IMPRINTS
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NEWSLETTER OF THE INSTITUTE FOR MATHEMATICAL SCIENCES, NATIONAL UNIVERSITY OF SINGAPORE

The Big IMS Move

The Institute for Mathematical Sciences (IMS) was formally established on 1 July 2000, and opened its doors to its first program one year later. In the beginning, much of the Institute's activities were held in borrowed premises. The IMS moved to its first home in 2001. The IMS was housed in two renovated two-storey colonial houses at 3 and 4 Prince George's Park (PGP). The two colonial houses then housed modest facilities which comprised one seminar room, one lounge for collaboration, shared offices for visitors as well as administration offices. An annex building was later added in 2003, which housed one large seminar room (ostensibly named "IMS Auditorium") in a stepped auditorium style featuring tiered seating as well as shared offices for visitors. For more than 20 years, as the IMS grew from infancy to adulthood, hordes of mathematicians, scientists and graduate students had their group photos taken with the iconic IMS house as the backdrop.

In 2023 under the leadership of the new IMS Director, Professor Shen Zuowei, plans were well underway for the new premises of the IMS at Block S17. The goal was to bring the Institute physically closer to the mathematical science faculty

From left to right: **Prof Sun Yeneng**, Dean, Faculty of Science; **Prof Shen Zuowei**, (Director, IMS); **NUS President Tan Eng Chye**; **Prof Chong Chi Tat** (Former Director, IMS); **Prof Louis Chen** (Founding Director, IMS)

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members on campus. Continuing our legacy, the aim for the new premises is to create a conducive environment which are even better than the old premises for fostering close interaction and collaboration amongst scientists for mathematical and interdisciplinary research.

A key design concept was to bring nature into the new IMS facility, largely inspired by the fact that the colonial houses were once surrounded by lush virgin forest. To encourage collaboration, the new facilities feature a large and open collaboration space with waving ceiling baffles mimicking clouds, as well as other shared spaces including a 16-seat hot desk area.

One key challenge was finding a suitable room style in a single level space to replace the much beloved "IMS auditorium" at PGP, which featured a seven-meter-long whiteboard and a large projection screen. The result is a 92-seat executive seminar room spread out in three-tiered seating which comes with an impressive movable whiteboard (even longer than that of the old auditorium) alongside dual projector screens.

The new premises was officially opened by the NUS President Tan Eng Chye on 12 February 2025. The day's celebration began with a welcome address by the Director, Shen Zuowei, followed by a speech delivered by NUS President Tan Eng Chye. Fittingly, the Institute's former Director, Chong Chi Tat, also delivered a speech on the past achievements of IMS.

On this milestone day, a simple ribbon ceremony and group photo took place with the new IMS main entrance signage as the backdrop. The new entrance signage spans two levels with the shape of Singapore formed by an aperiodic set of prototiles (referred to as an "einstein", meaning one stone in German). This new entrance will serve the backdrop for group photos of IMS programs for many years to come.

As the opening day coincided with the last day of the Chinese Lunar Year (元宵节), the reception began with a boisterous lo-hei (捞起, meaning "tossing up good fortune" in Cantonese) session, followed by a sumptuous lunch reception. For the afternoon, the audience enjoyed two lectures delivered by Shen Zuowei on "Mathematics in Data Science" and Gan Wee Teck on "The Local Langlands Correspondence".

It is with much confidence that the Institute will continue to shine and be a beckon for all, and in time to come, the IMS will celebrate its 50th anniversary.

IMS 10th Anniversary in 2010.

The first group photo in 2002 – Representation Theory of Lie Groups.


The last group photo taken 2024 – Algorithmics of Fair Division and Social Choice.


IMS
Institute for Mathematical Sciences


IMS
Institute for
Mathematical Sciences
National University of Singapore

Singularities in Fluids and General Relativity

From 16 December 2024 to 10 January 2025, the Institute hosted a program on "Singularities in Fluids and General Relativity". The organizers contributed to this invited article.

BY YAO YAO and AN XINLIANG *(National University of Singapore)*

Singularity formation arises in many fundamental equations in fluid dynamics and general relativity due to different mechanisms. Examples include the black hole formation in general relativity, shock formation and implosion in compressible fluid equations, and finite-time blow-up in incompressible fluid equations. Even though the equations are different, many of their analysis techniques are closely linked to each other. In view of the significant breakthroughs in the past decades, the institute hosted a three-week program on Singularities in Fluids and General relativity from 16 Dec 2024 to 10 Jan 2025. The aim of the program is to bring together experts on singularity formation that come from different areas of PDE and mathematical physics. We expect that the combined effort of the participants with complementary strength would generate new ideas and innovative strategies on these equations, as well as inspire young researchers participating in the program. We also hope the program would lead to new collaborations among researchers in the region and worldwide.

The first week of the program was a workshop on fluid equations, focusing on the old and new challenges in this field, such as questions on regularity, singularity and stability. There were 20 research talks by experts on

both incompressible and compressible fluid equations, based in Czech Republic, Hong Kong, Japan, South Korea, Spain, Switzerland, UK and USA, along with 6 short talks by junior researchers from Singapore, South Korea, USA. The talks in the workshop covered a broad range of topics of central interest in community, including finite-time singularities for incompressible fluid equations, boundary layer theory for Navier-Stokes equations, implosion in compressible fluid equations, and long-time behavior of fluid equations. The workshop was a great success, and has led to further collaborations among its speakers and participants.

The second week of the program consists of several mini-courses given by Xinliang An (NUS), Mahir Hadzic (University College London), Taoran He (NUS), In-Jee Jeong (Seoul National University) and Dawei Shen (Columbia University). Hadzic lectured on stability, oscillations, and damping in galactic dynamics, focusing on recent progress on the asymptotic stability for the gravitational Vlasov Poisson equation. Jeong lectured on the long-time behavior of 2D Euler equations, in particular on the topics of desingularization and vortex confinement. An, He and Shen gave an introduction to mathematical general relativity, then moved on to some recent results on stability of Minkowski space time and Kerr black holes. The lectures were very well received by the junior participants. During the week, 4 junior researchers from Singapore, South Korea and USA also presented their work.

The third week of the program was a workshop on black holes, naked singularities, shocks and implosions. There were 21 research talks by experts on mathematical general relativity, dispersive equations and compressible fluid equations, based in Australia, Canada, China, France, Germany, Greece, Singapore, Switzerland, UK and USA. The talks in this workshop covered many diverse phenomena on singularities in mathematical physics, such as black hole formation and naked singularity in general relativity, shock formations for compressible fluids, finite-time blow-up for wave maps, and the stability/instability of the singular solutions. The workshop also includes a colloquium talk by Jeremie Szeftel (Sorbonne Université) on the black hole stability problem. Overall, the workshop was very well received, and it offered a broad perspective on current research progresses on singularity formation.

Workshop on p-adic Geometry

8–22 Nov 2024

CO-CHAIRS:
Wee Teck Gan | National University of Singapore
David Hansen | National University of Singapore

This was an instructional workshop on p-adic geometry (broadly interpreted). There were four mini-courses by leading experts, presenting some of the latest developments in this exciting and fast-moving field. This workshop was partially funded by the gift from the GS Charity Foundation.

Algorithmics of Fair Division and Social Choice

25 Nov–13 Dec 2024

CO-CHAIRS:
Xiaohui Bei | Nanyang Technological University
Edith Elkind | University of Oxford
Warut Suksompong | National University of Singapore

This program aimed at studying questions pertaining to social choice and collective decision making, with a focus on algorithmic aspects. Topics covered encompass fair resource allocation, multiwinner voting, participatory budgeting, preference aggregation, coalition formation, facility location, matching, as well as implementations of algorithms for these tasks in the real world. Objectives of the program include bringing together researchers from different countries who are engaged in these fields, providing a forum for discussion and exchange of ideas, as well as promoting NUS and Singapore as attractive destinations for conducting cutting-edge research.

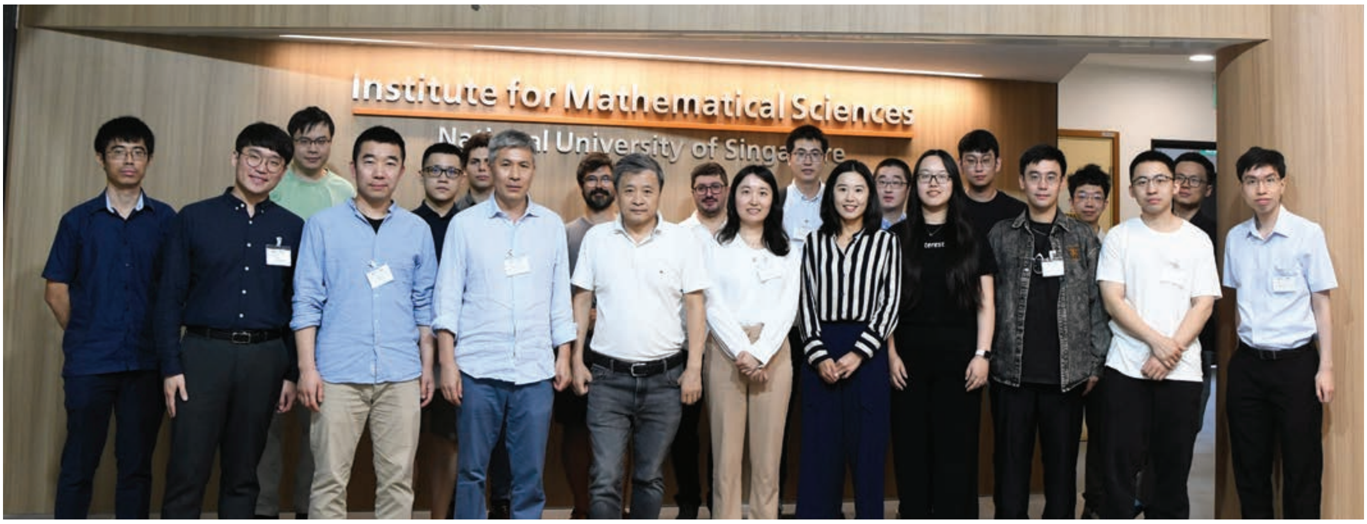
IMS Young Mathematical Scientists Forum — Applied Mathematics

6 – 9 Jan 2025

CHAIR:
Weiqing Ren | National University of Singapore

MEMBERS:
Weizhu Bao | National University of Singapore
Ying Chen | National University of Singapore
Hui Ji | National University of Singapore
Tan Minh Nguyen | National University of Singapore
Kim Chuan Toh | National University of Singapore
Xin Tong | National University of Singapore

The aim of this forum was to bring together a community of early-career scientists in the areas of applied mathematics from around the globe. This event will provide a platform for them to network, engage in enriching discussions, showcase their research, and exchange ideas with each other and faculty members from NUS. Twenty young scientists presented their research talks during the forum.



Special Colloquium Lecture by Wendelin Werner

8 Jan 2025

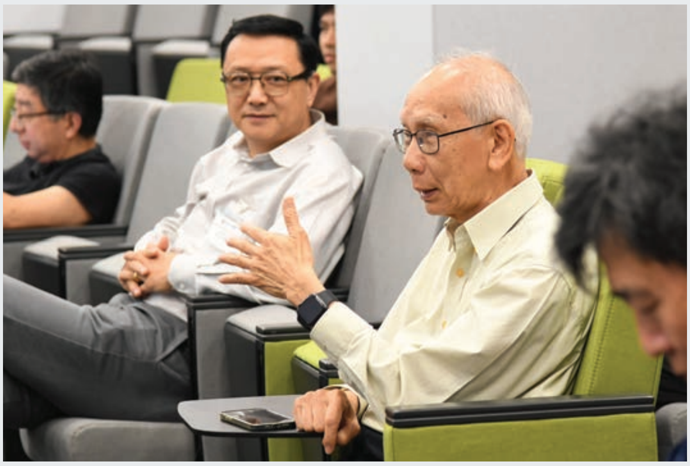
This was jointly organized with the Department of Mathematics, NUS and is an external engagement event of the Global Young Scientists Summit (GYSS) 2025. Professor Wendelin Werner (Fields Medal, 2006) delivered his talk on "From Fields to Random Geometric Structures and Back" to a rousing full house audience of more than 90.

Conference on Limits on Computability, Definability, Provability–Celebrating the Mathematical and Professional Contributions of Chong Chi Tat

9 –10 Jan 2025

CO-CHAIRS:
Jun Le Goh | National University of Singapore
Yue Yang | National University of Singapore

The two-day conference was jointly organized by the Department of Mathematics, the Faculty of Science, and the Institute for Mathematical Sciences. It featured a diverse program of talks by distinguished scientists from around the world. The event brought together Professor Chong's colleagues and friends (a total of twelve talks during the conference), many of whom have cherished his friendship and mentorship over the past five decades.



Workshop on Dynamics and Related Topics

04–06 March 2025

CHAIR:
Daren Wei | National University of Singapore

This symposium was organized with the Department of Mathematics at NUS. The aim of the symposium is to examine important recent developments on the homogeneous dynamics, zero entropy systems, anti-classification problem, and their various generalisations. A second aim was to facilitate the communications and international collaboration among the scholars in dynamics in the Asia region.

Frontiers of Statistical Network Analysis: Inference, Tensors and Beyond

12–30 May 2025

CO-CHAIRS:
Jialiang Li | National University of Singapore
Dong Xia | Hong Kong University of Science and Technology
Yuan Zhang | Ohio State University

The goal of this workshop was three-fold: to advance the field by exchanging ideas and deepening understanding; to facilitate future research by discussing challenges and establishing collaboration opportunities; and to benefit the local Singaporean audience and other participants through lectures from leading experts and emerging star scholars.

Research in Industrial Projects for Students (RIPS) 2025 – Singapore

19 May –18 Jul 2025

The Research in Industrial Projects for Students Program in Singapore (RIPS-SG) provides an opportunity for talented undergraduate students to work in international teams on a real-world research project proposed by sponsors. The student team, with support from their academic mentor and industry mentor, will research the problem and present their results, both orally and in writing, at the end of the program. A total of 15 students were selected for this program.

The sponsors of the research projects were MOH Office for Healthcare Transformation (MOHT), The Procter & Gamble (P&G) Singapore Innovation Center (SgIC), Cubist Systematic Strategies, and Home Team Science and Technology Agency (HTX).

Applied Geometry for Data Sciences Part II

2– 6 Jun 2025

CO-CHAIRS:
Han Fei | National University of Singapore
Wilderich Tuschmann | Karlsruhe Institute of Technology
Theodore Papamarkou | Zhejiang Normal University
Yuguang Wang | Shanghai Jiao Tong University
Kelin Xia | Nanyang Technological University
Rex Ying | Yale University

As the fourth paradigm, data-driven sciences can fundamentally change sciences and pave the way for a new industrial revolution. The remarkable achievements of AlphaFold models in deciphering protein folding marks a pivotal moment. The amazing performance of ChatGPT and Sora ushers in a new era for AI generated content and opens up a world of opportunities. Despite such strides, the ongoing challenge remains in developing highly efficient data representations and featurization models. Mathematically, computational and discrete geometries are powerful tools for data characterization, representation and modelling. Notably, geometric deep learning has played a pivotal role, elevating the capabilities of learning models to grapple with intricate geometric and topological structures inherent in complex datasets. The synergistic integration of geometric and topological methodologies with learning models holds immense potential to fundamentally reshape the landscape of data sciences. This workshop aimed to combine advanced geometric and topological tools with data-driven learning models.

AN INTERVIEW WITH YUJIRO KAWAMATA

by Chong Chi Tat

Yujiro Kawamata is University Professor at the University of Tokyo. He received his PhD from the University of Tokyo in 1980.

His research interest lies in algebraic geometry, particularly birational geometry. These include subadditivity of the Kodaira dimension, birational characterization of abelian varieties, vanishing theorem for cohomology of Q-divisors, minimal model program of algebraic varieties, adjunction theory of canonical divisors, deformation of algebraic varieties and singularities, semi-orthogonal decomposition of derived categories, and non-commutative deformation.

Kawamata was awarded the Mathematical Society of Japan Autumn Prize in 1988, and the Japan Academy of Sciences Prize in 1990 in recognition of his contributions to the minimal model program. In the same year, he was an invited speaker at the International Congress of Mathematicians. He was interviewed by Chi Tat Chong on 25 August 2023 when he participated as a Distinguished Visitor at the IMS program "Recent development in algebraic geometry" (14 August – 1 September).

IMPRINTS I Thank you for the opportunity to have a conversation. I'd like to start with your early life. How did you first get interested in mathematics?

KAWAMATA K Well, as a child, I enjoyed working with numbers, like addition and multiplication. I was intrigued by the various methods to perform calculations quickly. I attended school in Tokyo, and we had competitions for calculations and such. I remember it was around first grade, when I was about six years old, that I performed well in calculations. That's when I started to believe I had some aptitude for mathematics. I wasn't as proficient in other subjects and mathematics became the only subject I was good at. However, I didn't have any specific plans to become a professional mathematician. Essentially, mathematics remained my favorite subject throughout high school and university.

I When you entered the University of Tokyo, did you already decide to major in mathematics?

K No, not really. I liked mathematics and I spent a lot of time reading mathematics books. However, I also read books about other fields of science and literature. I enjoyed reading books in general. I was somewhat of an inactive person, so I just liked reading books at home.

I Did you have an interest in other sciences, such as physics, chemistry, or biology?

K Yes, I did. I also explored physics and even topics related to DNA during that period when they were gaining attention. However, I discovered that physics was too difficult for me. In my opinion, one can learn mathematics by only reading books, but one cannot learn subjects like physics from books alone. Since I could only rely on books for my education, mathematics naturally became more appealing to me.

I Were you interested in experiments in physics?

K I wasn't. During that period, a lot of people enjoyed building things like radios and basic computers. Many of these people were also good at mathematics. However, I wasn't skilled when it came to experiments.

I Can you recall any particular mathematics books you read that caught your interest?

K Initially, I learned from school textbooks. Interestingly, my older brother, who is seven years older than me, is a very organized person. He neatly arranged the books on the shelf, making it easy for me to borrow them. I started learning mathematics on my own. Later, when I was in high school, I read more advanced texts, and the most impressive book at that time was Lev Pontryagin's *Continuous Groups*. It had been translated into Japanese by Nagayoshi Iwahori, and I read the Japanese version.

I So at that time you were already conversant with analysis and algebra?

K Actually, I might not have understood everything in the book, but I was drawn to the introductory section. In the introduction, he discussed topology, defining what topology is and explaining concepts like neighborhoods and so on. And Pontryagin was blind. So he was explaining things that he couldn't see, in a completely abstract way. I was really attracted to his approach to defining topology. The way he defined topology is a little bit different from the Bourbaki definition. After that I read Bourbaki and all that during my university years. The Bourbaki definition is very symmetric and very elegant. Pontryagin's definition is more down to earth.

I When you went to the University of Tokyo as an undergraduate, were you already well prepared in mathematics?

K In some sense, yes. I skipped most of the lectures at the university because I was already familiar with introductory materials like calculus and linear algebra. Instead, I read more advanced books. But then it became more and more difficult to study alone. The more advanced courses began in the first semester of my second year. During that time, I was still at the junior level on the Komaba campus and professors from the main Hongo Campus came to teach us. It was quite a shock to me. They were really smart and knowledgeable. Until then, I had only been reading books written by Japanese mathematicians or translated books in Japanese, so my exposure to mathematics had been somewhat indirect. But from those courses, I saw real mathematics for the first time.

I In those days, was the curriculum flexible enough for a second year student to take more advanced courses?

K No, it wasn't. The curriculum was fixed. Everyone started to learn the same material until the fourth semester. There were about 50 students majoring in mathematics, although enrolment in the university was around 3000.

I spent the first two years studying at the College of General Education, at the Komaba campus, and then, in the third year at the Faculty of Science at the Hongo Campus. I liked the Komaba campus more than the Hongo campus. Hongo Campus feels like a company headquarter while the Komaba campus is like a park.

I When did you decide to stay for your Master and PhD?

K First, I made the decision to become a mathematician, which was the initial step. This decision was made in the fourth semester. So, I chose to pursue a career in mathematics, and then in my fourth year, I had to select a research advisor. That's when I chose Shigeru Iitaka.

I How old was Iitaka at that time?

K Iitaka was about 10 years older than me. I was around 21, and he was about 31. He was a young lecturer back then, not yet a full professor. He was highly energetic, quite the opposite of me. I believe I was drawn to his personality. He held strong views on mathematics, on the directions to be pursued, and so on. I think that's what led me to choose to work under his guidance.

I I believe I never met Professor Iitaka. In 1982, there was a conference in Singapore sponsored by JSPS[1] where 10 senior Japanese mathematicians came to Singapore. Iitaka was not on the delegation.

K He looked like a student, even looking younger than I back then. I believe he still looks younger than me now.

I When you started working on your PhD thesis, did you already choose a problem in the minimal model program?

K No, I did not. At that time, Kunihiko Kodaira was of course highly influential. When I first arrived at the Hongo campus, he was still a professor there. However, when I entered graduate school two years later, he was around 60 years old and had just retired and was relocating to Gakushuin University. During that time, the main focus was on the Master's program and I had to write a Master's thesis. My doctorate was somewhat of an addition. Nowadays, the PhD is the primary focus.

After completing my Master's thesis, I went to Germany, to the University of Mannheim where I did my PhD (called Doctor of Sciences at that time). My mentor was Herbert Popp while my adviser was still Iitaka. I believe Popp was German but was heavily influenced by Grothendieck. Although he wasn't one of Grothendieck's students, he had met him and had authored a book on the fundamental group [P] in the style of Grothendieck. During my time in Mannheim, there was also Eckart Viehweg who became my long-time friend.

I Was Viehweg a young mathematician back then?

K He was about five years older than I and was an assistant to Popp. And I was the second assistant to Popp. We were more like colleagues at that time. We discussed mathematics a lot.

I How did you decide to go to Germany for the PhD?

K Initially, after completing my Master's thesis, I worked as an assistant and was employed at the University of Tokyo. Then Popp visited Japan. He enjoyed Japan very much, particularly the Japanese martial arts Aikido, which he practiced during his time in Japan. He made frequent trips to Japan and extended invitations to Japanese mathematicians to visit Germany. I believe the first person from Japan to go to Mannheim was Kenji Ueno, followed by Yukihiko Namikawa, Takao Fujita... Later, Popp was looking for a new person to invite for a two-year visit. It was somewhat like a postdoc position, but without the requirement of a doctorate degree, and he invited me.

I started working on my PhD thesis when I was in Germany, but initially I worked on something different. My Master's thesis, which I wrote while in Tokyo, focused on the problem of additivity of the Kodaira dimension, which is called the Iitaka conjecture. Viehweg was the first to make a breakthrough in this direction. He used moduli space of curves to establish additivity of the Kodaira dimension for the fiber space version, and I extended it to the logarithmic version, so curves with boundaries. That served as my Master's thesis.

I At that time you had not met Viehweg yet?

K I hadn't met him in person yet, but I was familiar with Viehweg's name, thanks to his papers. I wanted to extend Kodaira's classification of surfaces to the logarithmic case with boundaries, and that became my second paper. That was the beginning of my work in the minimal model program. I extended the minimal model theory to the logarithmic case, and I found out that there were singularities. In the case of smooth surfaces, there are no singularities in the minimal model, but I realized that singularities do exist in the logarithmic case, which is now known as KLT singularity (Kawamata Log Terminal singularity). During that time, I obtained very primitive results in dimension two. This was my first paper [K1] after arriving in Mannheim (for [K1], I was much inspired by a paper by Zariski [Z], to which [Mu] is an appendix). And after that I worked on a different subject, which was about birational characterization of abelian varieties. This conjecture was proposed by Ueno, and I managed to solve it [K2].

The main point of the paper was the additivity of the Kodaira dimension. And that was proved by using the positivity of Hodge bundles. While Viehweg used the moduli space for curves, I replaced it with positivity. Now that we have the minimal model theory, there is a good theory of moduli spaces. So now there is a different approach.

Viehweg [V] and I [K2] proved the vanishing theorem, which was around 1980. I think this was my first paper after I returned to Japan. The vanishing theorem was somewhat of a small technical result. Our field has a program called classification of algebraic varieties, and a primary focus was on the study of fiber spaces. A fiber space is a family of varieties with degenerations. The main conjecture during that period revolved around the additivity of the Kodaira dimension. There were also positivity theorems and various topics related to moduli spaces.

I Did you discuss much with Iitaka, or did you just work on your own?

[1] Japan Society for the Promotion of Science

K I mostly work on my own, but I believe I was impressed by Iitaka's strong opinions. I recall Iitaka stating his belief that logarithmic surfaces are of dimension 2.5. Kodaira had studied dimension two, and the next step was dimension three. But dimension three is extremely difficult. So, as a first step, one should study 2.5. And actually this worked out well. In order to study this, I developed the logarithmic theory (of minimal models), which later became a core point in the minimal model program. It enables induction on dimension to be introduced. The minimal model theory now encompasses arbitrary dimensions. Induction sounds very natural but in reality working with higher dimension is very complicated and the logarithmic terminology helps a lot.

I Why do you think in minimal model theory dimension two was much simpler than higher dimensions? I mean, the Italian school did basically everything for dimension two. And once you go to a higher dimension, it becomes very tough. What's the key difference?

K It's simply a matter of dimension. So, in the two-dimensional case, one can only construct curves. In dimension three, one can construct either curves or surfaces. Now, if I construct a surface, it is very similar: it has co-dimension one. But if I construct only curves, then it becomes more challenging and requires completely new ideas. This is an example of how difficult it is to go up in dimension.

I Was Shigefumi Mori also working on this topic at that time?

K Yes, he introduced the theory of extremal rays. He proved that the first step contraction is possible. So, if the variety is not minimal, then one should contract it to a smaller variety. This is the minimal model program. Mori proved the first step—from a smooth threefold contracting a divisor to a point or a curve. But in that case, only co-dimension one can be contractible. No curve contraction occurs. With Mori contraction, a singularity appears, and then after that curve contraction appears, and it gets deeper and deeper…

I When did you first meet Mori?

K Mori was in Kyoto, and he is two years my senior. In 1976, I was a speaker in a large symposium organized by Masayoshi Nagata in Kyoto. Several distinguished mathematicians such as David Mumford, Philip Griffiths and Heisuke Hironaka were invited speakers at the conference. Shigeru Mukai and Mori were also there. That was a large conference for its time. Organizing big conferences posed challenges back then due to limited funding. Nowadays, there are big conference happening regularly, but during that period, such large-scale conferences were very rare.

I Where did the funding for the conference come from?

K From Toyosaburo Taniguchi. He was a textile industrialist who made generous donations. He appreciated mathematics and supported this conference as well as other subjects in mathematics.

I Coming back to the minimal model program, what do you think were the milestones of its development?

K There were significant developments in the 80s—contraction, extremal rays and so on had been completed. But a missing point was the existence of flips and termination of flips. And these two conjectures were too difficult in the 80s. Mori [Mo] solved it for the dimension three case. But his method did not work for higher dimension. Much later, in the 90s, Christopher Hacon and James McKernan [HM1, HM2] introduced a new idea, building on Vyacheslav Shokurov's ideas [S1, S2]. They proved the existence of flips. Subsequently, in the so-called BCHM paper [BCHM], along with Caucher Birkar and Paolo Cascini, they proved termination of flips in certain cases. That completed the program in the case of general type for arbitrary dimension. Now, there is still the remaining conjecture that it is only possible in the case of positive Kodaira dimension. Termination of flips is not completely solved except for some special cases. There is also another big conjecture called the "abundance conjecture". The minimal model only gives a numerical condition on the canonical divisor, which corresponds to curvature. But no geometric structure is known. The abundance conjecture says that geometrically one can construct a structure, something like the Kodaira elliptical surface case. So, in Kodaira's classification, there are surfaces of general type, elliptic surfaces, K3 surfaces and so on. Things like this are not completed yet. Only in dimension three is there a proof combining the results of Yoichi Miyaoka [Mi] and me [K8].

Birkar proved the so-called BAB conjecture. That is a kind of boundedness of each class. Now we have a rough classification according to the Kodaira dimension. And that opened the theory of moduli spaces. Now, we have moduli spaces for many varieties of arbitrary dimension. So the study of boundedness and moduli spaces is near completion.

I In recent years, your interests have shifted towards derived categories. How is it connected to the minimal model program?

K In a way, this is the finer structure. With numerous new researchers working in that area, it was becoming increasingly challenging to compete with them, which prompted me to shift my focus slightly. But as I was saying, it can be considered the finer structure. For example, these days I am spending more time studying three-dimensional objects. Three-dimensional varieties were very popular in the 80s. Now, by using derived categories and non-commutative geometry, I can investigate three-fold in a finer way.

I In your IMS lecture[2], you talked about deformation theory. It's a non-commutative case, right? How does it relate to the study of minimal models?

K They are connected indirectly. However, my work is inherently linked to it because my expertise primarily revolves around the minimal model program and birational geometry. Most of my work is related to this. Naturally, there are researchers, algebraists who have been working in non-commutative algebra for many years and possess a deeper understanding of the subject. I might be weaker in that aspect, but my perspective is different. In that sense, I believe that I can contribute.

I Among your achievements, which are the ones that you like most?

K Possibly the basepoint-free theorem [K7]. It's a core method within the minimal model program. So this gives me contraction. If a variety is not minimal, one needs to contract something. But contraction, algebraically, is quite a nontrivial statement. Topologically contraction is trivial, but algebraically the variety can be very rigid and resistant to contraction. To contract a variety, one needs to prove something algebraic. And this is the basepoint-free theorem. I discovered the proof during my time as a Miller fellow at Berkeley, which was around 1982.

I Did you begin working on the problem only after your arrival in Berkeley, or had you been thinking about it for a while? What was the inspiration that led to the solution?

K The finite generation of the canonical ring was the fundamental question in algebraic geometry. Mumford proved it in the case of dimension two (for general type surfaces) [Mu]. In dimension 2.5 (log surfaces) I proved it in the first paper in Mannheim [K1]. Indeed, when I explained the idea of minimal models of log surfaces to Popp, he asked me whether the finite generation follows (and it does). He was also a person with strong opinion (like Iitaka) and asked only important questions. So finite generation problem in dimension 3 was the next problem which was longstanding. My first paper in Berkeley [K5] gives an affirmative answer to the question of minimal models of general type three-folds.

I worked on this problem in Berkeley intensively. I read a paper by Bombieri [Bo] on pluri-canonical systems of general type surfaces, and somehow it helped. [K5] was my first result in Berkeley. The technique established in that paper became the base-point free theorem in [K7] and [K6].

I Was there anyone at Berkeley at that time with whom you interacted mathematically?

K Not really. All my life I work mostly alone. But Robin Hartshorne was my sponsor, and Shoshichi Kobayashi was also there. The basepoint-free theorem was an application of the Vanishing Theorem which was perhaps obtained the previous year. The Vanishing Theorem is very technical and an easy generalization of Kodaira's Vanishing Theorem. But how to apply it to higher dimension was the point.

A divisor is a basic technique in algebraic geometry. One can consider a line bundle attached to the divisor and consider the space of sections—global sections. And then, how many sections exist, and so on. This is a very basic problem in algebraic geometry. To find sections, I used the vanishing theorem. Because the higher cohomology vanishes, it is natural to look for a special section or something of that nature. But in practice, it is challenging to reach a geometric conclusion and I developed a theory for Q-divisors.

I Was it presented in a published paper?

K I submitted the first paper to the *American Journal of Mathematics*. It took a very long time to be reviewed. I believe the idea was rather novel, and initially, it seemed that they didn't immediately understand the significance of the results. Subsequently, I wrote a paper in the *Annals* [K7] which essentially presented an easy consequence of the techniques I had developed in the first paper.

I Why did you not submit your first paper to the *Annals*?

K Well, the conclusion of the second paper looked better. The paper published in the *American Journal*, while it was the core, didn't contain particularly surprising conclusions. The paper in the *Annals,* on the other hand, was published relatively quickly, and the one in the *American Journal* came out later.

[2] *Recent developments in Algebraic geometry*, Institute for Mathematical Sciences, National University of Singapore, 14 August—1 September 2023.
https://ims.nus.edu.sg/events/recent-developments-in-algebraic-geometry-arithmetic-and-dynamics2/

I There is this conjecture called the Miles Reid fantasy about moduli space of the Calabi-Yau manifold. Do you believe this conjecture to be true?

K I would like to believe this conjecture. I'm always on the optimistic side. For example, at the beginning of the minimal model program, many people were skeptical and believed that minimum models did not exist. I think that was the prevailing sentiment, especially in the early stages. Miles Reid was one of the people who influenced me most. He visited Japan when I was a Master's student. Although he's British, he pursued his PhD in France. At that time in the UK, there was no good advisor in algebraic geometry. So, he went to France and he studied under Pierre Deligne. After his PhD, he received a scholarship from the Royal Society and traveled to Moscow to work with Andrei Tyurin.

He then came to Japan, I believe for a year. He was a very kind person. For instance, he reviewed my Master's thesis and helped correct my English. And during the Nagata symposium I mentioned earlier, I remember that Miles was talking to Mumford a lot. He was describing the idea of a classification theory of algebraic varieties. At that time, people thought that classification was only feasible for surfaces, and higher dimension was considered impossible. Miles advocated his idea to Mumford, although Mumford might not have entirely agreed with that.

I Has Mumford also worked in the minimal model program?

K No. However, I believe he was at least interested in the results. In the appendix [Mu] to a paper of Oscar Zariski, he demonstrated the finite generation of canonical rings for surfaces, which serves as a key motivation for the minimal model program. Now it is proved in arbitrary dimension that canonical rings are finitely generated. This appendix by Mumford is the starting point for this problem. So in that sense Mumford should have had an interest in this, although he never directly worked on minimal models.

I You also worked on Iitaka's conjecture, was that a separate project?

K No, that was in fact the starting point. It revolved around the additivity of the Kodaira dimension, and it brought Viehweg's contributions in that field to our attention. Viehweg was the first to establish something non-trivial in that direction. Birkar also worked on the conjecture. In a sense the conjecture is a testing ground for many ideas and tools.

I Have you trained any PhD students in the minimal model program?

K My students have worked on various topics. I have had talented students, but they focused on different areas of research. For instance, Keiji Oguiso, who is now a professor at the University of Tokyo, was one of my former students and is present at this conference[3]. In fact, he's one of the organizers. I believe around 30 students have been supervised by me. Oguiso and Yukinobu Toda are very good. In fact many of them are quite good. But these two are quite outstanding.

I I notice that a number of mathematicians in Japan have authored books aimed at popularizing mathematics. Have you done so yourself?

K I haven't personally written such books, but some of my students have. For instance, one of my female students, Yukari Ito, who is married to Hiraku Nakajima, a professor at IPMU[4], has written such books.

I A few years ago, when I was in Tokyo, my Japanese host took me to a bookstore that primarily sold books in mathematics. I noticed numerous titles in Japanese.

K Yes, mathematics is quite popular in Japan. This is not the majority of people, but some people really like mathematics.

I If you were to write a book, for example, popularizing the minimal model program, do you think it would be of interest to them?

K I believe someone should undertake that task. I had one student Masao Morita who didn't complete graduate school, but he was very good at explaining mathematical concepts to the general public. He offers lectures on elementary topics and supports himself through tuition. He refers to himself as an independent mathematician, unaffiliated with any university, yet making a living by teaching online courses and giving popular lectures on mathematics.

I What do you think is the reason that two fields in particular—number theory and algebraic geometry, have attracted so many important Japanese mathematicians? Could this phenomenon be traced back to Teiji Takagi?

K Yes, Takagi had a significant influence on mathematics in Japan. He was probably the most

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[3] *Recent developments in Algebraic geometry*, Institute for Mathematical Sciences, National University of Singapore, 14 August—1 September 2023.
https://ims.nus.edu.sg/events/recent-developments-in-algebraic-geometry-arithmetic-and-dynamics2/

[4] Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo

influential. Many of Japan's leading mathematicians graduated from Tokyo, and Takagi was a professor in algebra there. He played a key role in establishing a tradition. Additionally, there was also Goro Shimura among several others. While number theory boasts a long standing tradition, the same cannot be said of algebraic geometry.

I When did Kodaira return to Tokyo?

K In the late 60s, during the period of student revolts. That was not good timing. He was around 50 at that time.

I Did he become a professor at the University of Tokyo?

K Yes, but he was already an associate professor before heading to the United States. I believe he held a position in Tokyo prior to the war, possibly in the physics department. Back then, there wasn't a strict separation between physics and mathematics. His father held a senior public sector role in the Nagano prefecture. When Tokyo was bombed, the entire department relocated to Nagano prefecture, and Kodaira helped a lot facilitating this evacuation. Kodaira was already a significant figure back then. During wartime, there was no communication with the outside world, and he developed his theory on Kähler manifolds. Immediately after the war, he moved to the United States.

I Where did he learn his mathematics?

K His teacher was probably Shokichi Iyanaga. But Kodaira was also kind of self-taught. He also studied in the physics department. He first entered physics as a student, and then joined the mathematics department. I believe he explored both subjects and then he went to the Institute for Advanced Study where he met Donald C. Spencer and developed deformation theory. He first went to Princeton, then Johns Hopkins and then Stanford. I met him when I was a student. He retired around the time I entered graduate school. I never took a course under him. He was very busy as Dean of the Faculty of Science. I did not receive any guidance from him.

I Would you say that the study of algebraic geometry in Japan was significantly influenced by his work?

K Yes, indeed. I read his papers and lecture notes. In fact, I studied algebraic geometry using his lecture notes as a textbook. When he returned to Japan, he delivered some excellent lectures, and his assistant transcribed them into lecture notes, handwritten in Japanese. These notes were later published by the Department of Mathematics, and I purchased and studied them. They were quite concise and straight to the point without lengthy and tedious introductions. That was very good.

I Were the handwritten manuscripts by Kodaira, Takagi, Iyanaga etc. kept in the mathematics library at the University of Tokyo?

K I think that Takagi's handwritten notes are there. The others, I am not so sure. And many theses are kept in the university library. In the old days, many people wrote their thesis in handwritten Japanese.

I Do you currently have students working under you?

K No, I'm not working with students. Tokyo University does not allow retired people to mentor students, and I am now officially retired. I hold the title of a university professor, which is an honorary lifetime title. I am employed at the university's central office. If I were to supervise students, it would have to be in an unofficial capacity. However, I believe students would not embark on such an adventure.

I Is the title of "university professor" the highest rank at Tokyo University?

K It's not the highest rank, but yes, it's an honor. There could be more than 10 in total currently, in various disciplines.

I Is any of your children a mathematician?

K None of them. I believe my oldest son is quite good in mathematics, but he didn't pursue it professionally.

I If you had the opportunity to start again, would you pursue a different career in view of recent exciting developments in science and technology such as artificial intelligence?

K I think that being a mathematician was the right choice, because I know that I am not good at other things.

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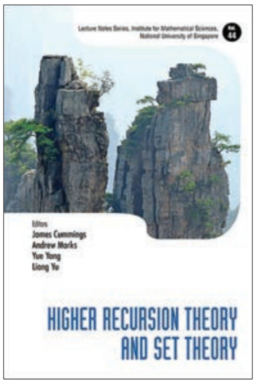

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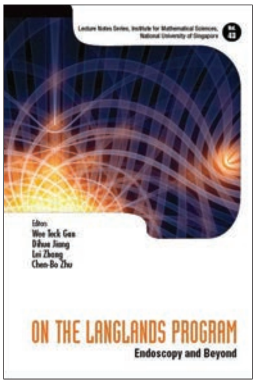

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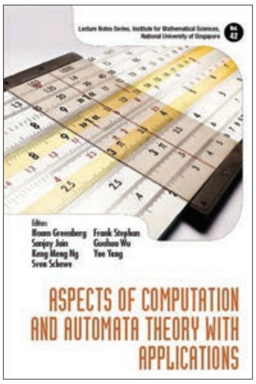

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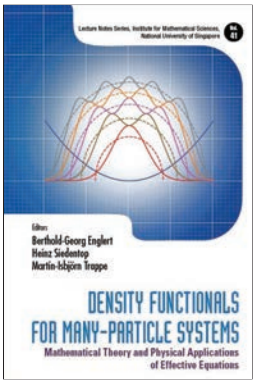

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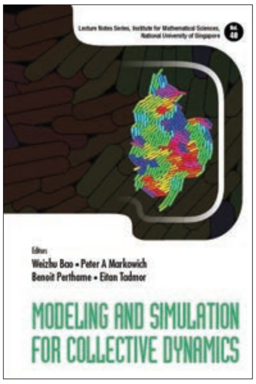

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