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Mathematical Methods for the General Relativistic Two-body Problem

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ANALYTICAL MODELING OF GRAVITATIONAL WAVES: A RECENT VIEW ON THE POST-NEWTONIAN FRAMEWORK

LAURA BERNARD

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Keywords: general relativity, post-Newtonian formalism, perturbation theory, two-body problem

Next generation of gravitational wave detectors will have a tremendous sensibility, allowing many more gravitational wave detections over a very large frequency band. In particular, LISA, the future space-based detector, will be able to detect gravitational waves from supermassive black hole binaries and extreme/intermetdiate mass ratio inspirals. In order to learn the most from GW detections, one of the most prominent challenge lies in our ability to produce a bank of extremely accurate gravitational waveforms for all the expected sources.

To provide a consistent and unified description of the different phases of the coalescence of a binary system: inspiral, merger and ringdown (IMR), different methods are used. To describe the merger phase, where the strongest gravitational phenomena take place, solutions to the full nonlinear gravitational field equations are needed, which have been obtained with numerical relativity tools. On the other hand, the ringdown and inspiral stages of the coalescence can be described using perturbative techniques. For the former, BH perturbation theory is used to describe the relaxation phase, with the black hole quasi-normal modes playing a fundamental role. Finally, the inspiral phase is very accurately modeled with the multipolar post-Minkowskian – post-Newtonian (mPM-PN) formalism. It consists in a multipolar and weak field expansion combined with a series expansion in small velocities.

In this talk, I gave an overview of the mPM-PN framework in GR. In particular, I highlighted the similarities with more recent diagrammatic approaches, that all rely on a hierarchy on scales allowing to solve the two-body problem in successive region of space, and I insisted on the synergies and complementary between the different approaches.

Then, I presented the current state of the art, insisting on some recent results:

- the radiation-reaction at 4.5PN in the Burke-Thorne gauge;
- the gravitational flux and waveform at 4.5PN;
- some tail and memory contributions from EFT techniques;
- the finite-size effects amplitude modes at 4.5PN and the spinning amplitude modes at 3PN.

Finally, I concluded with some prospects in view of the next generation of gravitational wave detectors, insisting on:

- the interplay between PN and scattering amplitudes results;
- the interplay between PN and self-force and numerical relativity results to build full IMR waveforms;

- the importance of hereditary effects (tails, memory);
- the importance of improving current waveforms by including spin precession, high eccentricity and dynamical effects.

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DYNAMICAL TIDAL RESONANCES IN EMRIS

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Classification AMS 2020:

Keywords:

Extreme-mass-ratio inspirals (EMRIs) are a key target for LISA, given that they are unique probes of the spacetime structure around the massive central black hole (BH) [1]. They will allow for extraordinarily precise tests of general relativity, since we will observe their intricate relativistic orbits over some $\sim 10^5$ orbits instead of $O(10)$ orbits in comparable-mass binaries observed by LIGO-Virgo-KAGRA. EMRIs are typically modeled as clean, isolated 2-body systems. Yet, in realistic galactic-center environments, other nearby stellar-mass objects can induce tidal resonances that leave an observable imprint on the gravitational waveform [2]. Without accounting for these resonant effects in our model, the utility of EMRIs as precision probes may be impeded. We may misattribute the effects to deviations from General Relativity or, worse, may not even be able to detect the EMRI. Conversely, if we can correctly model such tidal resonances, we unlock valuable information about the population of dark objects in the galactic core, otherwise difficult to access [3, 4].

Observational evidence has accumulated over the past years indicating the reality of such tidal perturbers with the observations of QPEs possibly describing the interaction of a stellar object with an accretion disk of a central massive BH [9, 10, 11, 12] and the discovery of new faint stars near SgrA* (such as S301 with a periapsis at only $\sim 260 M_*$ with M_* the mass of SgrA*[8]). While EMRI rates with or without perturbers remain highly uncertain [15, 16], this highlights the importance of correctly incorporating tidal resonances in EMRI models.

To calculate the influence of a resonance on the waveform one needs to know when the resonance occurs and what the size of the effect is, which we will refer to as the 'jump'. A resonance occurs when the orbital frequencies of the EMRI with respect to Boyer-Lindquist time $\omega_r, \omega_\theta, \omega_\phi$ and of the tidal perturber become commensurate, i.e.,

$$n\,\omega_r + k\,\omega_\theta + m\,\omega_\phi + s\,\omega_{\rm td} = 0 \tag{0.1}$$

for integers n, k, m, s. The tidal perturber in principle also has three orbital frequencies, but since we take it to be at distances $\mathcal{O}(100M)$, its Keplerian frequency $\omega_{\rm td}$ is sufficient to describe its motion. In our earlier work [2, 3, 4], we assumed that the perturber is sufficiently far to be considered stationary during the resonance time (effectively setting $\omega_{\rm td}$ to zero in the resonance condition above). For such a stationary perturber there are a priori $9 \times 9 \times 5 = 405$ possible resonances with $|k|, |n| \leq 4$ and $|m| \leq 2$ (restricting to the leading-order, quadrupolar tidal deformation).[1] But considering symmetry properties and the condition that the resonance should occur in the LISA band — say, pragmatically,

[1]The perturbation is modeled by the leading order quadrupolar deformation of the central massive BH, which naturally restricts $|m| \leq 2$.

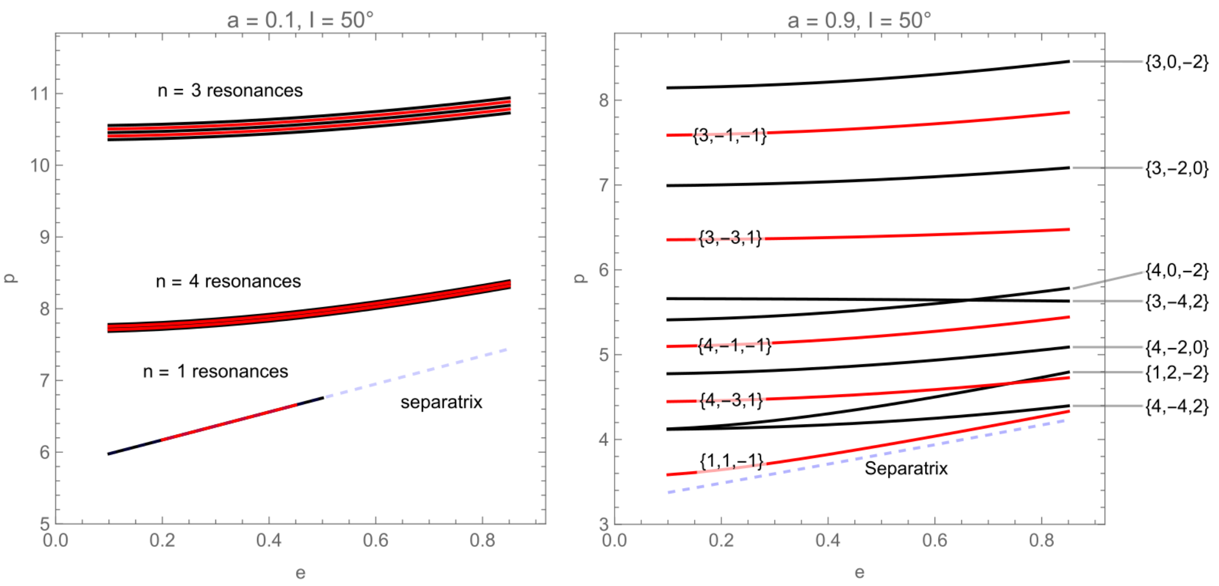

FIGURE 1. Resonance contours for a prograde orbit with inclination $50°$ in the eccentricity e - semilatus rectum p plane of the EMRI. The contour labels correspond to the resonance numbers $\{n, k, m\}$. The spin parameter of the central BH is set to $a = 0.1$ (left) and $a = 0.9$ (right). The blue dashed lines indicate the separatrix.

for a semi-latus rectum $< 100M$ — only 12 are allowable for a prograde orbit (and the same number for a retrograde orbit). The corresponding resonance contours are shown in Fig. 1. Resonances with the same n group together for small spin values of the central massive BH, while they diverge as the spin value increases (an effect somewhat reminiscent of Zeeman splitting). Both scenarios require extra care when implementing in Fast EMRI Waveforms (FEW) [17, 18, 19, 20]: for close resonances one has to make reasonably small steps to make sure one does not jump over a resonance contour, and when the resonances cross and overlap one has to account for both.

While for a typical EMRI evolution in the presence of a stationary tidal perturber, 12 resonances are possible, not all 12 resonances will be excited and produce observable imprints on the waveform. Whether a resonance is excited depends critically on the phase parameters with which the EMRI enters the resonance. Moreover, the impact on the waveform is also determined by the moment during the EMRI evolution it is excited. For instance, even if the instantaneous impact of the resonance is sizable, if the resonance occurs close to the separatrix, the impact on the waveform will be minimal as the resonance will only impact the final few orbits. Numerically evolving various EMRIs using the method of forced osculating orbital elements [21] seems to suggest that typically only 2-3 stationary resonances result in observable dephasing.[2]

Stationarity of the perturber is, however, not a good approximation: during a resonance for a perturber at realistic distances, the perturber typically completes 1-2

[2]Such a numerical evolution is computationally expensive (even when the radiation effects are modeled using 5PN fluxes as we did) and thus unsuitable for real-time waveform generation in data analysis pipelines.

orbital periods. Allowing for a dynamical perturber in principle also requires accounting for its possible eccentricity (known formation channels of tidal perturbers, such as two-body relaxation processes [13] and the Hills mechanism [14], suggest high eccentricities). Here, I will nevertheless assume that the perturber is in a circular orbit. This has the added benefit that we do not need to introduce any additional parameters compared to the stationary case.[3] The term $s\omega_{\rm td}$ in the resonance condition greatly enriches the allowed resonance structure: now there are more than 300 possible resonances in the LISA band (the exact number depends critically on the value of $\omega_{\rm td}$). Moreover, when the resonance occurs is no longer just determined by the EMRI evolution itself (i.e. $\omega_r, \omega_\theta, \omega_\phi$), but also depends on the additional parameter $\omega_{\rm td}$. A few exploratory numerical evolutions with the osculating elements code show some EMRI trajectories with 3 significant dynamic tidal resonances, but also some with as many as 17. This case is only just being explored and deserves further research. Key questions to address are: Which resonances dominate? Do we need to model all resonances for accurate phase modeling, or only the largest? How do self-force and tidal effects interact? Answering these questions and developing efficient implementations in FEW will be crucial for extracting maximal science from EMRI observations.

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HYBRID POST-NEWTONIAN/SELF-FORCE INSPIRAL AND TRANSITION-TO-PLUNGE WAVEFORMS

GEOFFREY COMPÈRE

Keywords: Gravitational waves, gravitation, black holes, binary mergers, inspiral, post-Newtonian, self-force

Credits: This talk is based on [1, 2, 3] with contributions from L. Honet, A. Pound, J. Matthews, B. Wardell, G. Piovano, M. van de Meent and N. Warburton.

Since the first detection of a gravitational-wave (GW) signal 10 years ago (GW150914) [4], the LIGO-Virgo-KAGRA (LVK) collaboration has now seen more than a hundred binary coalescence events among their first three observing runs [5]. With the upcoming release of the fourth version of the Gravitational-Wave Transient Catalog (GWTC-4) and the fifth observing run planned for 2027 [6], many more GW events will soon be reported or discovered by ground-based detectors. Together with the improvement of detectors' sensitivity, this spurs GW modelers to provide fast and faithful waveform models for parameter estimation studies and tests of general relativity [7, 8, 9, 10, 11, 12].

In particular, one specific event from the third observing run, GW191219_163120, has been estimated to come from a binary with mass ratio $\sim$1:27. Such a high mass ratio lies beyond what current models are able to cover [5, 13] and points to one of the LVK observational science short-term R&D objectives: providing fast and accurate waveform models for asymmetric-mass-ratio binaries [14].

On the other side, future space-based detectors such as LISA will detect GW signals in the millihertz spectrum [15], allowing us to observe signals emitted by extreme-mass-ratio inspirals (EMRIs). The joint use of space-based detectors with future third-generation (3G) ground-based detectors such as the Einstein Telescope [16] will enable the observation of sources such as intermediate mass ratio coalescences (IMRACs) across multiple bandwidths [15]. Those intermediate systems with mass ratios typically ranging from $\sim 1{:}10^2$ to $\sim 1{:}10^4$ currently lack accurate waveform models and constitute a real GW modeling challenge for 3G detectors.

The waveform modeling technique that naturally leverages the existence of two disparate masses is the gravitational self-force (GSF or SF) program, where the Einstein field equations (EFEs) and the orbital motion of the secondary black hole are expanded in the binary's small mass ratio. Recent milestones in the self-force community have been, for example, the construction of a first-post-adiabatic (1PA)/second-order self-force (2GSF) waveform model for spinning binaries with a slowly spinning primary black hole and rapidly spinning, precessing secondary [17] and the development of a fast, data-analysis-ready adiabatic (0PA) model for eccentric equatorial binaries with a rapidly spinning primary in the `FEW` python package [18], leveraging the SF multiscale expansion framework [19, 20, 21, 22] and hardware acceleration [23] for rapidly generating waveforms.

1. Transition-to-plunge

These pieces of work focus primarily on the inspiral phase of the binary, which is expected to be sufficient for modeling EMRI signals. In contrast, IMRACs usually have a merger that occurs in the frequency band of ground-based detectors [15]. Moreover, recent results in second-order self-force [24, 13] show that self-force models can be remarkably precise even at more comparable mass ratios $\sim$1:10 [17, 25], and for such systems the merger can always represent a significant fraction of the signal-to-noise ratio (SNR). Those considerations make it important to extend the multiscale self-force framework beyond the innermost stable circular orbit (ISCO), where the inspiral motion of the binary breaks down.

A recent work by three of us [26] extensively derived the self-force framework for the transition-to-plunge (or simply "transition") motion of nonspinning, quasicircular binaries. This work extended the results of Refs. [27, 28, 29] by including a treatment of the Einstein field equations and waveform generation on top of the orbital dynamics, while also reformulating the transition in the phase-space approach [21, 22, 30] that underlies the multiscale expansion's accuracy [13] and efficiency.

In GSF theory, the merger-ringdown part of the waveform is generated by the secondary' final, approximately geodesic plunge into the primary after it transitions across the ISCO [31, 32]. Again, three of us recently showed how to formulate this regime in the phase-space approach of the multiscale expansion [33] (building on Refs. [34, 35]). This created a unified framework for inspiral, transition, and plunge that can be carried (in principle) to any order in the small mass ratio. In Ref. [36], two of us employed that framework to generate subleading-order merger-ringdown waveforms in a test case of modified gravity.

These developments have paved the way for building the first inspiral-merger-ringdown (IMR) model for quasicircular, nonspinning binaries beyond leading order in GSF theory [37, 38, 39, 40]. (See Refs. [41, 42, 43] for earlier such waveforms at leading order, following an iterative method initiated in Ref. [44] rather than a multiscale approach.)

With a first complete beyond-leading-order GSF IMR waveform model for nonspinning binaries at hand, we now aim to to include additional physical parameters. The work presented in this talk represents one of the intermediate steps towards including the effects of the primary black hole spin in the IMR waveform model presented in Ref. [37, 38, 39, 40]: we derive and implement second-post-leading transition-to-plunge (2PLT) waveforms using the phase-space formalism for non-eccentric equatorial motion of a Schwarzschild secondary black hole around a primary Kerr black hole. Moreover, we build a composite waveform model that smoothly interpolates between an adiabatic (0PA) model in the early inspiral and a 2PLT transition model when reaching the ISCO using a matched asymptotic expansions procedure. We address and solve issues already raised in Ref. [26] about the accuracy of such composite models due to early-time transition residuals in the dynamics.

2. HYBRID MODELS

There has been substantial recent progress toward more faithful waveform models in much of the binary parameter space, but all these models have limitations in the high-$\mathring{q}$ regime [45]. In PN theory, waveforms have been pushed to 4.5PN beyond leading order [46, 47, 48]; however, PN rapidly loses accuracy at high $\mathring{q}$ because the number of orbital cycles in the strong-field regime scales linearly with $\mathring{q}$. Links between scattering binaries and gravitationally bound systems [49, 50, 51, 52, 53, 54, 55, 56] have also allowed PM scattering calculations to inform models of inspirals [57, 58, 59, 60, 61], but application of these ideas to asymmetric systems is still in a germinal stage [62, 63, 64, 65, 66, 67]. The SXS collaboration's catalog of NR waveforms now contains 4170 simulations, including 164 with mass ratios $\mathring{q} > 8$ [68, 69, 70], and work on the high-$\mathring{q}$ regime is ongoing [71, 72, 73]; however, NR is still currently limited to mass ratios $\lesssim 20$, and it is not feasible for NR to explore the whole high-$\mathring{q}$ parameter space (and effectively impossible to model EMRIs with NR) due to the quadratic scaling of NR runtime with $\mathring{q}$ [74].

In principle, the challenges of high-$\mathring{q}$ modeling are met by SF theory, in which the small, secondary object is treated as a source of perturbations on the spacetime of the larger, primary black hole, and the spacetime metric is consequently expanded in powers of the small mass ratio $\mathring{\varepsilon}$. This approach has reached recent milestones in both accuracy and efficiency. By combining a multiscale formulation of the Einstein field equations [19, 20, 75, 21, 22, 76] with GPU acceleration, the FastEMRIWaveforms (FEW) package [77, 23, 78, 18] can generate long, LISA-length waveforms in tens of milliseconds. At the same time, the most advanced SF models have proved highly accurate for all mass ratios $\mathring{q} \gtrsim 10$ [13, 25, 79]. However, current SF models remain severely limited in their coverage of the binary parameter space, particularly for spinning and precessing systems.

It is generally accepted that, in order to meet LISA requirements, it is necessary and sufficient to go to second order in the SF expansion of the EFEs [45, 80, 81]. This is motivated by the fact that the phase of the GW signal admits an expansion of the form [19, 21]

$$\varphi(t, \mathring{\varepsilon}) = \frac{1}{\mathring{\varepsilon}}\varphi_{(0)}(\mathring{\varepsilon}t) + \varphi_{(1)}(\mathring{\varepsilon}t) + \mathcal{O}(\mathring{\varepsilon}), \tag{2.1}$$

where the first term of the expansion is the adiabatic (0PA) phase and the second term is the first post-adiabatic (1PA) correction. The former depends on the dissipative piece of the first-order self-force (1SF), while the latter depends on the full first-order self-force as well as the dissipative piece of the second-order self-force (2SF) [19, 20, 21].

Currently, the only available 1PA model is restricted to the case of nonspinning, quasicircular inspirals [13]. 0PA models are available for generic binaries involving a spinning primary, but they are limited to weak fields and small eccentricities [82] or else to equatorial systems whose orbital angular momentum is aligned with the primary's spin axis [18]. 0PA models also fall short of the necessary accuracy requirements for EMRIs. For IMRIs and other less extreme binaries, which will be observable when the two bodies are at much larger separations, even a 1PA model loses accuracy [25].

A recent Bayesian analysis [83] confirmed that neglecting 1PA corrections introduces significant biases on the parameter estimation for EMRIs and IMRIs. However, it also showed that these biases can be mitigated or entirely eliminated by approximating the

1PA terms with PN data. This is the starting point of our work: *to combine SF and PN results to construct a model that accurately covers the whole range of mass ratios* $10 \lesssim \mathring{q} \lesssim 10^6$ *and particularly covers the spinning binaries for which there are no complete 1PA models.*

More concretely, we seek to build a hybridized SF+PN model that achieves the following:

(1) To model EMRIs with sufficient accuracy for LISA, the model should be "exact" (accurate to 6 or more digits [84]) in its 0PA information and should be as complete as possible in its 1PA information. Since 0PA effects [75, 85, 86, 87], first-order conservative self-force effects [88, 89], and all linear-in-secondary-spin effects [90, 91, 92, 93, 94, 22] can now be calculated in SF theory for generic orbital configurations around a spinning primary, completing a hybrid EMRI model for spinning binaries requires using a PN approximation to the missing second-order dissipative self-force effects.

(2) To be efficient enough for LISA data analysis and to dovetail with the prevailing EMRI modeling program, the model should take the multiscale form [21, 22] that is compatible with the FEW rapid waveform-generation software package [77, 23, 78, 18].

(3) To be sufficiently accurate for long signals that extend into the weak field, in the mass-asymmetric but non-EMRI regime $10^{-4} \lesssim \mathring{\varepsilon} \lesssim 0.1$ [18], the model must contain terms beyond 1PA order [25]. More generally, for the purpose of achieving high accuracy over the broadest possible range of signals, all available PN information should be included.

(4) Following the principle of parsimony, we also aspire to keep the model as conceptually simple as possible and built entirely from first principles, with no calibration to NR data.

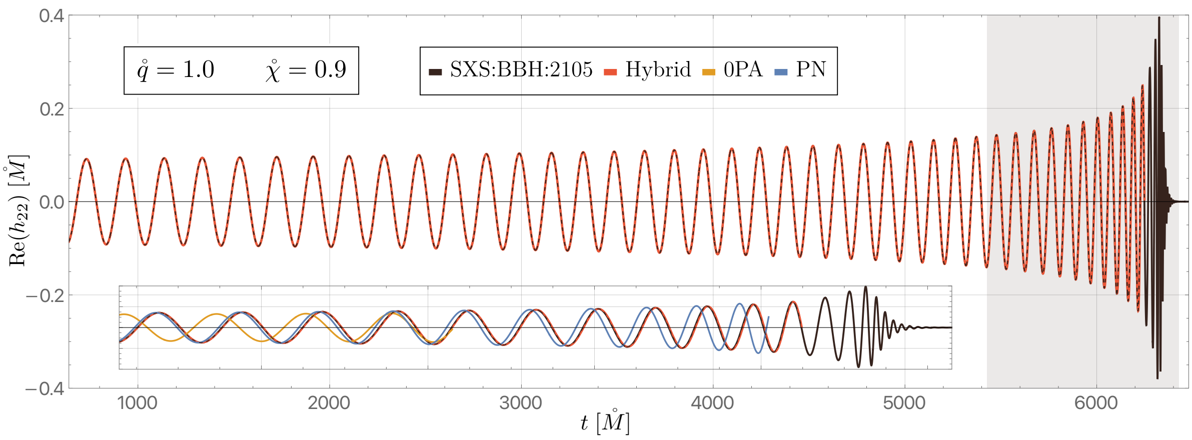

FIGURE 1. Self-force/post-Newtonian hybrid waveform (in red) and NR waveform SXS:BBH:2105 (in black) [95] for a quasicircular binary with primary spin $\mathring{\chi} = 0.9$ and mass ratio $\mathring{q} = 1$. The inset zooms in on the shaded gray region close to the merger. The hybrid model is described in the core of this article. We also display 0PA and 4PN waveforms for comparison (in orange and blue, respectively), aligned with the NR waveform at the same (early) reference time as the hybrid waveform.

In this talk, we develop a model achieving each of these objectives in the case of a nonspinning secondary object on a quasicircular orbit around a spinning primary black hole. Our model is restricted to the inspiral regime, but it could be extended to the merger-ringdown regime using the framework in Refs. [26, 33]. When applied for mass ratios $\mathring{q} \leq 15$, we find that our hybrid model matches NR inspiral waveforms far more accurately than either the 0PA or PN models taken individually. We find excellent numerical agreement with SXS simulations even at comparable mass ratios, as illustrated in Fig. 1 for an equal-mass, rapidly spinning binary.

Like the multiscale approach as a whole, our formulation (i) is modular, immediately improvable as PN and SF data advances, and (ii) will ultimately enable rapid generation of long waveforms for generic, eccentric, precessing binaries through seamless integration with the FEW package. We hence expect that our approach will provide accurate, efficient models of IMRIs and serve as accurate stand-ins for EMRI models until complete 1PA results are available.

In the Letter [1], we further extend our model to include additional SF information, and we provide a more thorough accuracy benchmarking against both NR and other models.

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THE PROSPECTS AND CHALLENGES OF SCIENCE WITH LISA EMRI OBSERVATIONS

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Extreme-mass-ratio inspirals (EMRIs), the inspirals and mergers of compact objects (usually black holes but white dwarfs or neutron stars are also possible) with a massive black hole (MBH) in the centre of a galaxy, are a key source for the future space-based gravitational wave (GW) detector LISA [1], recently officially adopted by ESA and due to be launched in 2035. A typical EMRI generates many hundreds of thousands of cycles of gravitational radiation detectable by LISA, all of which are generated while the small object is very close to the central black hole. This signal thus encodes detailed information about the properties of the black hole and its immediate environment [2].

EMRIs may form in a number of different ways. The classic channel is driven by two-body relaxation. MBHs in galactic centres are usually surrounded by a cluster of stars. These stars interact with each other gravitationally, and close encounters can perturb the orbits of compact objects so that they pass very close to the central MBH. If this happens, energy and angular momentum are radiated from the orbit into GWs, leaving the compact object on an orbit that is bound to the MBH. The compact object then gradually inspirals into the central MBH via GW emission, eventually forming an EMRI. This details of this process depend on the physics of nuclear stellar clusters (see [3] for more details). Other EMRI formation channels include the Hills mechanism [4], in which a binary star system is tidally disrupted when it passes close enough to the MBH, ejecting one component as a hypervelocity star and leaving the other bound to the central black hole where it becomes an EMRI, or the tidal stripping of a giant star. In the latter case, the envelope of the giant star is removed by the tidal interaction, and the core of the star is then left on an orbit around the black hole such that it eventually inspirals as an EMRI [5]. Stars might also form directly in-situ in the vicinity of a black hole via fragmentation of a massive accretion disc [6], with their remnants becoming EMRIs.

This variety in formation channels offers uncertainty for LISA, but also highlights the discovery potential of the mission, as LISA will provide the first measurements of the relative rates of EMRIs from these different channels. Over recent years, we have also made electromagnetic observations of systems that are believed to be related to EMRIs. Around 20 hypervelocity stars have been observed in our galaxy whose trajectories are consistent with formation via the Hills mechanism in the Galactic centre or in the centre of the Large Magellanic Cloud [7]. In addition, transient electromagnetic emission from MBHs has been observed that is believed to be associated with the same dynamical processes that create EMRIs. This includes tidal disruption events, which are the detonations of stars perturbed onto orbits that pass close to MBHs, and quasi-periodic eruptions/oscillations (QPEs/QPOs). The leading model to explain QPEs is that the emission is triggered by a compact object orbiting close to the MBH passing through the accretion disc of the MBH as it orbits [8]. Objects on such orbits will later become EMRIs, offering the prospect of multimessenger observations of the same system [9]. These electromagnetic observations have provided EMRI rate estimates that are consistent with theoretical models, providing greater confidence that LISA will observe a significant number of EMRI events over the mission lifetime.

Estimating how many EMRI events that LISA will observe requires three ingredients — the rate of EMRIs in galactic nucleii, as a function of the MBH properties; the number density of black holes in the range $10^4 M_\odot \lesssim M \lesssim 10^7 M_\odot$ to which LISA is sensitive; and the detectability of EMRIs in LISA data, as a function of the system parameters. There are large uncertainties in the first two ingredients in particular. For the rate per galaxy, uncertainties arise not only because of the variety of different EMRI formation channels, but also because the physics of stellar clusters is complex and difficult to simulate. Matching of scales between the Newtonian many body dynamics that dominates far from the black hole, and the relativistic dynamics in the close vicinity of the MBH, create computational difficulties that lead to large uncertainties in predicting the proportion of compact objects passing close to the MBH that plunge directly into the black hole rather than inspiraling. The rate at which compact objects are brought from larger radii in the galaxy to the vicinity of the MBH, to replenish the compact objects falling into the MBH, is also hard to model. The space density of MBHs in the mass range relevant to LISA is not much better constrained, as these MBHs are very difficult to observe electromagnetically, once again providing LISA with a rich discovery space at the cost of uncertain rate predictions. One thing comparatively well constrained is the sensitivity of LISA to EMRIs. Even there the signal-to-noise ratio required for a confident detection is not known, due to data analysis uncertainties, and computations rely on having models for the EMRI signals that are both physically accurate and sufficiently cheap to evaluate that the whole parameter space can be explored. EMRI waveform modelling is a subject of intense study, as reported elsewhere at this meeting, and fast, physically accurate models are now available [10], but only for a restricted portion of the parameter space. Approximate models [11] must therefore be used to assess the EMRI detection horizon, introducing a factor of two uncertainty in the rates. Given all these uncertainties the estimated EMRI detection rate ranges from a few to a few thousand events per year [12], with a best guess of ~ 100 EMRIs yr^{-1}. Recent work has attempted to address some of the astrophysical uncertainties [13]. This has not significantly changed the rate estimates, but has found that the ratio of inspirals to direct plunges might be higher than previously thought, and EMRIs may have higher residual eccentricity at plunge [14], creating further modelling challenges.

Hundreds of EMRI observations would offer a rich range of possibilities for science. The $\sim 10^5$ waveform cycles observable from a typical EMRI allow the parameters of the system to be measured with unprecedented accuracy. EMRI observations will constrain the intrinsic parameters of the system (mass and rotation rate of the MBH, mass of the smaller object, and properties of the orbit, such as inclination and eccentricity) to precisions of 10^{-6}–10^{-4}, sky locations to a few square degrees and luminosity distances to a few percent [12, 15]. These precisions arise from the large number of observed waveform cycles and are achieved even for events at the threshold of detection. Through these precise measurements, EMRI observations will provide unique information about the properties of quiescent MBHs in the relatively low redshift Universe, the relative importance of different EMRI formation channels and the physical processes that govern the dynamics in dense stellar systems [16]. In addition, EMRIs can probe the immediate astrophysical environments of their host MBH. As we know from observations of QPEs, EMRIs can occur around MBHs with accretion discs. When the smaller object passes through the MBH disc as it orbits, it experiences a drag which changes the orbital trajectory. In general, any effect that leads to a significant orbital dephasing over the observation can be detected, meaning effects at the level of 10^{-5}. It has been shown that EMRIs occurring in systems with discs can provide measurements of the disc density profile [17] and, for EMRIs on eccentric orbits that transition from supersonic to subsonic motion over the inspiral, also measurements of the disc viscosity and accretion rate [18]. For the same reason, EMRIs also provide very sensitive probes of the spacetime structure outside the MBH and can thus be used for fundamental physics, to test for consistency of the MBH geometry with that of a Kerr black hole, as predicted by GR. EMRIs are sensitive to the smallest absolute deviation of any GW probe. The natural ways to build alternative theories to GR are to include quadratic and higher order curvature terms in the action, or to introduce couplings to additional fields. EMRIs are not a particularly good probe of higher curvature deviations, as these are suppressed for MBHs relative to stellar-origin black holes. However, EMRIs are excellent probes of scalar-tensor theories of gravity, for which the signature is dominated by scalar charge accumulating on the small object, and therefore it is the curvature of the smaller black hole that is most important [19]. Finally, EMRIs can also be used as standard sirens, to probe the expansion history of the Universe, by combining the luminosity distance measurement from the GWs with an electromagnetic redshift. EMRIs are unlikely to have direct counterparts, so the most promising approach to cosmography is via cross-correlation of EMRI localisation volumes with galaxy catalogues. This could provide few percent measurements of the Hubble constant and ten percent measurements of Ω_m when all the EMRIs observed over the mission are combined. These constraints are further improved when combined with measurements from MBH binary mergers observed by LISA at higher redshift [20].

To realise this exciting scientific potential it is necessary to identify and characterise the EMRIs in the LISA data and this poses significant challenges. LISA will have a source-dominated data stream with thousands of sources of many different types simultaneously present and overlapping in time and frequency, the instrumental noise will not be known a priori and the data will be contaminated by glitches and gaps. These complexities mean that LISA data analysis requires a simultaneous *global fit* to all the sources of all the different types plus the instrumental noise. Global fits are under development by several groups and have successfully analysed simplified data containing only MBH binaries, galactic binaries and stationary Gaussian noise [21]. Inclusion of EMRIs in these fits poses additional challenges, since the same complexity of the EMRI signals that allows the remarkable precision of parameter measurement introduces a complex structure in the likelihood space, with many secondary modes and a primary mode occupying a tiny fraction of the prior volume [22]. The successful recovery of isolated EMRI signals in simplified data has already been demonstrated [23], and new searches are being developed designed to tackle more realistic data sets [24]. Over the coming decade we expect to bring these various pieces of work together into a final LISA global fit pipeline, able to find and fit all of the sources in the data. Then we will finally be ready to deliver the precision measurements required to realise LISA's revolutionary scientific potential.

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PUTTING THE HYPE IN HYPERBOLIC BLACK HOLE SCATTERING

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This talk summarises recent advances in the modelling of hyperbolic black hole scattering, a scenario of increasing interest for the gravitational wave community. While these scattering encounters have not been observed with current detectors [1, 2], multiple works have suggested that they may be observed with future detectors [3, 4]. Furthermore, scattering encounters provide a useful theoretical laboratory for testing and comparing different approaches to the two-body problem in General Relativity. The talk will highlight the key findings from recent studies in post-Minkowskian, self-force, Numerical Relativity, and Effective One Body theory, and their implications for our understanding of black hole dynamics.

Post-Minkowskian (PM) theory is a weak-field approximation where the spacetime is expanded order-by-order in Newton's constant G. PM calculations have seen significant progress in recent years due to the adaptation of advanced techniques from particle physics to the two-body problem. The state-of-the-art calculations have now reached the 5th order in G up to linear order in the mass ratio [5].

Self-force (SF) theory is an alternative expansion in the mass ratio of the two bodies, which is valid at all separations if the mass ratio is sufficiently small. While SF on bound orbits have been extensively studied, the case of unbound orbits has been restricted to a scalar field toy model in order to develop the necessary computational infrastructure [6]. These calculations have been compared to PM results showing good agreement in the weak-field regime [7]. SF information has also been used to improve the accuracy of PM results in the strong-field regime by incorporating information about the scatter-capture separatrix. This resummation has been shown to significantly improve the accuracy of PM results across all separations [8].

In order to study strong-field interactions in the comparable mass regime we must use Numerical Relativity (NR) where the full Einstein equations are solved on a computer. While progress in NR was fueled by the need to model late-inspiral and merger of quasi-circular binaries, recent works have simulated hyperbolic black hole encounters. These simulations have been used to extract the scattering angle with recent results being the first to extend the simulations up to mass ratios of 1:10 and measure disparate scattering angles of each black hole due to assymetric gravitational wave emission [9].

Finally, the Effective One Body (EOB) formalism provides a framework to model the two-body problem across all mass ratios and separations by mapping it to an effective one-body problem. Recent work compared different models of hyperbolic scattering within the EOB framework to NR results, finding that the most recent EOB models show very good agreement with NR across a range of mass ratios and impact parameters [9].

This comparison has also highlighted areas where further improvements can be made, particularly in the case of spinning black holes where so-called “evolution” models show large discrepancies with NR results [9].

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THE DDPC AND EMRI WAVEFORM MODELLING: STRUCTURE, ROLES, AND ROADMAP

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1. Introduction

The European Space Agency's (ESA) Laser Interferometer Space Antenna (LISA) is a space-based gravitational-wave observatory scheduled for launch in 2035. The Distributed Data Processing Centre (DDPC) is one of two primary ground segments in the LISA mission, responsible for transforming raw telemetry data (level 0) into a scientifically validated catalogue of gravitational-wave sources (level 3). The DDPC provides the full analysis chain from instrument calibration to the production of a final catalogue, functioning alongside ESA's Science Operations Centre (SOC). An independent pipeline run by the NASA Science Ground Segment (NSGS). The DDPC is comprised of a set of Coordination Units (CUs), each addressing a distinct stage of the data-processing pipeline or providing support to the pipeline's development and testing. This paper outlines the DDPC structure, the function of each CU, and the roadmap for the Waveform Coordination Unit (CU Wav), with emphasis on the Extreme Mass Ratio Inspiral (EMRI) subunit.

2. DDPC Structure and Coordination Units

The DDPC comprises multiple CUs aligned with successive data-processing levels and key operational domains:

- **CU L01 (Level 0 - Level 1):** Converts raw spacecraft telemetry into time-delay interferometry (TDI) channels.Tasks include data cleaning, instrument calibration, and laser-noise suppression via TDI combinations, and the development of the L0-L1 pipeline, 'Lolipops'.
- **CU L2A (Alerts):** Implements low-latency source detection and preliminary parameter estimation, and provides alerts for multi-messenger astronomy.
- **CU L2D (Deep):** Develops and hosts the Global Fit Pipeline which uses Bayesian inference for joint analysis of all detectable sources. The current focus is on massive black hole binaries (MBHBs) and galactic binaries (GBs), with planned inclusion of EMRIs, stellar-origin black-hole binaries, stochastic backgrounds, and non-astrophysical artifacts such as glitches and data gaps.
- **CU L3C (Catalogue):** Builds the final catalogue of sources, selecting the most relevant parameters and metadata for community dissemination.

- **CU SIM (Simulations):** Generates synthetic datasets for validation. This includes both instrument noise and mock populations of gravitational-wave sources. They produce the LISA Data Challenges, providing controlled environments to test pipelines and models.
- **CU Wav (Waveforms):** Manages waveform model quality assurance via waveform reviews, defines waveform conventions, and develops a Waveform Generator.

Each CU operates semi-autonomously but is interlinked through shared interfaces and review procedures, ensuring that the pipeline components integrate smoothly with one another.

3. CU Wav

CU Wav is responsible for the verification and standardization of waveform models used across DDPC pipelines. Importantly, it is **not** responsible for the development of the waveform models themselves. Its central activities are:

(1) **Waveform Review Process:** An internal peer-review system assessing code quality, robustness, and compatiability with DDPC piplinees and hardware (rather than physical accuracy). Reviews include code inspection, compilation tests across platforms, pertubation tests, injection and recovery parameter estimation tests, and convention consistency checks.
(2) **Waveform Generator Development:** Creation of a standardized interface that allows all CUs to call waveform models in a uniform way. The generator currently supports Phenom and SEOB models for MBHBs, models for GBs, and models for the LISA instrument response, but will be extended to include more source types.
(3) **Conventions:** Establishing consistent parameter definitions and conventions across waveform models and providing conversions between conventions.

4. The EMRI Subunit

The EMRI subunit within CU Wav currently includes 14 members with expertise ranging from post-adiabatic (1PA) waveform modelling to EMRI search and parameter estimation. The group meets bi-weekly and coordinates through dedicated communication channels. Its primary goal is to establish the validation framework and deliver a roadmap for EMRI waveform readiness for the LISA data-analysis pipelines. The subunit's current focus is on vacuum General Relativity models within the FastEMRIWaveforms (FEW) framework [1]. Models from different frameworks can also be reviewed if they are to be utlised within the DDPC.

5. EMRI Waveform Review Programme

The subunit has defined a staged review plan:

(1) **Eccentric Kerr Adiabatic Model (0PA)** [2]**:** first waveform review for an EMRI waveform model, starting September 2025.
(2) **1PA Quasi-Circular Model with Spin** [3]**:** review scheduled for mid-2026.
(3) **Spherical (Circular and Inclined) 0PA Model:** requires the inclusion of polar modes and improved interpolation; review planned for late 2026.

(4) **Generic (Eccentric + Inclined) 0PA Model:** development of efficient 4D interpolation methods; review targeted for 2027.

Parallel efforts include implementing secondary-spin effects, orbital resonances, and hybrid approaches combining gravitational self-force (GSF) and post-Newtonian (PN) inputs. These developments aim to support a fully realistic EMRI waveform model in time for the LISA Critical Design Review (CDR) in 2032, including spinning secondaries and resonant dynamics.

6. Timeline and Milestones

The EMRI subunit roadmap aligns with the overall DDPC schedule:

- 2025–2026: Establish waveform-review protocols, finalize Waveform Generator specifications, and validate early EMRI models (Eccentric Kerr 0PA, 1PA Circular).
- 2026–2028: Expand reviews to spherical and generic waveform models, consolidate waveform conventions.
- 2029–2030: Deliver a validated, efficient EMRI waveform models compatible with the Global Fit pipeline, in time for catalogue generation before the LISA Critical Design Review (CDR 2032).

Intermediate milestones include integration into the Mojito (2026) and Long Island Iced Tea (2030) data challenges (2025–2027).

7. Outlook and Community Involvement

LISA's success in detecting and characterizing EMRIs depends on community-driven model development. The DDPC provides the framework, but the broader research community must contribute by:

- Developing FEW or developing alternative frameworks.
- Producing robust, well-documented, and efficient codes that can tile parameter space with GSF information.
- Advancing 2nd order GSF and PN-based hybrid models.
- Implementing secondary-spin and resonance effects.
- Exploring beyond-GR or environmental effects that can be modeled within the two-timescale adiabatic framework.

Collaborators are encouraged to engage early so models can be reviewed and integrated within the DDPC schedule.

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The Hyperboloidal Framework in the gravitational self-force programme

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Overview

The hyperboloidal framework has emerged as a powerful geometric and numerical approach for treating wave propagation on black-hole backgrounds [1–5]. Its central idea—rooted in Penrose's conformal treatment of infinity [6]—is to combine a conformal mapping of the spacetime [7–9] and foliate the conformal manifold with spacelike hypersurfaces that asymptote to future null infinity while remaining regular across black-hole horizons. This construction enables the simultaneous resolution of near-horizon and asymptotic regions within a single coordinate patch and eliminates the need for artificial outer boundaries, thereby avoiding spurious reflections and boundary-condition systematic errors.

Hyperboloidal methods play an increasingly important role in the modelling of gravitational radiation from black-hole systems, especially in regimes requiring extreme accuracy or controlled asymptotics. For perturbative approaches, including first- and second-order metric perturbations and gravitational self-force computations, the hyperboloidal formulation offers an attractive alternative to standard choices of coordinates based on Boyer–Lindquist or Schwarzschild slicing. It provides a unified description of both the black-hole exterior region (up to the event horizon) and the radiation zone (extending to $\mathscr{I}^+$), with all relevant geometric structures incorporated directly at the level of the differential equations.

Perturbation theory and the role of hyperboloidal slicing. Schematically, Einstein's equations take the form

$$\Box_g \Phi = S + \mathcal{N}(\Phi, \partial\Phi), \tag{1}$$

for a given field Φ representing some combination of metric or curvature components. This form emphasises the wave-equation character dictating the dynamics. Nonlinear couplings between Φ and its lower-order derivatives are captured by $\mathcal{N}$, whereas S represents possible sources. For the two-body problem in the extreme-mass-ratio regime, S assumes the form of distributional sources (e.g. a point particle) within the gravitational self-force programme [10].

Given the small parameter $\epsilon = m/M \ll 1$ naturally arising in extreme-mass-ratio inspirals, one can solve eq. (1) perturbatively by expanding the metric as

$$g = g_o + \epsilon g^{(1)} + \epsilon^2 g^{(2)} + \cdots \tag{2}$$

In the linear regime, the primary task is to solve

$$\Box_{g_o} \Phi^{(1)} = S^{(1)},$$

with g_o a stationary black-hole metric. At higher order, the source terms depend on the lower-order solutions; any systematic error in the first-order calculation therefore propagates and may jeopardise the second-order problem. This makes control of asymptotic behaviour and boundary regularity essential.

Hyperboloidal coordinates provide this control: the principal part of the transformed wave operator becomes singular at both the event horizon and $\mathscr{I}^+$, and the physically relevant ingoing/outgoing boundary conditions become directly encoded as regularity conditions. In particular, the radiative degree of freedom can be extracted unambiguously at null infinity without the need for extrapolation. Ref. [2] gives a detailed and updated review of the fundamentals of this research programme.

Self-force calculations. At first order, the gravitational self-force programme requires solving the linearised field equations with a distributional worldline source. These equations are solved either in the time or frequency domain. Exploring symmetries of the background spacetime, one typically projects the equations into a harmonic basis [11], decomposing the problem into a set of equations for individual (ℓ, m)-modes characterising the angular structure of the field. This strategy leads to $(1{+}1)D$ wave equations in the time domain, or $1D$ ordinary differential equations in the frequency domain. Recently, there has been interest in treating the equations within an m-mode strategy [12, 13], thereby avoiding the decoupling between the radial and polar angular directions. In all the above-mentioned strategies, regularisation schemes are required to obtain physical observables at the particle's position [10].

Over the past years, the hyperboloidal framework has been consistently adapted to the needs of first-order calculations within the gravitational self-force programme. Initial efforts concentrated on toy models given by a scalar field on the Schwarzschild background, with a particle in a circular orbit. In this context, successful results have been demonstrated in the time domain [14–16] and in the frequency domain [13, 17].

Currently, the research programme aims to include scenarios of increasing relevance to gravitational-wave physics. The challenges include: (i) extension of the framework beyond scalar-field toy models into the gravitational case; (ii) considering the Kerr solution for the background metric; and (iii) modelling the particle's trajectory in more intricate orbits. Ref. [18] made significant progress in tackling step (i) for circular orbits in the Schwarzschild background, by solving the first-order metric perturbation in the Lorenz gauge within the hyperboloidal framework. For step (ii), the infrastructure put forward in ref. [13] was developed with a direct extension to Kerr in mind, and work in this direction is in progress. Finally, step (iii) constitutes one of the major challenges. In this context, the next section introduces initial ideas for treating the problem of particles on eccentric equatorial orbits.

Elliptic-coordinate mapping for future m-mode PDE solvers

For eccentric-orbit self-force calculations, the particle librates between radial turning points σ_- and σ_+. A promising strategy for improving spectral resolution in the near-particle region is to introduce an *elliptic coordinate mapping* adapted to this libration domain. The idea is to reinterpret the physical hyperboloidal coordinates (σ, y) in an excision region around the particle's trajectory in terms of shifted elliptical coordinates (μ, ν) centred at the midpoint of the libration interval.

Construction of the map. Standard planar elliptic coordinates are defined by

$$x = a\cosh\mu\cos\nu, \qquad y = a\sinh\mu\sin\nu,$$

with $\mu \in [0, \mu_0]$ and $\nu \in [0, \pi]$. The parameter μ_0 fixes the size of the excision region and world tube around the particle.

To adapt this map to the physical hyperboloidal coordinates, we shift the x coordinate to be centred at the midpoint of the libration region σ_0, and we place the focal points at the turning points $\sigma_\pm$, by imposing $\sigma_\pm - \sigma_0 = \pm a$.

This strategy leads to the physical coordinate map

$$\sigma = \frac{\sigma_+ + \sigma_-}{2} + \frac{\sigma_+ - \sigma_-}{2}\cosh\mu\,\cos\nu, \quad y = \frac{\sigma_+ - \sigma_-}{2}\sinh\mu\,\sin\nu, \tag{3}$$

which naturally parametrises grid points near the particle worldline in a similar way to the polar coordinate map used for circular orbits (see fig. 1). Its introduction into future hyperboloidal m-mode solvers provides a potential geometrical strategy and may help bridge time-domain and frequency-domain approaches within a unified framework. A possible enhancement may require adjusting the elliptic coordinates to a system adapted to the particle's co-moving frame [12, 13].

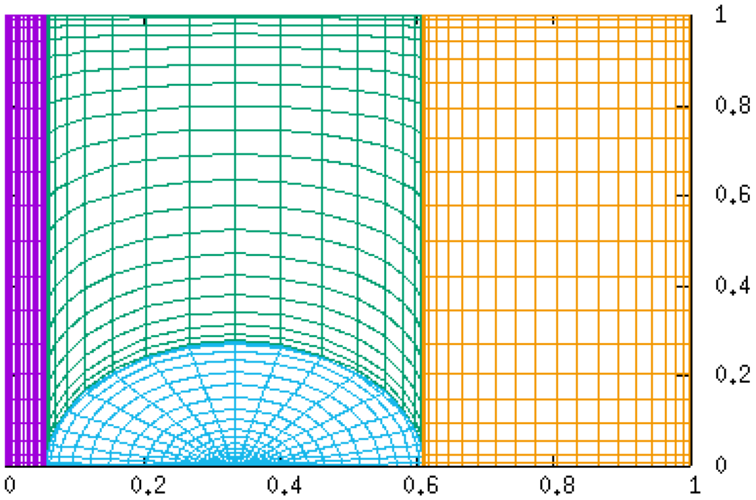

FIGURE 1. Elliptic coordinates mapped into hyperboloidal (σ, y) coordinates as a potential geometrical strategy for an m-mode solver for an eccentric equatorial orbit.

CONCLUDING REMARKS

The hyperboloidal framework, together with modern spectral solvers and geometric domain decompositions, offers a promising path toward next-generation self-force calculations. The incorporation of elliptic-coordinate mappings represents a natural and flexible extension of the method, potentially enabling accurate and efficient solutions of the m-mode elliptic PDEs associated with eccentric inspirals. This line of development is particularly timely given the demands of future space-based detectors such as *LISA*, where controlled asymptotics, high-accuracy mode extraction, and robust treatment of distributional sources will be indispensable.

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COMPUTATIONAL ADVANCES IN SELF-FORCE: BUILDING A BRIDGE BETWEEN THEORY AND WAVEFORM MODELING

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Classification AMS 2020: 83C35, 83C25

Keywords: perturbation theory, gravitational waves, multiscale expansions

1. Background

The multiscale self-force (MSF) framework provides an efficient and systematic method for modeling compact binaries with disparate masses and their associated gravitational-wave signals [5]. In this framework, the binary's mass ratio $\epsilon = m_2/m_1$ is treated as a perturbative parameter, and the full spacetime metric $\mathrm{g}_{\mu\nu}$ is decomposed as

$$\mathrm{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} = \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + \cdots , \tag{1.1}$$

where $g_{\mu\nu}$ is the background metric of the larger mass m_1 and $h_{\mu\nu}$ encodes the perturbations sourced by the smaller body m_2. Due to the gradual inspiral of disparate mass systems and their quasi-periodic nature, these perturbations are further decomposed into "fast" oscillating phases $\phi_a \doteq (\phi_r, \phi_\theta, \phi_\phi)$ and "slowly"-evolving orbital phase-space variables J_b (e.g., energy, angular momentum, frequencies),

$$h^{(n)}_{\mu\nu} = \sum_{k^a} h^{(n)k^a}_{\mu\nu}(J) e^{-ik^a\phi_a}, \tag{1.2}$$

where $k^a \doteq (k_r, k_\theta, k_\phi)$. The amplitudes $h^{(n)k^a}_{\mu\nu}(J)$ are parametrized purely by J_b, and once computed across this space, they determine the inspiral dynamics, fast phase evolution, and resulting gravitational-wave signals order by order in ϵ.

At leading-order in this multiscale expansion, the system undergoes adiabatic radiation-reaction in which time-averaged gravitational wave fluxes [which can be computed from the asymptotic amplitudes of $h^{(1)k^a}_{\mu\nu}(J)$] drive the secular decay of the binary orbit over the inspiral timescale $T_{\mathrm{insp}} \sim M/\epsilon$. These leading-order dynamics—often referred to as adiabatic or 0-post-adiabatic (0PA) order—contribute $O(\epsilon^{-1})$ cycles to the gravitational wave signal. At subleading or 1PA order, conservative corrections from $h^{(1)}_{\mu\nu}$ and dissipation driven by $h^{(2)}_{\mu\nu}$ contribute $O(1)$ cycles to the gravitational wave phase. MSF waveform models must therefore include both 0PA and 1PA contributions in order to achieve the subradian phase accuracy required to meet the science goals of both ground- and space-based observatories.

2. Results

Current MSF waveform models incorporating 0PA and 1PA effects have successfully described the inspiral of black hole binaries with mass ratios as comparable as $\epsilon \sim 0.1$ [6]. However, these models, and the associated precomputed 0PA and 1PA data, remain restricted to non-spinning black holes on quasi-circular orbits. Recent work has extended the framework to eccentric binaries with spin, but only at the 0PA level, leaving the 1PA data required for subradian accuracy still unavailable [1]. A central challenge lies in the computationally intensive offline stage in which the amplitudes of $h^{(1)}_{\mu\nu}$ and $h^{(2)}_{\mu\nu}$ must be computed across a large region of parameter space. This precomputed database then serves as the essential input for the subsequent rapid online generation of waveforms. Extending these 0PA and 1PA datasets to more realistic eccentric, spinning, and precessing systems is an interesting theoretical challenge and crucial for the next generation of gravitational-wave astronomy.

In this talk, I outline how recent computational advances are bridging the gap between the theoretical foundations of the MSF framework and the practical generation of 0PA and 1PA data products for accurate MSF waveform models. I review which specific information from $h^{(1)}_{\mu\nu}$ and $h^{(2)}_{\mu\nu}$ needs to be computed and stored across the parameter space, with a particular focus on

- 0PA fluxes, computed from the asymptotic amplitudes of $h^{(1)}_{\mu\nu}$, with a focus on the results of Refs. [1, 2];
- 1PA redshift corrections $\langle z_1 \rangle$, computed from the local behavior of $h^{(1)}_{\mu\nu}$ along the worldline of m_2, with a focus on the results of Refs. [3]; and
- 1PA dissipative corrections, which are derived from the asymptotic amplitudes of $h^{(2)}_{\mu\nu}$, with a focus on the results of Refs. [6].

I highlight recent progress in computing these quantities, enabled in part by the development of open-source tools such as the MATHEMATICA packages in the Black Hole Perturbation Toolkit [7] and the PYTHON library `pybhpt` [3, 4]. These resources are making it increasingly feasible to extend 0PA and 1PA data into new regions of parameter space, as evidenced by the recent work of Refs. [1, 3]. I also discuss ongoing advances in constructing second-order perturbations in Kerr spacetime—an essential milestone for producing subradian (1PA-accurate) waveforms for binaries with spinning and precessing black holes.

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FIX THE FRAME, RESOLVE THE MEMORY: THE BONDI–SACHS GAUGE IN BLACK HOLE PERTURBATION THEORY

ANDREW SPIERS

Keywords: Black hole perturbation theory, Bondi–Sachs gauge, BMS symmetries, gravitational wave memory, second-order self-force

Understanding gauge and frame dependence is crucial for extracting physical observables from black hole perturbation theory (BHPT) [1], especially as calculations extend to second-perturbative order. At second order, constructing gauge-invariant quantities is challenging. Quantities that are gauge-invariant at linear order become gauge-dependent when extended to second order. The gauge dependence of second-order calculations introduces scope for working in poorly behaved gauges, which produce singular source terms. Near future null infinity generic gauges produce singular sources (so-called infrared divergences) such that integrals do not converge [4].

Further subtleties arise in the gauge choice of second-order calculation near future null infinity. Gravitational-wave memory is frame-dependent and will be detectable with next-generation detectors such as LISA [**?**]. When measuring gravitational waveforms or comparing waveforms between models, not including the memory effect consistently can cause errors. Gravitational memory effects are associated with choices of gauge near future null infinity; working in consistent gauges between models, and the frame of gravitational detectors, allows one to consistently incorporate gravitational memory effects [6].

To address these challenges, we construct a perturbative treatment of the Bondi–Sachs (BS) gauge on Kerr spacetime, accompanied by a BMS frame-fixing scheme. The BS gauge enforces the canonical Bondi–Sachs falloff and determinant conditions on the metric near future null infinity, ensuring that the retarded-time foliation and luminosity distance are uniquely defined. Our formalism allows one to transform to the BS gauge with a prescribed BMS frame from any initial gauge at first order. The BMS frame corresponds to a choice of supertranslation and Lorentz frame that fixes the residual asymptotic freedom of the BS gauge, thereby providing an unambiguous notion of angular coordinates and Bondi time. Our method provides a consistent way to extract memory effects associated with the BMS frame. Additionally, the BS gauge avoids infrared divergences, and using our first-order gauge fixing scheme, we define second-order gauge invariants near future null infinity.

Our derivation begins with the Kerr metric expressed in BS coordinates, following the prescription in Bai et al. [2]. The Kerr BS coordinates are defined using an asymptotic expansion in terms of Boyer–Lindquist coordinates. Hence, these Kerr BS coordinates are only exactly in BS form at future null infinity; within the interior, they are only approximately in BS form in the large radius limit. Nonetheless, this approximate BS structure suffices for our analysis, and the asymptotic expansion can be extended as required.

We define the *perturbative BS gauge* by specifying gauge conditions on the first-order metric perturbation. The transformation to this gauge is described by a vector field satisfying a hierarchical set of first-order ODEs along outgoing null rays, with boundary conditions applied at future null infinity satisfying the BS gauge. Introducing an asymptotic expansion of the metric perturbation reduces these equations to an algebraic form. For practical implementation, we recast the gauge vector in Boyer–Lindquist coordinates and decompose it into Newman–Penrose components.

Residual freedom in the perturbative BS gauge corresponds to the BMS frame. We analyse this freedom in our perturbative BS gauge and give a BMS frame fixing scheme based on choices for the Bondi mass aspect and the Wald–Zoupas angular momentum aspect on a given retarded time-slice at future null infinity. Our procedure constrains the supertranslation, boosts, and rotation degrees of freedom, leaving only the background Killing symmetries (time translations and axial rotations), which can be fixed by waveform alignment. The resulting prescription fully determines the integration constants in our perturbative BS gauge transformation vector calculation.

Although our explicit construction is developed at first order, we leverage it to define second-order gauge-invariant quantities near null infinity. We use the second-order Weyl scalars

$$\{\psi^{(2)}_{4L},\ \psi^{(2)}_{4},\ \psi^{(2)}_{0L},\ \psi^{(2)}_{0}\}$$

are invariant under purely second-order gauge vector transformations [7, 8]. By fixing the first-order gauge with our perturbative BS formalism, these scalars become genuine gauge invariants (invariant under first- and second-order gauge transformations) between asymptotically flat gauges. Hence, applying our BS gauge vector calculation allows one to construct second-order gauge invariants associated to the BS gauge using gauge fixing.

Implementing our BS gauge fixing scheme has further advantages for second-order calculations. Re-expressing the second-order Teukolsky equation [7, 8] in a gauge-fixed form, associated with an asymptotically flat gauge (such as the perturbative BS gauge), naturally avoids infrared divergences and provides a consistent framework to extract gravitational-wave memory. Our methods will be used to help compute second-order self-force calculations in Schwarzschild and Kerr spacetime and enable systematic comparisons between second-order results and other approaches, including post-Newtonian and post-Minkowskian theory, ringdown calculations, and numerical relativity.

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INTEGRABILITY OF THE RELATIVISTIC TWO-BODY PROBLEM

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Classification AMS 2020: 83C10, 83C57, 83C35, 70H06, 70H08, 70H40, 70H33, 37J06, 37J35, 37J40

Keywords: General relativity, Black holes, Neutron stars, Gravitational waves, Compact objects, Binary inspirals, Equations of motion, Integrability, KAM theory, Multiple scale perturbation methods

The relativistic two-body problem is central to gravitational wave astrophysics, particularly for modeling compact binary inspirals and the resulting gravitational-wave signals. It represents one of the simplest dynamical problems in general relativity, yet it is significantly more complex than its Newtonian counterpart due to the inherent length scale set by the gravitational radius and the presence of spin and radiation-reaction effects. This abstract summarizes a presentation on the integrability of such systems under various approximations, emphasizing the role of symmetries, perturbations, and dissipative effects.

We begin with a review of *Hamiltonian integrable dynamics*, where a system with N degrees of freedom is Liouville integrable if there exist N functionally independent, commuting constants of motion. According to the Liouville-Arnold theorem [1], the motion is confined to invariant tori and can be solved by quadratures. In action-angle coordinates $(\mathbf{J}, \boldsymbol{\theta})$, the Hamiltonian becomes $H(\mathbf{J})$, and the equations of motion simplify to:

$$\dot{\boldsymbol{\theta}} = \frac{\partial H}{\partial \mathbf{J}} \equiv \boldsymbol{\Omega}, \quad \dot{\mathbf{J}} = 0. \tag{0.1}$$

The *Kolmogorov-Arnold-Moser (KAM) theorem* ensures that most invariant tori survive under small perturbations of size ϵ. However, tori satisfying a local resonance condition $\boldsymbol{\Omega} \cdot \boldsymbol{k} = 0$, where $\boldsymbol{k}$ is an integer vector, may be destroyed and replaced by nonlinear oscillations of amplitude $\sim \sqrt{\epsilon}$. Between the surviving tori and the resonant oscillations, a thin chaotic layer generically emerges. Although the resonance condition is fulfilled on a dense set in phase space, only resonant tori with small $\boldsymbol{k}$ vectors are practically relevant. This is because the amplitude of the resonant oscillations scales with the size of the $\boldsymbol{k}$-harmonic of the perturbation with respect to θ, which decays exponentially as $\sim e^{-C|\boldsymbol{k}|}$ for large $|\boldsymbol{k}|$. This allows for a practical cutoff in resonance analysis.

In conservative approximations of the relativistic two-body problem, integrability is preserved up to certain post-Newtonian (PN) orders. Systems of two point particles with only orbital degrees of freedom are integrable due to Poincaré symmetries, yielding six commuting integrals: energy, total linear momentum, one component of the total angular momentum, and its magnitude. However, the inclusion of spin introduces additional degrees of freedom and potential non-integrability. Known integrable cases include Kerr geodesics [2], spinning test particles [3, 4, 5], and spinning compact binaries at general mass ratios in certain PN regimes [6, 7].

Actions serve as coordinate-independent invariants that bridge PN and test-particle limits. These quantities depend only on the homotopy class of phase-space loops and are known to match across regimes [8]. However, integrability is fragile: tidal interactions, internal modes (e.g., f-modes in neutron stars), and environmental effects generically break it [9, 10].

For inspiraling binaries, we define integrability as the existence of a convergent two-timescale expansion up to the transition to plunge. The equations of motion can then be cast in action-angle variables, and near-identity transformations can be applied to eliminate angle dependence from the right-hand sides [1]:

$$\dot{\mathbf{J}} = \epsilon \mathbf{g}_1(\mathbf{J}) + \epsilon^2 \mathbf{g}_2(\mathbf{J}) + \ldots, \tag{0.2}$$

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\Omega}(\mathbf{J}) + \epsilon \mathbf{f}_1(\mathbf{J}) + \ldots \tag{0.3}$$

Transient resonances from gravitational self-force (GSF) corrections can violate this structure [11]. In particular, resonant terms in the perturbation break the convergence of the near-identity expansion. A specific feature of the dissipative case is the dependence of the dissipation on the so-called resonant phase [12]. Resonances caused by other perturbative sources also disrupt the transformation, requiring alternative treatments.

The analysis reveals that switching between formulations of the equations of motion near resonances is necessary when $\boldsymbol{\Omega} \cdot \mathbf{k} \sim \epsilon^{\beta}$, with $\beta \in (0, 1/2)$ depending on the scheme [13, 14].

In conclusion, integrability persists in many regimes of the relativistic two-body problem but is not guaranteed. Understanding its limits is essential for accurate modeling of gravitational wave sources. Existing schemes can handle weak integrability breaking, but future work should focus on self-consistent evolution through resonances, gauge-invariant formulations, and a systematic mapping of resonant effects.

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PROBING FORMATION CHANNELS OF EXTREME MASS-RATIO INSPIRALS

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Classification AMS 2020: 83C35

Keywords: Extreme Mass-ratio Inspiral, Active Galactic Nucleus

In this talk, I discuss how to probe formation channels and environmental effects of extreme mass-ratio inspirals (EMRIs), which are one of the main extragalactic sources of space-borne gravitational wave detectors. I have focused on two major channels - dry EMRIs that are produced through gravitational scatterings in nuclear star clusters and wet EMRIs that have Active Galactic Nucleus (AGN) playing a critical role to transport compact objects to the vicinity of massive black holes. The main content of this talk is based on our recent publications [1, 2, 3].

1. Direct measurement

The main astrophysical environmental effects of EMRIs may come from accretion disks, nearby stellar-mass objects, and/or dense dark matter distribution. On the one hand, these environmental objects may affect the gravitational waveform through the tidal resonance, as first discussed in [4, 5]. On the other hand, they may introduce additional dissipation channels, and also directly modify the total flux of the gravitational wave radiation during the secular evolution (as the background spacetime deviates from Kerr). We have been developing a general formalism to analyze the long-term secular motion of EMRIs in a perturbed black hole spacetime. Taking the scenario for a rotating black hole surrounded by a Axion cloud (as dark matter candidate), we manage to compute the extra scalar radiation due to the presence of the cloud [2]. In [3] we present the formalism to compute the modification of gravitational wave flux is the background deviates from Schwarzschild. The next step is to combine both effects due to scalar and (extra) gravitational wave radiation to obtain the EMRI waveform with these clouds, in order to allow future detections.

2. Indirect measurement

If the environmental effects, such as the disk effects are not directly observed in a set of EMRIs, we may still probe their population properties by studying the distribution of the key system parameters. With the population model developed in [1], we show the eccentricity of dry EMRIs are mostly (more than 99% percentile) larger than 0.01. The wet EMRIs, on the other hand, tend to have much smaller eccentricities because of disk dissipation, but they are not zero due to disk turbulence effects and multi-body mean-motion resonance in the disk. The turbulent eddies within a disk are generally associated with density fluctuations, so that they introduce fluctuating gravitational force on the EMRI object. The population model predicts that the resulting eccentricity should be mostly smaller than 0.01. In addition, the population model also predicts that a significant fraction of disks may host more than one stellar-mass object for $r \sim \mathcal{O}(10^2)$

gravitational radii. These object likely form mean-motion resonance pairs during their migration within the disk, and excite eccentricity to the level of $\mathcal{O}(10^{-4})$. As a result, eccentricity is a key observable to distinguish wet and dry formation channels. More importantly, it may be used to probe the level of turbulence within an AGN disk.

There are other observables that show distinct distribution in different EMRI formation channels, including the inclination angle, the mass, etc. In particular, the wet EMRI mass distribution is affected by EMRI capture probability onto the disk, the accretion process, and the chance of forming pairs and merge within the disk. The resulting mass distribution may show rich and different signatures compared with the mass distribution of black holes within LIGO-Virgo catalogs. So a precise measurement on the mass distribution may being valuable information on the evolution of stellar-mass black holes within AGN disks.

3. Summary

We have shown the properties of formation channels of EMRIs can be measured through both direct and indirect methods. In particular, the study in [1] represents the first step of probing formation mechanisms through population modelling. In the future, there are many unanswered questions and open discovery space for both directions.

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