an interacting Bose gas. To be precise, starting from many-body quantum mechanics, where the problem is linear and regular but involving non commutative operators, we justify the emergence of the Φ^4_3 measure as a semiclassical limit which captures the formation of Bose–Einstein condensation just above the critical temperature. We employ and develop several tools from both stochastic quantization and many-body quantum mechanics. Since the quantum problem is typically formulated using a nonlocal interaction potential, our first key step involves approximating the Φ^4_3 measure through a Hartree measure with nonlocal interaction, achieved by developing new techniques in paracontrolled calculus. The connection between the quantum problem and the Hartree measure emerges through a variational interplay between classical and quantum models.

In particular, I will talk about recent result on singular kernel including Yukawa kernel, which has the same small scale behavior as Coulomb potential.