Relative Langlands Program Minicourse

5–9 Jan 2026

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Abstracts

Mini Courses

Relative Duality for Symmetric Spaces

David Nadler
University of California, Berkeley

This will be an introduction to relative duality for symmetric spaces $X = G/K$. The emphasis will be on examples such as $\mathrm{GL}_{2n}/(\mathrm{GL}_n \times \mathrm{GL}_n)$ and $\mathrm{GL}_{2n}/\mathrm{Sp}_{2n}$, and the relation to the geometry of real groups. Based on prior and ongoing joint work with Tsao-Hsien Chen, Sanath Devalapurkar, Mark Macerato, and John O'Brien.

Some references: arXiv:2403.13995, arXiv:2207.04078

Lecture 1. Real-symmetric correspondence for loop spaces
Lecture 2. Introduction to relative Satake categories
Lecture 3. Examples of relative Satake equivalences

Relative Trace Formulas, Endoscopy, and Duality

Spencer Leslie
Boston College

TBA

Coulomb Branches in Geometry and Representation Theory

Harold Williams

University of Southern California

In this minicourse we will survey the theory of Coulomb branches, geometric objects attached to certain supersymmetric quantum field theories. Over the past several decades, Coulomb branches have played an increasingly prominent role in geometry and representation theory, as they unify and generalize a wide range of structures of independent mathematical interest. Moreover, this physical perspective has proven to be a rich source of new ideas and conjectures, even concerning classical and well-studied objects.

Quantum Groups and Relative Langlands

Valentin Buciumas

Pohang University of Science & Technology

Recently, there have been several instances where quantum groups have been connected to ideas appearing in the (relative) Langlands program. In this mini-course, I will give a brief introduction to quantum groups and explain some of their appearances in function theoretic versions of relative Langlands. The focus will be on the fundamental local equivalence, which relates the spherical Gelfand-Graev module of a metaplectic p-adic group to the representation theory of Lusztig's quantum group at a root of unity. Time permitting, I will discuss the Gaiotto conjectures and some combinatorial interpretations of these results.

Research Short Talks

Twisted Osborne Conjecture and n-homology of Unipotent Arthur Packets

Chang Huang

Tsinghua University

We prove a twisted version of the Osborne conjecture obtained by Hecht and Schmid in their 1983 Acta Mathematica paper. Bergeron and Clozel (2013) have considered a special case, and we generalize their method to our setting. Based on this twisted Osborne conjecture, one can verify the compatibility between (leading isotypic component of) n-homology and twisted endoscopic transfer. It contributes to the construction theory of unipotent Arthur packets.

On the Waldspurger's Identity

Cheng Chen

Centre national de la recherche scientifique

Waldspurger's identity plays a crucial role in the theory of endoscopy. In particular, over non-Archimedean local fields, Waldspurger's identity and the Fundamental Lemma together imply the existence of the endoscopic smooth transfer. I will introduce the Archimedean version of Waldspurger's identity and some of its applications—such as the compatibility between endoscopic smooth transfer and the Fourier transform on Lie algebras. This work is joint with Z. Luo.

Classification of Hyperspherical Data and their Duals

Guodong Tan

National University of Singapore

The recent joint work of Ben-Zvi, Sakellaridis and Venkatesh (BZSV) proposed a duality of the relative Langlands problem through the concept of hyperspherical data (G, H, ι, S). This talk focuses on the classification of such data and the construction of their duals. I will discuss joint work with Chen Wan and Lei Zhang regarding both classical and exceptional cases.

Relative Kazhdan-Lusztig Isomorphism for the pair $\mathbf{GL}_2n$, $\mathbf{Sp}_2n$

Guy Shtotland

Ben-Gurion University

The Kazhdan–Lusztig isomorphism, relating the affine Hecke algebra of a p-adic group to the equivariant K-theory of the Steinberg variety of its Langlands dual, played a key role in the proof of the Deligne–Langlands conjectures concerning the classification of tamely ramified irreducible representations. For a spherical variety X, we can construct two modules over the affine Hecke algebra: the first by considering Iwahori invariant functions on X, and the second using relative Langlands duality and equivariant K-theory. It is natural to expect a relation between them. I will discuss the relation in the case of $X = \mathrm{GL}_{2n}/\mathrm{Sp}_{2n}$ and its application t.

Hausdorffness of Certain Nilpotent Cohomology Spaces

Hao Ying

Zhejiang University

Equipped with the natural subquotient topology, the Lie algebra homology and cohomology spaces associated with smooth representations of real reductive groups are locally convex topological vector spaces, which may not be Hausdorff. The Hausdorff property of these spaces plays a significant role in representation theory—for instance, in Casselman's comparison theorem. In this talk, we will introduce the nilpotent Lie algebra homology and cohomology spaces associated with smooth representations of compact Lie groupsand then show that these spaces are indeed Hausdorff. This is joint work with Fabian Januszewski and Binyong Sun.

Fargues's Categorical Conjecture for Elliptic Parameters for SL(n)

Kenta Suzuki
Princeton University

Fargues and Scholze give a geometric construction of L-parameters attached to smooth irreducible representations of p-adic groups and furthermore predict an enhancement to a category equivalence. I will explain two approaches to prove Fargues and Scholze's functor is an equivalence on elliptic parameters for SL(n). The first approach is character-theoretic, using Fu's recent result on the stability of Fargues-Scholze's L-packets. The second approach follows Gaitsgory and Raskin's proof of the geometric Langlands conjectures for groups with disconnected center, using the 2-Fourier-Mukai transform. As a consequence, we prove Fargues and Scholze's construction gives a bijection between supercuspidal representations of SL(n) and (enhanced) elliptic L-parameters.

The Local Twisted Gan-Gross-Prasad Conjecture

Le Nhat Hoang
National University of Singapore

The Gan-Gross-Prasad (GGP) conjecture studies a family of restriction problems for classical groups and proposes precise answers to these problems using the local and global Langlands correspondences. It also has a twisted variant in the equal-rank Fourier-Jacobi case, which is called the twisted Gan-Gross-Prasad conjecture. In this talk, motivated by the works of J.-L. Waldspurger and R. Beuzart-Plessis in Bessel models, I will introduce a local trace formula approach and how to use it to prove the twisted Gan-Gross-Prasad conjecture for tempered representations over nonarchimedean fields.

Geometric Realizations of Local Arthur Packets

Mishty Ray
The University of British Columbia

Local Arthur packets are sets of representations of p-adic groups that help us realize important classes of automorphic representations. Vogan's geometric perspective on the local Langlands correspondence attaches to each enhanced Langlands parameter a perverse sheaf on the Vogan variety for a fixed infinitesimal parameter. Following his work with Adams and Barbasch on real groups, Vogan suggested a geometric analogue to local A-packets for p-adic groups. Cunningham et al reformulated this proposal by using the vanishing cycles functor on equivariant perverse sheaves on the Vogan variety. In this talk, I will set up this geometric perspective, report on the status of Vogan's conjecture, and mention related results.

Beilinson-Bloch-Kato Conjecture for Polarized Motives

Peng Hao
Massachusetts Institute of Technology

The Beilinson—Bloch—Kato conjecture vastly generalizes the (rank part of the) BSD conjecture for modular elliptic curves. Based on previous results of Liu-Tian-Xiao-Zhang-Zhu, we prove the BBK conjecture at analytic rank zero for certain conjugate self-dual motives (U(N)-motives), e.g. those coming from odd symmetric powers of a non-CM modular elliptic curve. As an application, we obtain examples of non-trivial high dimensional varieties for which all conjectures of Hodge, Tate, and Beilinson—Bloch—Kato are true for large ℓ, namely, powers of certain non-CM elliptic curves. The proof uses theta lifts extensively, and the same trick applies in the orthogonal setting.

Computing Microstalks in the Betti Automorphic Category

Swapnil Garg
University of California, Berkeley

TBA

Automorphic Periods and Sum of L-values

Weixiao Lu
Massachusetts Institute of Technology

The relative Langlands duality posed by Ben-Zvi-Sakellaridis-Venkatesh predicts that periods of automorphic form can sometimes equal to sum of several special values of automorphic L-functions rather than a single one. In this talk, I will present explicit evidence for this phenomenon. I will discuss families of automorphic periods on general linear groups and classical groups that are designed to detect specific types of Eisenstein series. These periods indeed evaluate to finite sums of L-values. This is joint work with Guodong Xi.

Beyond Endoscopy via the Trace Formula

Yuhao Cheng
Tsinghua University

At the beginning of this century, Langlands introduced a strategy known as *Beyond Endoscopy* to attack the principle of functoriality. Altuğ studied GL_2 over $\mathbb{Q}$ in the unramified setting. In the first paper, he isolated the trace of the trivial representation in the elliptic part of the trace formula, and used this isolation to recover the Kuznetsov bound in the second paper. In the last paper, he gave a new proof of the bound of the average of the trace of Hecke operator. We will generalize the three papers to the case over $\mathbb{Q}$ with ramification. In particular, we isolate the non-tempered part in the elliptic part, prove the Kuznetsov bound in the ramified case, and bound of the average of the trace of Hecke operator for modular forms with arbitrary level. Moreover, we present an asymptotic formula for the elliptic part in the general case.

Volume of Shtukas and Derivatives of the Zeta Function

Zeyu Wang
Massachusetts Institute of Technology

For a proper algebraic variety X equipped with a line bundle L, one can define the volume of X with respect to L. When X is the moduli space of G-Shtukas over a curve and L is the determinant line bundle, this definition no longer applies, as the Shtuka space is non-proper. In the work of Feng–Yun–Zhang, an ad hoc notion of volume was introduced in the case where the modification type is minuscule, and this volume was shown to encode higher derivatives of the Zeta function of the curve.

In this talk, I will present a generalization of this picture to arbitrary modification types.
