# The 4th Australia-China-Japan-Singapore-US Index Theory Conference–Analysis and Geometry on Manifolds

04 Aug 2025–08 Aug 2025

July 25, 2025

# Abstracts

#### **Reflections on Mathai's Career**

6 Aug 11.30 am

4 Aug 9.30 am

Peter Bouwknegt The Australian National University, Australia

To be updated

# Singular Metrics of Nonnegative Ccalar Curvature and RCD

Xianzhe Dai

University of California, Santa Barbara, USA

The positive mass theorem of Schoen-Yau and Witten is one of the landmark results and it is closely related to the existence of positive scalar curvature metrics. Various motivations lead to consideration of singular metrics. I will first review these and recall some earlier work on on positive mass theorem and Geroch type theorem for singular metrics, including our work with Yukai Sun and Changliang Wang for isolated conical singularity. Then I will discuss our very recent work on singular metrics with nonnegative scalar curvature with small singularity, in particular confirming Schoen's conjecture for isolated singularity on spaces which are connected sums with the torus. Crucial to our approach is the novel connection with RCD spaces, singular spaces having some weak Ricci curvature bound. This part of the work is joint with Changliang Wang, Lihe Wang, and Guofang Wei.

#### Covering Complexity, Scalar Curvature, and Quantitative Index Theory

Hao Guo

Tsinghua University, China

I will discuss how quantitative index theory provides a link between a certain type of topological complexity of a manifold and lower bounds on positive scalar curvature. The work, which is carried out in the spin case and uses Dirac operators, involves two ingredients. The first is a pairing between two flavours of quantitative K-theory, where the filtrations are given respectively by propagation of operators and by Lipschitz control of vector bundles. The second ingredient is a vanishing theorem for the quantitative version of the higher index of Dirac operators. This is joint work with Guoliang Yu.

## Index Formula for Quarter-plane Toeplitz Operators via Extended Symbols

Shin Hayashi

Aoyama Gakuin University, Japan

We consider index theory for Toeplitz operators on a discrete quarter-plane of two-variable rational matrix function symbols. Index theory for such operators has been investigated by Simonenko, Douglas-Howe, Park and index formulas are obtained by Coburn-Douglas-Singer, Duduchava. We revisit Duduchava's idea and discuss an index formula for quarter-plane Toeplitz operators from a geometric viewpoint. By using a factorization of matrix-valued functions (called the Wiener-Hopf factorization) and analytic continuations, we see that the symbols of Fredholm quarter-plane Toeplitz operators, initially defined on a two-dimensional torus, can canonically be extended to some three-sphere, and show that their Fredholm indices coincide with the three-dimensional winding numbers of extended symbols. 7 Aug 3.30 pm

6 Aug 10.30 am

#### Noncommutative Geometry of the Satake Compactification

7 Aug 2.30 pm

Nigel Higson

The Pennsylvania State University, USA

A basic organizing principle in representation theory for real reductive groups, discovered by Harish-Chandra, is that a tempered irreducible unitary representation is either square-integrable modulo center, or embeddable in a representation that is parabolically induced from such. This is one of two foundational ideas underpinning the Plancherel formula, the other being that a real reductive group possesses square-integrable representations precisely when it possesses a compact Cartan subgroup. Square-integrable representations have been analyzed in detail from a geometric point of view, starting with the work of Parthasarathy and Atiyah-Schmid and culminating in the work of Lafforgue. Harish-Chandra's first principle has received less attention, but in my lecture I shall indicate how, by attaching a C\*-algebra to the Satake compactification of G/K, one may give a short and conceptual noncommutative-geometric proof. This is joint work with Jacob Bradd and Bob Yuncken.

## The Equivariant Fried Conjecture for Suspension Flow

7 Aug 11.30 am

#### Peter Hochs

Radboud University, Netherlands

Analytic torsion was introduced by Ray and Singer as a way to realise Reidemeister-Franz torsion analytically. (The equality was independently proved by Cheeger and Müller.) The Ruelle dynamical zeta function is a topological way to count closed curves of flows on compact manifolds. The Fried conjecture states that, for a suitable class of flows, the Ruelle dynamical zeta function has a well-defined value at zero, and that the absolute value of this value equals analytic torsion. With Hemanth Saratchandran, we defined equivariant versions of analytic torsion and of the Ruelle dynamical zeta function, which incorporate actions by possibly noncompact groups on possibly noncompact manifolds. This leads to the question under what conditions the resulting equivariant version of Fried's conjecture is true. With Chris Pirie, we have recently obtained positive results on a basic class of flows: suspension flows of isometries. This includes results in the noncompact setting.

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# Algebraic Topology of 24 Dimensional String Manifolds

Ruizhi Huang

Chinese Academy of Sciences, China

String manifolds of dimension 24 are of special interest in geometry and topology. For instance, there is the famous Hirzebruch's prize question concerning the realization of certain string manifold at dimension 24. Mahowald-Hopkins partially solved the question in homotopy theory.

In a recent work joint with Fei Han, we find representatives of an integral basis of the string cobordism group at dimension 24. As applications, we showed various Rokhlin type divisibility theorems, and confirmed a string version of Farrell-Zdravkovska-Yau conjecture at dimension 24. Our results also provide potential clues to understand a question of Teichner and Zhang.

## Amenability vs Nonamenability in Covering Geometry

Tsuyoshi Kato Kyoto University, Japan

In this talk, I will explain some extreme natures of distribution of zeros of vector fields and fixed point sets of self maps on amenable or nonamenable covering spaces.

#### Tensor Networks in Condensed Matter Physics and Subfactors 8 Aug 9.30 am

Yasuyuki Kawahigashi The University of Tokyo, Japan

We have a notion of a 4-tensor in 2-dimensional topological order in condensed matter physics recently. Mathematically, this is simply a map from the direct product of four finite sets to the complex number field with some properties. We then identify this with a bi-unitary connection that appears in subfactor theory and is a variant of quantum 6j-symbols, a generalization of a notion in representation theory of quantum groups. We then translate various notions in the two studies in both directions and have deeper understanding of their mathematical structures. 4 Aug 2.30 pm

# Stable Homotopy Theory of Invertible Quantum Spin Systems

8 Aug 10.30 am

Yosuke Kubota

Kyoto University, Japan

In the past decade, A. Kitaev proposed that the set of invertible gapped quantum spin systems would form an Omega-spectrum. This conjecture has potentially significant application to the study of SPT phases. In this talk, I explain a mathematically rigorous realization of this proposal with the language of operator algebra and coarse geometry. The relationship with index theory and K-theory is also briefly discussed. This talk is based on the preprint arxiv:2503.12618.

#### TBC

Qin Li

Southern University of Science and Technology, China

To be updated

#### Dualities among W-algebras with N=2 Supersymmetry 5

Andrew Linshaw

University of Denver, USA

I will introduce vertex algebras, which first appeared in physics in the 1980s and were axiomatized by Borcherds in his proof of the Moonshine conjecture. I will then explain some dualities among certain vertex algebras known as W-algebras which were conjectured in 2017 by the physicists Gaiotto and Rapcak, and proven in my recent work with Creutzig. Similar dualities between W-algebras with N=2 supersymmetry were conjectured by Prochazka and Rapcak in 2018, generalizing earlier conjectures of Candu and Gaberdiel in 2013 and Ito in 1991. I will discuss the proof of these conjectures and some of their applications in recent joint work with Creutzig, Kovalchuk, Song, and Suh. 5 Aug 4.30 pm

5 Aug 2.30 pm

## Small Scale Index Theorem, Scalar Curvature and Gromov's Simplicial Norm

Qiaochu Ma

Texas A&M University, USA

Scalar curvature encodes the volume information of small geodesic balls within a Riemannian manifold, making it, to some extent, the weakest curvature invariant. This raises a natural question: what topological constraints does scalar curvature impose on manifolds? In this talk, we will show that for a manifold with a scalar curvature lower bound, the simplicial norm of certain characteristic classes can be controlled by its volume and isoperimetric constant. This is joint work with Guoliang Yu.

## Generalized Positive Scalar Curvature on Spin<sup>c</sup> Manifolds

5 Aug 9.30 am

Jonathan Rosenberg University of Maryland, USA

Thanks to the work of Gromov–Lawson, Schoen–Yau, Stolz, and many others, there is a highly developed theory of which closed spin manifolds (in dimensions > 4) admit positive scalar curvature. This theory is closely linked to the geometry of the spin Dirac operator. We describe a parallel theory for Spin<sup>c</sup> manifolds, which is linked to the geometry of the Spin<sup>c</sup> Dirac operator. However, it involves not the ordinary scalar curvature, but what we call the \*generalized scalar curvature\*, which also incorporates the curvature of the Spin<sup>c</sup> line bundle.

We explain how most of the results for spin manifolds have parallels in this setting, though sometimes the proofs are rather different. This is joint work with Boris Botvinnik and Paolo Piazza.

## Automorphic Forms and the Sup Norm Problem : A Survey

Jyotirmoy Sengupta

5 Aug 11.30 am

India Association for the Cultivation of Science, India

In this talk we will review automorphic forms starting from the most classical case of holomorphic modular forms on the upper half-plane in C and the sup norm problem for them. Time permitting we will describe some preliminary results for this problem in the case of Hermitian modular forms of degree 2. The latter is part of an ongoing work with my collaborators, S. Gun and B. Paul. 4 Aug 3.30 pm

### The Large-Time Behavior of the Heat Kernel on Symmetric Spaces and Bismut's Formula

Yanli Song

Washington University in St. Louis, USA

Let G be a connected linear real reductive group with maximal compact subgroup K. In this talk, I will present an approach to studying the large-time behavior of the heat kernel on the symmetric space G/K using Bismut's formula. I will explain how Bismut's formula provides a natural link between index theory and Vogan's minimal K-type theory in representation theory, extending the work of Atiyah and Schmid on discrete series representations. I will also discuss applications of this formula to the Novikov–Shubin invariants of locally symmetric spaces. This talk is based on joint work with Shu Shen and Xiang Tang.

## Llarull's Theorem on Odd Dimensional Manifolds and Spectral Flow

Guangxiang Su

Nankai University, China

In this talk, we first review Llarull's theorem and then discuss the spectral flow proof of the odd dimensional case and its extension on noncompact manifolds. This talk is based on joint works with Yihan Li, Xiangsheng Wang and Weiping Zhang.

## Superconnection and Orbifold Chern Character

Xiang Tang Washington University in St. Louis, USA

The orbifold Chern character is a fundamental invariant for complex vector bundles on an orbifold. In this talk, we will extend this construction to coherent sheaves on complex orbifolds, following the ideas of Bismut, Block, Shen, and Wei on flat antiholomorphic superconnections. We will present a Riemann-Roch-Grothendieck theorem concerning the orbifold Chern character for orbifold embeddings. This result enables us to establish the uniqueness of the orbifold Chern character. 7 Aug 9.30 am

4 Aug 10.30 am

## Macroscopic Index and Fractionally Quantized Traces

5 Aug 3.30 pm

Guo Chuan Thiang BICMR, Peking University, China

I will present some new trace formulae which unify and generalize that appearing in Roe's partitioned manifold index theorem, and the Kubo formulae used in quantum Hall physics. Using coarse (co)homology and K-theory, we prove that these trace formulae are universally quantized and dependent only on large-scale geometric input data. Furthermore, guided by the fractional quantum Hall effect, we find that there exist "hidden" fractionally quantized trace formulae, by exploiting the principal function theory of Carey-Pincus and Helton-Howe in a new way.

#### Lie Groupoid Structures on Donaldson Moduli Spaces

Bryan Bai-Ling Wang

5 Aug 10.30 am

7 Aug 4.30 pm

#### The Australian National University, Australia

Donaldson moduli spaces of instantons on four-manifolds are fundamental in differential geometry and gauge theory, but their structure is often complicated by quotient singularities and bubbling phenomena in the compactifications. We will provide Lie groupoid structures on Donaldson moduli spaces and their bubble tree compactifications, providing a smooth and categorical framework that captures their stratified and singular nature. This construction lays the groundwork for future applications, including the definition of K-theoretical Donaldson invariants, the geometric realization of  $\mu$ -maps. Some applications will be discussed. This is based on joint work with Bohui Chen and Shuauge Qiao.

## Euler Characteristic, Equivariant K-homology and Higher Kazhdan Projections

Hang Wang

East China Normal University, China

We compute the equivariant K-homology class of the de Rham operator using Witten deformation, leading to an explicit formula for the Euler characteristic and an equivariant Poincaré–Hopf theorem. For virtually free groups, we show that the combinatorial Euler characteristic maps under the Baum–Connes assembly to the K-theory class of higher Kazhdan projections, which decompose as alternating sums of averaging projections over finite subgroups. This yields concrete computations and non-vanishing results for delocalized  $\ell^2$ -Betti numbers. This is joint work with Hongzhi Liu, Shaocong Xiang, Zijing Wang and with Sanaz Pooya, Baiying Ren.

### Coarse Baum–Connes Conjecture for $l^q$ Coarsely Embeddable Spaces

Jinmin Wang

Chinese Academy of Sciences, China

Roe's algebra is C\*-algebra associated to a proper metric space, which reflects its large scale features. The coarse Baum–Connes conjecture provides a framework to compute the K-theory of Roe's algebra by showing that the coarse assembly map is an isomorphism. In 2015, Chen–Wang–Yu proved the injectivity of the coarse assembly map for spaces that coarsely embed into  $l^q$ . In this talk, we will address the surjectivity for such  $l^q$  coarsely embeddable spaces. Our strategy is to construct a Bott–Dirac operator with an  $l^q$  potential on finite dimensional Euclidean space. This approach also leads to an  $l^p$ -version of the coarse Baum–Connes isomorphism for  $l^q$ coarsely embeddable spaces. The talk is based on joint work with Zhizhang Xie, Guoliang Yu, and Bo Zhu.

## Quasi-representations and a K-theoretic Invariant

Jianchao Wu

Fudan University, China

Generalizing the intriguing phenomenon of Voiculescu's almost commuting unitary matrices is the notion of a quasi-representation of a (discrete) group. As demonstrated in the work of Exel and Loring on Voiculescu's example, there may be topological obstructions to perturbing quasi-representations into genuine representations—this is where (topological or operator) K-theory enters the picture. Previous studies have mostly focused on fundamental groups of finite CW complexes. In this talk, we introduce the notion of a character as a more general and refined invariant for quasi-representations of discrete groups. The talk is based on a joint project with Shmuel Weinberger and Guoliang Yu. 8 Aug 3 pm

8 Aug 11.30 am

#### On a Family of Non-formal Star Products of Entire Functions

6 Aug 9.30 am

Siye Wu

National Tsing Hua University, Hsinchu

In deformation quantisation, star product is normally defined as a formal power series. When it is convergent, the star product is called non-formal. We show that for entire functions belonging to various Banach, Fréchet and more general spaces, it is possible to define a family non-formal star products that includes normal ordering with respect to certain complex structures. In the latter case, the star representation gives rise to a family of quantum Hilbert spaces that form a projectively flat vector bundle. This is a joint work with A. Yoshioka.

## On Gromov's Dihedral Rigidity Conjecture of Scalar Curvature 8 Aug

4 pm

#### Zhizhang Xie

#### Texas A&M University, USA

In this talk, I will present my joint work with Jinmin Wang and Guoliang Yu on a new index theorem for manifolds with singularities, such as manifolds with corners and, more generally, manifolds with polyhedral-type boundary. As an application, we obtained a positive solution to Gromov's dihedral rigidity conjecture. This conjecture concerns comparisons of scalar curvature, mean curvature and dihedral angles for compact manifolds with polyhedral-type boundary, and has very interesting implications in geometry and mathematical physics. Further developments of this new index theorem have led us to a positive solution of Gromov's flat corner domination conjecture. As a consequence, we answered positively a long standing conjecture in discrete geometry — the Stoker conjecture.

## Degenerations of Algebraic Manifolds and Analytic Torsions of Nakano Semi-positive Vector Bundles

4 Aug 11.30 am

Ken-Ichi Yoshikawa

Kyoto University, Japan

For a degeneration of projective algebraic manifolds over a Riemann surface and a Nakano semi-positive vector bundle twisted by the relative canonical bundle on the total space of the family, we study the asymptotic behavior of analytic torsion of the fibers as a function on the base curve. Our main result is that the logarithm of the analytic torsion admits an asymptotic expansion, whose dominant term is given by the logarithmic singularity and whose subdominant term is given by the log-log singularity. The coefficient of the logarithmic divergence is given by an integral of certain characteristic classes associated with the semi-stable reduction of the family. On the other hand, the coefficient of the log-log-singularity is more subtle. When the singular fiber has at most isolated singularities, then, up to a nowhere vanishing continuous function, the logarithm of the ratio of two analytic torsions (one for the trivial line bundle and the other for an ample line bundle) is expressed as the ratio of the determinants of certain period integrals and has at most a log-log-singularity. We conjecture that the ratio of these analytic torsions is comparable to the product of all small eigenvalues of the Laplacian acting on the canonical forms. Assuming this conjecture is true, we would like to explain that our formula for the ratio of two analytic torsions may be viewed as a multiplicative analogue of the McKean-Singer formula in a certain sense. If time permits, we will present an evidence supporting this conjecture.

# Nonpositively Curved and Infinite Dimensional Spaces and the Novikov Conjecture

8 Aug 2 pm

Guoliang Yu Texas A&M University, USA

In this talk, I will explore how the geometry and analysis of nonpositively curved, infinite-dimensional spaces can be applied to investigate the Novikov conjecture for diffeomorphism groups. I will aim to present the material in a way that is accessible to graduate students.