

Relative Langlands Program Conference

12–16 January 2026

January 12, 2026

Abstracts

On Norms on Harish-Chandra Modules

Bernhard Krötz

Universität Paderborn

Joint with Joseph Bernstein, Pritam Ganguly, Job Kuit and Eitan Sayag. Based on <https://arxiv.org/abs/2510.09370>. The Casselman-Wallach theorem is a foundational result in the theory of representations of real reductive groups connecting algebraic representations to topological representations. We provide a quantitative version of this theorem. For that we introduce the notion of *Sobolev gap* for a Harish-Chandra module. This is a new invariant whose finiteness is highly non-trivial. We determine the Sobolev gap for representations in the unitary dual of the group $(2,)$ and establish uniform finiteness results in general for representations of the discrete series and the minimal principal series. We use these notions to reformulate and extend classical results of Bernstein and Reznikov concerning automorphic functionals with respect to cocompact lattices. In particular, we prove an abstract convexity bound which applies to automorphic functionals with respect to general lattices in $(2,)$ and is independent of the type of unitarizable irreducible Harish-Chandra module. Finally, we offer an extensive list of open problems.

Archimedean Modular Symbols and Automorphic Periods

Binyong Sun

Zhejiang University

Modular symbols are cohomological interpretations of period integrals for cohomological automorphic representations. The Archimedean modular symbols, which are linear functionals on certain relative Lie algebra cohomology spaces, capture the Archimedean behavior of modular symbols. We define automorphic periods by investigating the rationality of these Archimedean modular symbols. This construction is an analogue of Deligne's periods for critical pure motives. By using Archimedean modular symbols and automorphic periods, together with the rationality of certain Eisenstein cohomology spaces and cuspidal cohomology spaces, we obtain rationality results for critical values of standard L-functions and Rankin-Selberg L-functions.

These results align with Blasius's conjecture. The talk is based on some recent works joint with Dihua Jiang, Yubo Jin, Jiang-Shu Li, Dongwen Liu, and Fangyang Tian.

Automorphic Unfolding and Higher Teichmüller Theory

Eric Yen-Yo Chen

EPFL

In higher Teichmüller theory, one is interested in identifying special components in real character varieties of a closed oriented surface. We observe an analogy between this procedure and automorphic unfolding, which can be made precise under the nonabelian Hodge correspondence. Finally, based on recent joint work with Enya Hsiao and Mengxue Yang, we characterize and study an underlying "codimension 2" relative Langlands duality in the Dolbeault setting.

Nontempered Local Gan-Gross-Prasad Conjecture for Unitary Cohomological Representations of $U(n,1)$

Hang Xue

The University of Arizona

We characterize when there is a continuous invariant linear form for unitary cohomological representations of $U(n, 1) \times U(n-1, 1)$. This is compatible with the prediction of the nontempered local GGP conjecture. We borrow some ideas from our previous proof of the tempered case using theta lifts. The main issue, however, is not the characterization itself, but rather to establish various irreducibility results of the full theta lifts.

Symmetric Bow Varieties and Examples of S-dual Pairs

Hiraku Nakajima

Kavli IPMU

We define S-dual of Hamiltonian spaces by a modification of the definition of Coulomb branches of 3d SUSY gauge theories by Braverman-Finkelberg and myself. Bow varieties, studied in the context of Coulomb branches, give examples of S-dual pairs of Hamiltonian spaces. We further consider involutions on bow varieties, whose fixed point loci are symmetric bow varieties. They give examples of S-dual pairs, related to symmetric spaces.

Towards the Bar Involution for Double Affine Hecke Algebras

Ivan Loseu

Yale University

Double affine Hecke algebras were introduced in 1995 in his study of Macdonald polynomials. In recent years it has been observed (by Braverman, Kazhdan, Muthiah, Patnaik and others) that they share many features with the affine Hecke algebras. In particular, one could ask whether there is the bar involution on these algebras. Partial answers to this question were obtained by Muthiah (2019) and Hebert-Phillipe (2024) using a more combinatorial approach. I will report on our joint work in progress with Davis and Morton-Ferguson, where we use a coherent geometric approach to define a version of the bar involution and study its properties. Time permitting, I will indicate some potential (still mysterious) connections to affine & Dolbeault geometric Langlands correspondence/ Bezrukavnikov equivalence.

Theta Liftings in Higher Chow Groups for Orthogonal Shimura Varieties

Liang Xiao

BICMR, Peking University

In the celebrated theory of arithmetic theta liftings, a.k.a. the Kudla program, a key component is to prove the modularity of the generating series with values in (arithmetic) Chow cycles. In this talk, we report on a joint work in progress with Haocheng Fan, Wenxuan Qi, Peihang Wu, and Yichao Zhang, in which we propose a parallel story for higher Chow cycles. In the talk, I will focus on explaining the basic framework, which involves a (somewhat) new proof of the modularity in the cycles case and a mild generalization of Borcherds products. Assuming a yet-missing technical input on cohomology vanishing, this is expected to construct a modular generating series with values in higher Chow groups of orthogonal Shimura varieties, and to relate their regulators with the special values of L-functions, as predicted by Beilinson's conjecture.

Relative Langlands Duality for $osp(2n+1—2n)$

Michael Finkelberg

*Einstein Institute of Mathematics
The Hebrew University of Jerusalem*

This is a joint work with A.Braverman, D.Kazhdan and R.Travkin. We prove that the S-dual of $SO(2n+1) \times Sp(2n)$ acting in the tensor product of their tautological representations, is the symplectic mirabolic space: the tautological representation of $Sp(2n)$ times the cotangent bundle of $Sp(2n)$.

Periods of Automorphic Forms and Motivic Periods

Michael Harris

Columbia University

I will be reporting on recent results with Grobner, Lin, and Raghuram on the relation between the expressions of critical values of automorphic L-functions in terms of automorphic periods — integrals of automorphic forms over group-theoretic cycles — and the conjectural expression in terms of Deligne’s motivic periods. We use Eisenstein cohomology and the Ichino-Ikeda identity for unitary groups to provide simultaneous proofs of automorphic versions of Deligne’s conjecture on critical values of Rankin-Selberg L-functions, for sufficiently regular motives, and of predicted identities between periods of automorphic representations of different unitary groups with the same base change to $GL(n)$.

Local-global Principles for Periods of Automorphic Forms

Nadir Matringe

NYU Shanghai

The Waldspurger local-global principle for toric periods of cuspidal automorphic representations of inner forms of $GL(2)$ naturally extends in two directions. The first one is the Gross-Prasad conjecture for special orthogonal groups, whereas the second, more straightforward, leads to the Guo-Jacquet conjecture, so far proven under local restrictions. I will present a work in collaboration with Omer Offen and Chang Yang, which provides a local-global principle for period integrals attached to three families of symmetric subgroups of inner forms of $GL(n)$, generalizing that of Waldspurger for the trivial twisting character. This also gives a complete proof of the direct implication of the original Guo-Jacquet conjecture.

Generalized Fourier Transforms and Minimal Representations

Nadya Gurevich

Ben-Gurion University of the Negev

As a part of their far-reaching program Braverman and Kazhdan have defined a family of generalized G -equivariant Fourier transforms (GFT) acting on the space $L^2(X)$, where X is a basic affine space for the group G . In the talk we show that GFT in some cases are closely related to the action of an involutive element in the minimal representation of a bigger group H . This generalizes the fact that the longest Weyl element in the Weil representation of $\widetilde{Sp}(2n)$ acts by the classical Fourier transform on a n -dimensional vector space.

The talk is based on joint works with David Kazhdan and with Wee Teck Gan.

Equidistribution and Moments of L-functions

Paul Nelson

Aarhus University

I will survey how integral representations and period formulas recast equidistribution problems (for instance, involving unipotent shears on $(GL(2n), GL(n))$) as moment problems for automorphic L-functions, a mechanism originating in earlier work of many authors. I will recall rank-one prototypes in which the varying vectors are pure translates of fixed ones, as well as the microlocal framework from my joint work with Venkatesh on GGP periods such as $(U(n+1), U(n))$, where recent progress in homogeneous dynamics now yields effective consequences. I will then indicate a higher-rank variant, joint with Subhajit Jana, in which $(GL(m), GL(n))$ moments appear as L^2 -norms of shears of fixed automorphic forms; here, implications for moments and subconvexity remain conditional on dynamical input not yet available.

The Ramanujan and Density Conjectures

Peter Sarnak

Princeton University

TBA

On the Surjectivity of the Local Genestier-Lafforgue Parameterization

Raphael Beuzart-Plessis

Institut de Mathématiques de Marseille (I2M)

Genestier-Lafforgue and Fargues-Scholze have constructed a semi-simple local Langlands correspondence for reductive groups over local fields of positive characteristic.

In this talk, I will explain how, assuming a version of the stable (twisted) trace formula for base change over a function field, one can show the surjectivity of this parametrization for unramified groups and when the characteristic does not divide the order of the Weyl group. This is based on joint work with Michael Harris and Jack Thorne.

Periods and Shimura Correspondences

Solomon Friedberg

Boston College

The BZSV framework systematically describes a class of periods of automorphic representations. These periods should be nonzero precisely when the given representation is a (specific) Langlands lift from another group and if so be related to L-values. The generalized Shimura correspondence (still mostly conjectural) proposes a different set of lifts of automorphic representations, this time from covers of reductive groups to reductive groups. In this talk I will describe some examples of periods that should detect Shimura correspondences, and then discuss joint work with Omer Offen concerning a specific period, which we study by the relative trace formula.

Relative “Beyond Endoscopy” and Separation of the Ramanujan Spectrum

Yiannis Sakellaridis

John Hopkins University

In the development of the Beyond Endoscopy program to date, a lot of effort has been focused on finding a geometric expression for the “Ramanujan” spectrum of the Arthur–Selberg trace formula, i.e., removing the non-tempered Arthur packets. An early suggestion by Sarnak was to circumvent this problem altogether, by using the Kuznetsov formula instead, where only the Ramanujan spectrum appears.

I will report on joint work with Chen Wan, where we directly compare the two trace formulas for $GL(n)$, including explicit calculations of the “transfer operators” that isolate the non-Ramanujan spectrum in low rank. For $GL(2)$, our work generalizes Rudnick’s PhD thesis, and this operator coincides with the Fourier transform on the affine parameter space of orbital integrals, which has already appeared in works of Frenkel–Langlands–Ngô and Altug; but in higher rank it has a different form. Further motivation for this work comes from the Relative Langlands program, where there is a hope that non-standard comparisons of (relative) trace formulas could be used to prove conjectural relationships between periods of automorphic forms and special values of L-functions.

Period Integrals attached to Strongly Tempered BZSV Quadruples

Zhengyu Mao

Rutgers University

In this talk we explore the strongly tempered BZSV quadruples. We connect the period integrals arising from these quadruples to the periods that were studied before, for example the ones arising from Rankin–Selberg theory. We also explain how problems on period integrals for BZSV quadruples can be related to the ones attached to the strongly tempered quadruples.

Theta Correspondence and Springer Correspondence

Zhiwei Yun

Massachusetts Institute of Technology

The exact form of the theta correspondence between representations of an orthogonal group and a symplectic group over a finite field was conjectured by Aubert-Michel-Rouquier, and proved by S-Y. Pan, and then by Ma–Qiu–Zou using different methods. In this talk I will give a geometric description of the theta correspondence between unipotent principal series representations in terms of the Springer correspondence. The construction fits well into the general framework of relative Langlands. This is joint work with Jiajun Ma, Congling Qiu and Jialiang Zou.

Shimura Varieties and Twisted Period Integrals

Zhiyu Zhang

Stanford University

Waldspurger formula (1985) relates a twisted family of Hecke period (1936) and L-functions. Because of the twist, its proof is completely different and there is no direct relation to Hecke period. Thanks to the twist, it has deep applications, such as equidistribution theorem, BSD conjecture and quadratic twists of elliptic curves. In this talk, I will explore more non-classical period integrals (involving twists and theta series), and formulate conjectures around applications in arithmetic and geometry. After some new examples, I will focus on an explicit yet not well-understood example - the twisted Gan-Gross-Prasad (GGP) conjecture on Asai L-functions.

In a joint work with Lu and Wang, we prove it under mild assumptions on ramifications, using new relative trace formulas and other tools. I will formulate an arithmetic analog on Shimura varieties using non-classical cycles and mixed geometric theta series. I will provide a relative trace formula approach, and prove a twisted AFL based on new mirabolic cycles. Because of the twist, there is no direct relation to arithmetic GGP conjecture and AFL. Thanks to the twist, I will discuss applications to Asai motives and triple products of elliptic curves.
