Workshop on Applications of Model Theory in Complex Geometry and Differential Algebra

7–11 July 2025

July 4, 2025

Abstracts

Projective Curves and Weak Second-order Logic

10 July
1.30 pm
IMS ESR

Alessandro Berarducci
University of Pisa, Italy

This is joint work with Francesco Gallinaro. Given an algebraically closed field K of characteristic zero, we study the poset Pos(K) of irreducible proper algebraic subsets (points and curves) of the projective plane over K. Answering a question of Marcus Tressl, we prove that the poset interprets the field K and the subring of integers, and it is in fact bi-interpretable with the two-sorted structure (K, Fin(K)) consisting of the field K and a sort for its finite subsets. We prove that the theory of the poset depends on the transcendence degree of K, so for instance the complex numbers and the algebraic numbers have non-elementary-equivalent posets. Taking for K the complex numbers, we give a complete recursive axiomatization of (K, Fin(K)) modulo the theory of the integers. We also show that the integers are stably embedded in this structure.

Stable D-finite Functions

7 July
5 pm
S17-04-05

Shaoshi Chen
Chinese Academic of Sciences, China

In 2022, the speaker initialized a dynamical aspect of symbolic integration by studying stability problems in differential fields and characterized certain classes of stable elementary functions. In this talk, we will continue the dynamical studies of symbolic integration by focusing on the stability problems on D-finite functions. We introduce the notion of stability index in order to investigate the order growth of the differential operators satisfied by iterated integrals of D-finite functions and determine bounds and exact formulae for stability indices of several special classes of differential operators. With the basic properties of stability index, we completely solve the stability problem on general hyperexponential functions.

Definable Rings, Associative Algebras, and O-minimality

11 July
4 pm
IMS ESR

Analisa Conversano
Massey University, New Zealand

We show that in an o-minimal structure every non-null definably connected ring defines an infinite field, and it is, essentially, a product of finite-dimensional associative algebras over non-isomorphic definable real closed fields. Several definability issues will be discussed.

Images of Algebraic Sets under Lattice Quotients

9 July
4 pm
S17-04-05

Spencer Dembner
Stanford University, USA

Let $X \subset \mathbb{C}^n$ be an algebraic variety, and let Λ be a discrete subgroup. Then we are interested in characterizing the topological closure of X in $\mathbb{C}^n/\Lambda$. If Λ is cocompact and the quotient is a complex torus, then a theorem of Peterzil-Starchenko expresses the closure in terms of families of translated subtori. We show that a similar result holds without cocompactness, as long as Λ satisfies the weaker assumption that its real and complex spans agree. In addition, we show a similar result for sets definable in the real numbers. This is joint work with Hunter Spink.

Definable $\mathcal{C}^r$ Structures on Definable Topological Groups: d-minimal Case

10 July
2.30 pm
IMS ESR

Masato Fujita
Japan Coast Guard Academy, Japan

In 1988, Pillay proved that a group definable in an o-minimal structure can be equipped with a definable topology under which multiplication and inversion are continuous. In 2012, Wencel generalized Pillay's result to the case where the structure has a dimension function satisfying several conditions proposed by van den Dries in 1989 and the additional condition called continuity property. For instance, definably complete locally o-minimal structures satisfy these conditions.

In this talk, we consider d-minimal structures in which a standard dimension function may fail to satisfy the continuity property. The speaker proposed a new measure called partition degree, which enjoys a property like the continuity property. Using this, he showed that a definable topological group has a definable $\mathcal{C}^r$ structure. In this talk, the speaker will introduce basic properties of partition degrees and the strategies of their proofs.

Likely Intersections in Powers of the Multiplicative Group

11 July
2.30 pm
IMS ESR

Francesco Gallinaro
University of Pisa, Italy

Given an algebraic subvariety W of some power G of the multiplicative group over an algebraically closed field, and an algebraic subtorus H of G of dimension at least equal to the codimension of W, what can we say about the intersections between W and the cosets of H? We show that if W satisfies the condition of geometrical non-degeneracy, which broadly means that intersections of W with cosets of algebraic subtori are "generically typical", then there is a finite set F of subtori such that if H is not contained in any subtorus from F then W intersects every coset of H. We use an approach based on non-standard analysis and tropical geometry. This is joint work with Gabriel Dill.

Forking in Differential Fields of Positive Characteristic

7 July
4 pm
S17-04-05

Piotr Kowalski
University of Wrocław, Poland

I will report on joint work in progress with Omar León Sánchez and Amador Martin-Pizarro. Let DCF_p be the model companion of the theory of differential fields of characteristic p>0. Shelah proved in 1973 that the theory DCF_p is stable. We give an algebraic description of the forking independence relation in DCF_p by introducing a new differential algebraic notion of differential "transcendental imperfectness". We also show that types in DCF_p over algebraically closed sets (in the home sort) are stationary and give an example of a differential equation whose solution set is strictly disintegrated; that is, it has the induced structure of a pure set.

Minimality of Difference-differential Equations

7 July
2.30 pm
IMS ESR

Wei Li

Chinese Academy of Sciences, China

We analyze the behavior of systems of algebraic differential equations when considered as systems of difference-differential equations, with special emphasis on systems which define strongly minimal sets relative to the theory $\mathrm{DCF}_{0,n}$ of differentially closed fields of characteristic zero with n distinguished commuting derivations. We show that if X is a strongly minimal set relative to $\mathrm{DCF}_{0,n}$ defined by a finite system of algebraic partial differential equations and the forking geometry on X is trivial (also called "disintegrated"), then X remains minimal when regarded as definable set relative to the theory $\mathrm{DCFA}_{0,n}$ of difference-differentially closed fields of characteristic zero with n commuting derivations. We illustrate this theorem by describing the possible difference-differential equations consistent with differential equations of the form $y' = f(y)$ where f is a monic cubic polyomial over differential constants. This is a report on joint work with Thomas Scanlon.

Sharply o-minimal Structures

11 July
1.30 pm
IMS ESR

Dmitry Novikov

Weizmann institute of Science, Israel

Sharply o-minimal structures are a subclass of o-minimal structures characterized by more rigorous, quantitative finiteness properties. In this discussion, I will define these structures and explore recent advancements and applications of this theory.

Definable Groups in Geometric Fields with a Generic Derivation

7 July
1.30 pm
IMS ESR

Anand Pillay

University of Notre Dame,USA

We prove that a definable group Gamma definably embeds in a group G definable in the underlying field (generalizing the old theorem that a differential algebraic group differential algebraically embeds in an algebraic group). The case of Gamma finite-dimensional was done by Peterzil, Pillay, Point, giving also a closer connection between Gamma and G. The infinite-dimensional makes heavy use of results in Hrushovski-Rideau-Kikuchi's Metastable groups, Valued fields.

This is joint work with Silvain Rideau-Kikuchi and Francoise Point.

Semi-constant Families of Semi-abelian Varieties

9 July
2.30 pm
IMS ESR

Harry Schmidt
University of Warwick, UK

I will report on joint work with Gareth Jones in which we prove an effective version of the Zilber-Pink conjecture in Poincaré bi-extensions of elliptic curves.

Equidistribution of Polynomially Bounded o-minimal Curves in Homogeneous Spaces

9 July
1.30 pm
IMS ESR

Nimish A. Shah
The Ohio State University, USA

We show that on a finite volume homogeneous space of a linear algebraic group G, any trajectory of a "non-contracting" curve in G definable in a polynomially bounded o-minimal structure gets equidistributed with respect to a homogeneous measure. This extends the speaker's earlier work on polynomial trajectories, generalizing Marina Ratner's work on unipotent flows.

A crucial new ingredient in the proof is demonstrating that such curves satisfy the (C,alpha)-good property, which is akin to the Remez inequality.

This is a joint work with Michael Bersudsky and Hao Xing.

Continuous Selection Problems in Tame Expansions of Real Closed Fields

10 July
5 pm
S17-04-05

Athipat Thamrongthanayalak
Chulalongkorn University, Thailand

Let X and Y be sets. A set-valued map from X to Y is a map from X to the power set of Y. For a set-valued map T, we say that a function $f\colon X \to Y$ is a selection of T if $f(x) \in T(x)$ for every $x \in X$. In 1956, E. Michael presented a sufficient condition on the existence of continuous selections of set-valued maps. The given construction involves an infinite iterated procedure which makes the selection far removed from the set-valued map arose. This gives rise to the question: "If T is well-behaved in some prescribed sense, can we find a sufficient condition that guarantees the existence of continuous selections that are similarly well behaved?" Here, we study the above problem in tame expansions of real closed fields. More precisely, let $\mathfrak{R}$ be an expansion of a real closed field R. We may equip R with the order topology. Let T be a

set-valued from a closed subset of R^m to R^n. The main question is that "what is a sufficient condition that guarantees the existence of a continuous selection of T that is definable in the expansion $(\mathfrak{R}, T)$?" In this talk, we will discuss findings on this problem.

Transseries and Hardy Fields

8 July
1.30 pm
IMS ESR

Lou van den Dries
The University of Illinois Urbana-Champaign, USA

This will be a survey of our work on these topics. No familiarity with transseries or Hardy fields is required, but I mention here that both are natural settings for solving ordinary differential equations and studying their asymptotics (and for much more). Transseries are formal objects, and Hardy fields consist of actual functions (or rather germs of such). Around 2015 we showed that the differential field of transseries has a good model theory. We went on to prove that sufficiently rich Hardy fields have the same (complete) model theory, with interesting consequences even on the level of linear differential equations. More recently we extended this even to *analytic* Hardy fields. Some of the ideas involved go back to Sturm and Liouville (1830's).

This is joint work with Aschenbrenner and Van der Hoeven.

Tameness, Complexity, and the Space of Quantum Field Theories

9 July
5 pm
S17-04-05

Mick van Vliet
Utrecht University, Netherlands

Recently, model theory has found novel applications in physics: it has been proposed that tameness could be a physical principle, suggesting that functions arising in physical theories should be definable in a sharply o-minimal structure. The definable sets in these structures admit an effective measure of complexity which is consistent with logical operations. In this talk, I will review these developments, focusing on applications to the space of quantum field theories. The emphasis will be on connections to quantum gravity, where finiteness plays a central role. Along the way, I will highlight several mathematical questions which arise from this perspective.

Trace Definability

Erik Walsberg

10 July
1.30 pm
IMS ESR

I will discuss trace definability, with a focus on examples arising from differential algebra.
