Tutorial on applications of model theory in combinatorics and valued fields
16–20 June 2025

Abstracts

Abstracts are updated as of 19 June 2025

Franziska Jahnke
University of Münster, Germany

Model-theoretic Transfer Principles in Valued Fields

How similar are the p-adic numbers and the Laurent series over a finite field? Or more generally, how much arithmetic can be transferred between two (henselian) valued fields with the same residue field and value group? The aim of this course is to give an introduction to Ax–Kochen/Ershov principles and their applications to non-archimedean geometry. An AKE principle allows us to reduce questions about certain henselian valued fields to corresponding questions about their residue fields and value groups. The classical Ax–Kochen/Ershov Theorem gives an isomorphism between ultralimits of p-adic fields and Laurent series fields Fp((t)) (assuming CH), and was applied to resolve Artin's Conjecture "asymptotically". We will survey and prove various AKE type results, focusing in particular on perfectoid fields. Here, tilting allows to transfer results between certain henselian fields of mixed characteristic and their positive characteristic counterparts and vice versa. We present a model-theoretic approach to perfectoid fields, showing that the preservation of first-order properties via tilting can also be understood in terms of an AK/E principle. In particular, this yields simple proofs of the Almost Purity Theorem for valuation rings and the Fontaine-Wintenberger Theorem.

Back to Contents Page

François Loeser
Sorbonne University, France

Distinguished Visitor Lecture Series
Pseudo-finite Fields, Motives and Integrals

Pseudo-finite fields were introduced by J. Ax as a first-order axiomatization of finite fields. Together with J. Denef, we assigned to a definable set over a pseudo-finite field a geometric object, called a virtual Chow motive, that encapsulates its number of points. This device can be used to construct a motivic measure for definable sets over non-archimedean local fields for which a generalization of the classical AxKochen Theorem holds. More generally, together with R. Cluckers we proved a general transfer principle allowing to transfer identities between functions defined by integrals over non-archimedean local fields of characteristic zero to local fields of positive characteristic and vice versa. As we proved together with R. Cluckers and T. Hales, this principle can be for instance applied to the Fundamental Lemma of Langlands-Shelstad, an identity between orbital integrals whose proof by Ngô achieved considerable fame. More recently we have been able with A. Forey and D. Wyss to use some of these methods to obtain a motivic enhancement of this identity.

Back to Contents Page

Simon Machado
ETH Zürich, Switzerland

Model Theory and Approximate Algebraic Structures

This mini-course will explore how ideas from logic—particularly model theory— can provide powerful tools for tackling key problems in additive combinatorics.

Additive combinatorics investigates “approximate” algebraic substructures within groups, rings, and varieties. Over the past three decades, it has grown into a vibrant and central area of mathematics, with deep connections to number theory, graph theory, harmonic analysis, dynamics, and beyond.

While additive combinatorics is typically finitary and quantitative, model theory brings a qualitative, structural lens to mathematics. Remarkably, techniques from model theory—especially those involving pseudo-finite structures and stability—have led to some of the most striking results in additive combinatorics. In fact, certain foundational theorems in the field are known only through model-theoretic proofs.

We will focus on a few core examples, with particular emphasis on the structure of approximate subgroups as developed by Breuillard–Green–Tao and Hrushovski. We will also touch on incidence geometry, almost-representations, and Brunn—Minkowski-type inequalities in $SO_3(\mathbb{R})$. Rather than focusing on a fixed set of technical tools, the course will aim to develop the structural connections between model theory and additive combinatorics, showing how deep insights can emerge at their intersection.

Back to Contents Page