Arithmetic Dynamics and Diophantine Geometry

25 Aug 2025–29 Aug 2025

August 27, 2025

Abstracts

Mini Courses

Adelic Line Bundles over Quasi-projective Varieties

Ziyang Gao
University of California, Los Angeles, USA

We will give an introductory mini-course on adelic line bundles over quasi-projective varieties.

Zariski Dense Orbit Conjecture

Junyi Xie
Peking University, China

The Zariski dense orbit conjecture was proposed by Zhang, Amerik-Campana and Medvedev-Scanlon, which answers a basic question in arithmetic dynamics: When do we have a Zariski dense orbit? In general, this is still widely open but much progress has been made in this conjecture. Examples include the case of endomorphisms on projective surfaces and the case of birational maps of positive entropy in dimension 3. In this lecture, we will introduce the statement, history and the current main approach to the Zariski dense orbit conjecture.

Julia Sets, Energy Pairing and Uniform Boundedness for $z^2 + c$

Hexi Ye

Zhejiang University, China

Let $f_c(z) = z^2 + c$ be a family of quadratic polynomials—one of the most extensively studied families in complex dynamics. In this mini-course, we will introduce fundamental concepts such as Julia sets and energy pairings for this dynamical family, both in the Archimedean and non-Archimedean (p-adic) settings. We will then combine these tools to obtain an effective uniform bound on the number of common preperiodic points shared by any pair of dynamical systems within this family.

Research Talks

Elliptic Surfaces, Equidistribution, and Bifurcations

Laura DeMarco

Harvard University, USA

In joint work with Mavraki a few years ago, we studied the arithmetic intersection numbers of sections of elliptic surfaces, defined over number fields. One consequence was a Bogomolov-type extension (and new proof) of a theorem of Barroero and Capuano, addressing a case of the Zilber Pink conjectures. I will describe the underlying geometric features of this problem and formulate related open questions about families of maps (dynamical systems) on P^1.

Multiplier Spectrum and Moduli Space of Rational Maps

Zhuchao Ji

Westlake University, China

Multiplier spectrum is a natural invariant defined on the moduli space of rational maps of fixed degree. A celebrated theorem of McMullen says that multiplier spectrum determines the conjugacy class of rational maps up to finitely many choices when the flexible Lattès maps are excluded. We give a new proof of McMullen's theorem without using quasiconformal techniques. Moreover, we show that for generic rational maps, multiplier spectrum uniquely determines its conjugacy class. The talk is based on joint works with Junyi Xie.

The Story of the Ceresa Cycle

Wanlin Li

Washington University at Saint Louis, USA

The Ceresa cycle is an algebraic 1-cycles associated to a smooth algebraic curve. It is algebraically trivial for a hyperelliptic curve and non-trivial for a very general complex curve of genus >2. Given an algebraic curve, it is an interesting question to study whether the Ceresa cycle associated to it is rationally or algebraically trivial. In this talk, I will discuss some methods and tools to study this problem. Moreover, I will discuss some recent exciting developments on the study of algebraic cycles stemmed from the interest on this cycle.

Lower Bounds for Galois Orbits of Pre-periodic Points and Canonical Heights of pcf Poynomials

Harry Schmidt

University of Warwick, UK

In joint work with Philipp Habegger we prove lower bounds for the Galois orbit of pre-periodic points in terms of their period and pre-period length. These bounds hold for pcf polynomials with integral barycentre and I will demonstrate in my talk that they are too strong to hold for all polynomials. This is in fact reminiscent of the situation for Latté's maps and elliptic curves with CM. We also prove lower bounds for the canonical height of such polynomials with the additional condition that they have to be subhyperbolic. This latter part of our work, I will discuss if time permits.

Effective Equidistribution in Homogeneous Spaces and Restricted Projection Theorems

Lei Yang

National University of Singapore, Singapore

I will talk about recent developments on effective versions of Ratner's equidistribution theorem. I will explain the connection between quantitative behavior of unipotent orbits and restricted projection theorems from harmonic analysis. Based on joint work with Elon Lindenstrauss, Amir Mohammadi and Zhiren Wang.

Degenerations of Abelian Varieties by Ultrafilters

Jit Wu Yap

Harvard University, USA

Recently, Luo employed ultrafilters to construct degenerations of arbitrary sequences of rational maps on $\mathbb{P}^1$. This was later formalized by Favre-Gong in the framework of Berkovich spaces. We will explain how to adapt Favre-Gong's construction to the setting of abelian varieties and discuss some applications to uniformity problems. This is joint work with Nicole Looper.

Algebraic Families of Weakly Polarized Endomorphisms

Yugang Zhang
Paris-Saclay University, France

For a complex algebraic family of weakly polarized endomorphisms, one can define the associated canonical height function. Although the Northcott property fails in this setting, we prove that certain weaker forms of it still hold. As an application, we verify a conjecture of Kawaguchi and Silverman for a class of well-behaved automorphisms of projective varieties over a complex function field, including loxodromic automorphisms of surfaces.

Lightning Talks

When do Two Iterated Rational Functions have Finitely many Common Zeros?

Chatchai Noytaptim

University of Waterloo, Canada

In 2017, Hsia and Tucker proved—under compositional independence assumptions—that there are only finitely many irreducible factors of the greatest common divisors of two iterated polynomials with complex coefficients. In addition, Hsia and Tucker posed a question and asked whether a finiteness result of common zeros holds true for iterated rational functions with complex coefficients. In recent joint work with Xiao Zhong, we have answered the question in affirmative. In fact, the question is true except for special families of rational functions of degree one. In this lightning talk, I will highlight main tools to tackle the question.

Rational Self-maps of Complex Projective Surfaces with a Regular Iterate

Sina Saleh

Harvard University, USA

Let Φ be a rational self-map of a variety X defined over an algebraically closed field K. We say that Φ is *eventually regular* if there exists an iterate Φ^k that is regular for some $k \geq 1$. It is conjectured that if X is sufficiently "rigid" and a rational map Φ is eventually regular, then strong conclusions about the nature of Φ can be drawn. In particular, it was recently shown by Bell, Ghioca, and Reichstein that if Φ is an eventually regular rational self-map of a semiabelian variety G defined over an algebraically closed field of characteristic zero, then either Φ preserves a non-constant fibration or Φ itself is regular. In this talk, we focus on the case where X is a projective surface over the complex numbers and present analogues of the results of Bell, Ghioca, and Reichstein in this setting.

Adelic Line Bundles and Beilinson-Bloch Height

Yinchong Song

Peking University, China

We give a criterion when an hermitian line bundle on quasi-projective varieties is adelic, then we use it to study the Poincare line bundle related to Beilinson-Bloch heights.

Arithmetic Degrees are Cohomological Lyapunov Multipliers

She Yang

Peking University, China

For endomorphisms of projective varieties, we prove that the arithmetic degree of a point with Zariski dense orbit must be a cohomological Lyapunov multiplier of the dynamical system. We will apply our result to deduce a corollary towards the dynamical Mordell-Lang conjecture.

This is a joint work with Jiarui Song and Junyi Xie.

Quantitative Mordell Conjecture

Jiawei Yu

Peking University, China

The uniform Mordell conjecture, proved by Dimitrov-Gao-Habegger and Kuhne, asserts that a curve over a number field of genus $g > 1$ has at most $c_1 c_2^r$ rational points, where r is the Mordell-Weil rank and c_1, c_2 are inexplicit constants depending only on g. We make these constants explicit and confirm a conjecture that c_2 converges to 1 as g tends to infinity. This is a joint work with Xinyi Yuan and Shengxuan Zhou.

Preimages Question and Dynamical Cancellations

Xiao Zhong

University of Waterloo, Canada

Let X be a projective variety and f be a dominant endomorphism, both of which are defined over a number field K. Matsuzawa, Meng, Shibata, and Zhang proposed the preimages question which asks whether the tower of K-points

$$Y(K) \subseteq (f^{-1}(Y))(K) \subseteq (f^{-2}(Y))(K) \subseteq \cdots$$

eventually stabilizes, where $Y \subset X$ is a subvariety invariant under f. Restricting to a special setting, the preimages question becomes a dynamical cancellation question first studied by Bell, Matsuzawa and Satriano: whether there exists a positive integer s_0 such that for all $x, y \in X(K)$ satisfying $f^s(x) = f^s(y)$ for some $s \geq 0$, we necessarily have $f^{s_0}(x) = f^{s_0}(y)$. In this talk, I will introduce some progress toward these questions.
