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# Shilpak Banerjee Indian Institute of Technology Tirupati, India

(Anti-)classification Results in Dynamical Systems and Ergodic Theory

A fundamental theme in dynamics is the classification of systems up to appropriate equivalence relations. For instance, the equivalence relation of topological conjugacy preserves the qualitative behaviour of topological dynamical systems. Smale's celebrated program proposes to classify topological or smooth dynamical systems up to topological conjugacy. In Ergodic Theory the isomorphism problem dates back to von Neumann's foundational paper and asks to classify measure-preserving transformations up to measure isomorphism. These classification problems not only turn out to be hard but sometimes even to be impossible. In this talk, we give an overview of classification as well as (anti-)classification results and present some related projects.

#### Jianyu Chen *Soochow University, China*

Length Spectrum Rigidity for Piecewise Analytic Bunimovich Stadia

Motivated by the famous question of M. Kac: "Can you hear the shape of a drum?", the length spectrum rigidity problem for billiard tables has been intensively studied. In this talk, we shall first give a brief introduction on the background and recent development of this topic, and then explain a recent joint work with Vadim Kaloshin and Hong-Kun Zhang on the length spectrum rigidity for piecewise analytic Bunimovich stadia.

# Nhat Minh Doan National University of Singapore, Singapore

Punctured Torus Groups, Generalized Gauss Maps, and Billiards

We consider specific families of objects that can be parametrized by rational numbers between 0 and 1 from different perspectives: geometric, group-theoretic, and dynamical. Our focus is on understanding how rational numbers are divided into orbits based on different parameters within this space. Specifically, we show how various questions such as the pseudomodularity of punctured torus groups, the classification of generalized Gauss maps on the unit interval, and the study of billiard dynamics on ideal hyperbolic quadrilaterals are related. This is a joint work in progress with Ser Peow Tan and Daren Wei.

#### Changguang Dong, *Nankai University, China*

Rigidity of Conditional Measures for Certain Skew Products

We will discuss old and new properties of skew product systems, we will focus on the rigidity phenomenon of the conditional measures on fibers. Based on joint work with Kanigowski.

# James Farre *Max Planck Institute for Mathematics in the Sciences, Germany*

Long Curves and Random Hyperbolic Surfaces

We will fix some topological data, a pants decomposition, of a closed surface and build hyperbolic structures by gluing hyperbolic pairs of pants along their boundary. The set of all hyperbolic metrics with a pants decomposition having a given set of lengths defines an immersed torus in the moduli space of hyperbolic metrics—a "twist torus." Mirzakhani conjectured that as the lengths of the pants curves tend to infinity, that the corresponding twist tori equidistribute in the moduli space. We address Mirzakhani's conjecture and explain how to import tools in Teichmüller dynamics on a moduli space of singular flat surfaces with cone points to dynamics on the moduli space of hyperbolic surfaces equipped with a measured geodesic lamination. This is joint work with Aaron Calderon.

# Anish Ghosh *Tata Institute of Fundamental Research, India*

A Generalized Levy-Khintchine Theorem and a Conjecture of Y. Cheung

The famous Levy-Khintchine theorem is a beautiful limiting law for the denominators of the convergents of the continued fraction expansion of a real number. In a recent breakthrough, Cheung and Chevallier (Annales l'ENS, 2024) extended this theorem to higher dimensions. I will discuss a joint work with Gaurav Aggarwal in which we answer several questions related to Levy-Khintchine theorems in one dimension as well as higher dimensions. In particular, we resolve a conjecture of Yitwah Cheung and answer a question posed by Cheung and Chevallier.

#### Svetlana Katok *The Pennsylvania State University, USA*

Towards Reduction Theory Conjecture for Fuchsian Groups

Based on extensive numerical experiments, Don Zagier conjectured that for any finitely generated Fuchsian group of the first kind there is a partition of the boundary of the hyperbolic plane (circle at infinity) and a Bowen-Series-like boundary map acting in a piecewise manner by generators of the group such that its two-dimensional natural extension has an attractor with finite rectangular structure which every point enters in finite time. The finite rectangular structure property along with other properties (conjecturally equivalent to it) form, in Zagier's terminology, a reduction theory for the group. He conjectured that the rectangular structure persists even when the partition points used in defining the boundary map are perturbed in a continuous manner. I will talk about several results, joint with Adam Abrams and Ilie Ugarcovici, in various combinations, where the finite rectangular structure property was proved. For the modular group, for all (a,b)-continued fractions algorithms, where a and b are real numbers satisfying  $b \rightarrow =1$ , ab <= -1, for surface groups, for an open set of partitions, and recently, for all finitely generated Fuchsian groups with at least one cusp, a large class of Fuchsian group which contains all subgroups of the modular group, congruence or not. I will also talk about applications of the reduction theory to symbolic coding of geodesics.

# Minsung Kim *KTH Royal Institute of Technology, Sweden*

Anisotropic Spaces and Nil-automorphisms

Interconnections between parabolic and hyperbolic dynamics have been recently studied regarding renormalization techniques. In particular, they were observed in the deviation of ergodic integrals and solving cohomological equations for some parabolic flows (cf. Liverani-Giulietti, Adam-Baladi, Faure-Gouëzel-Lanneau, and Butterley-Simonelli, etc.).

In this talk, we introduce the geometric anisotropic Banach spaces on Heisenberg nilmanifolds. This construction shows how the Ruelle resonances for the transfer operator associated with the renormalization map (partially hyperbolic nil-automorphism) are related to the invariant distributions for cohomological equations of Heisenberg nilflows. This is joint work with Oliver Butterley.

# Qiao Liu *Xiangtan University, China*

Local Rigidity of Certain Solvable Group Actions on Tori

We study rigidity property of aZine actions of a semidirect product of Z and R on tori generated by toral automorphism and linear flow. Under mild assumption, such actions exhibit a weak version of local rigidity, i.e., any smooth perturbations close enough is smoothly conjugate to the aZine action up to constant time change.

# Didac Martinez-Granado University of Luxembourg, Luxembourg National University of Singapore, Singapore

Metrics and Reparameterizations for Hyperbolic Groups

I will present the "space of metrics of a group", a metric space parameterizing the geometric actions of an arbitrary hyperbolic group on Gromov hyperbolic spaces. Even for the surface group case, this space is much larger than the classical Teichmüller space, encompassing negatively curved Riemannian metrics, geodesic currents, random walks, and more. I will explore how Green metrics—those associated with admissible random walks on the group—are dense in the space of metrics. As an application, I will show that, for a closed negatively curved Riemannian manifold, there exists a natural isometry between the closure of the space of reparameterizations of the geodesic flow and the closure of the space of metrics. This is joint work with Stephen Cantrell and Eduardo Reyes.

# Sabyasachi Mukherjee *Tata Institute of Fundamental Research, India*

#### Combining Rational maps and Kleinian Groups as Algebraic Correspondences

Various connections and philosophical analogies exist between two branches of conformal dynamics; namely, rational dynamics on the Riemann sphere and actions of Kleinian groups. It was envisaged by Fatou in the 1920s that these conformal dynamical systems can be studied in the common framework of iterated algebraic correspondences. In the 1990s, Bullett and Penrose produced first examples of algebraic correspondences that arise as combinations of quadratic polynomials with the modular group. We will expound a general mechanism to produce algebraic correspondences that combine Fuchsian genus zero orbifold groups with generic complex polynomials having connected Julia sets, and describe their parameter spaces. Time permitting, we will touch upon ergodic theory and equidistribution results for such correspondences.

#### Kyewon Koh Park *Ajou University, S. Korea*

Some Properties of Entropy Zero Systems

Entropy is an isomorphism invariant which measures the chaoticity of a dynamical system. In the study of general group actions, entropy zero systems arise more naturally with some chaoticity of the subgroup actions and they exhibit diverse and interesting dynamics. To have better understanding of these systems we need to study the examples of entropy zero Z-actions and investigate some of their properties including their complexities.

#### Zhenqi Wang *Michigan State University, USA*

Global Smooth Rigidity for Toral Automorphisms

Let  $f: T^{d} \to T^{d}$  be a  $C^{\infty}$  diffeomorphism whose linearization  $L \in GL(d, \mathbb{Z})$  is very weakly irreducible. Let H be a conjugacy between f and L. We show that if H is  $C^{1+Holder}$ , then f is  $C^{\infty}$  conjugate to L. In particular, if L hyperbolic and H is  $C^{1}$  then f is  $C^{\infty}$  conjugate to L. As an application, we improve regularity of the conjugacy to  $C^{\infty}$  in prior local and global rigidity results. This is a joint work with B. Kalinin, V Sadovskaya.

#### Weisheng Wu *Xiamen University, China*

On the Uniqueness of Equilibrium States for Geodesic Flows

We show the uniqueness of equilibrium states (including measure of maximal entropy) for

- (1) geodesic flows on certain manifolds without conjugate points;
- (2) frame flows on certain manifolds of nonpositive curvature.

If time permits, we also discuss some applications to the problem of counting closed geodesics.

# Junyi Xie *Peking University, China*

Algebraic Dynamics and Recursive Inequalities

We get three basic results in algebraic dynamics:

- (1) We give the first algorithm to compute the dynamical degrees to arbitrary precision.
- (2) We prove that for a family of dominant rational self-maps, the dynamical degrees are lower semi-continuous with respect to the Zariski topology. This implies a conjecture of Call and Silverman.
- (3) We prove that the set of periodic points of a cohomologically hyperbolic rational selfmap is Zariski dense. In fact, for every dominant rational self-map, we find a family of recursive inequalities of some dynamically meaningful cycles.

Our proofs are based on these inequalities.

# Shucheng Yu, University of Science and Technology of China, China

An Asymptotic Estimate of Extreme Value Laws for Horocycle Flows

The classical logarithm law of Sullivan gives the rate of the deepest cusp excursion of generic geodesics on a non-compact hyperbolic surface of finite area. Going beyond logarithm law, one can ask a more refined question of determining the limiting distribution of the measure of points which derivate from the expected excursion rate by a fixed quantity. This is the so called extreme value law (if exists). Using connections with continued fractions, Pollicott obtained an exact extreme value law for geodesics on the modular surface. For horocycle flows, the logarithm law was proved by Athreya and Margulis for the modular surface and Kelmer and Mohammadi for a general hyperbolic surface. More recently, Kirsebom and Mallahi-Karai obtained an extreme value law for horocycle flows on the modular surface. Their result was then generalized by Marklof and Pollicott s result which gives an asymptotic estimate to the limiting distribution function in their extreme value law. This is work in progress with Andreas Strömbergsson.

# Tengren Zhang National University of Singapore, Singapore

Patterson-Sullivan Measures for Relative Anosov Groups

Relative Anosov groups are a class of subgroups of a semisimple Lie group G that include all Anosov subgroups, and all geometrically finite subgroups when G has rank one. We prove that under some mild conditions on G, the Poincare series associated to a relative Anosov subgroup (and a linear function on the Cartan subspace of G) diverges at its critical exponent if its critical exponent is finite. As a consequence of this, we deduce uniqueness and ergodicity results for the associated Patterson-Sullivan measure whose dimension is the critical exponent. This is joint work with Richard Canary and Andrew Zimmer.