<u>Abstracts</u>

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Leonardo Abbrescia Georgia Institute of Technology, USA

Energy estimates for shock formation to the full 3D compressible Euler system without assuming additional regularity on the vorticity and entropy.

All known proofs of shock formation to the compressible Euler system in multiple spatial dimensions require the use of a "geometric" coordinate system in which the solution looks rather smooth relative to differential operators tailored to the geometric coordinates. Although this naively leads to a derivative loss in the energy estimates, Christodoulou found a way to recover the loss in his historic 2007 work on irrotational and isentropic relativistic Euler. For the full Euler system, one also must worry about a loss of derivatives in the vorticity and entropy. This was resolved by in 2018 and 2022 by Luk-Speck up to the first point of gradient blow-up and extended to a full connected component of the singular boundary by prior work joint work with Speck.

All these results required that the Cauchy data for the vorticity and entropy be as regular as the velocity, which limits the potential application to the shock development problem where the regularity across the shock is determined by the Rankine-Hugoniot jump conditions. In this talk, we will present a new robust method of closing the energy estimates without requiring additional regularity on the vorticity and entropy that could, in principle, be used to solve the unrestricted shock development problem. These energy estimates will be explained in the context of the construction of the Cauchy horizon.

Xinliang An *National University of Singapore, Singapore*

On Black Hole Formation

In this talk, I will report several recent results toward black hole formation and naked singularity censoring.

John Anderson Stanford University, USA

Shock formation for the Einstein—Euler system

In this talk, I hope to describe elements of proving a certain stable singularity formation result for the Einstein—Euler system, which is the topic of work in progress with Jonathan Luk. I will first describe where this fits into the big picture of the study of multidimensional shocks, and why it is appropriate to call this a shock formation result. Then, I will try to describe some of the main ideas that go into proving shock formation results. Finally, I will try to describe some of the main difficulties that arise in the case of Einstein—Euler.

Stefanos Aretakis *University of Toronto, Canada*

Observational signatures for extremal black holes

I will present observational signatures for extremal black holes. These signatures rely on the precise late time asymptotics for solutions to the wave equation on such backgrounds. I will also present asymptotics for subextremal backgrounds.

Singularities in Fluids and General Relativity Workshop Two

Sanchit Chatuverdi Stanford University, USA

Zero viscosity limit of 1D viscous conservation laws at the point of first shock formation.

Despite the small scales involved, the compressible Euler equations seem to be a good model even in the presence of shocks. Introducing viscosity is one way to resolve some of these small-scale effects. In this talk, we examine the vanishing viscosity limit near the formation of a generic shock in one spatial dimension for a class of viscous conservation laws which includes compressible Navier Stokes. We provide an asymptotic expansion in viscosity of the viscous solution via the help of matching approximate solutions constructed in regions where the viscosity is perturbative and where it is dominant. Furthermore, we recover the inviscid (singular) solution in the limit, and we uncover universal structure in the viscous correctors. This is joint work with John Anderson and Cole Graham.

Greg Fournodavlos University of Crete, Greece

A localized construction of Kasner-like singularities

In this talk, we will give an overview of Kasner-like singularity constructions and present recent joint work with Nikos Athanasiou on localization procedure. We work with a first order symmetric hyperbolic formulation of the Einstein equations, in the second fundamental form of the Gaussian time slices and its spatial connection coefficients. Unlike in previous state of the art works, elliptic estimates are not needed to complete the construction which makes it easier to localize the argument. Also, the asymptotic data for the problem are explicitly constructed.

Dejan Gajic *Universität Leipzig, Germany*

Quasinormal modes on asymptotically flat spacetimes

Quasinormal modes (QNMs) are characteristic damped oscillations that are expected to play an important role in the late-time dynamics of perturbations of asymptotically flat, stationary black hole spacetimes measured at null infinity, before inverse-polynomial tails become dominant, and form a key element in the comparison of experimental detections of gravitational waves with theoretical predictions. In order to investigate the validity of this expectation mathematically, it is necessary to develop a suitable characterization of QNMs on Kerr black holes. In this talk, I will discuss a recent approach that is centered around a uniform analysis of QNMs on Kerr-de Sitter black holes in the limit of the cosmological constant going to zero. This talk is based on joint work with C. Warnick.

Joachim Krieger *École Polytechnique Fédérale de Lausanne, Switzerland*

Finite time bubble trees for the energy critical wave maps.

I will discuss a recent result, obtained in joint work with Jacek Jendrej(Paris), on finite time blow up solutions for the 2 co-rotational wave maps from $^{2+1}$ into S² which asymptotically decouple into two distinct bubbles collapsing at different rates and a radiation part.

Junbin Li *Sun Yat-sen University, China*

Gravitational waves perturbations to perfect fluid naked singularities

In this talk, we will discuss progress on trapped surface formation around spherically symmetric self-similar perfect fluid naked singularities due to small perturbations from gravitational waves (in a Holder space), which can be viewed as its low regularity instability in the full Einstein-Euler system.

Warren Li *Princeton University, USA*

BKL bounces outside homogeneity

Regarding solutions to the Einstein equations in General Relativity, physicists Belinski, Khalatnikov and Lifshitz (BKL) proposed an ansatz for general spacetime metrics near (spacelike) singularities. They suggest that, near the singularity, the evolution of the spacetime geometry at different spatial points decouples and is well-approximated by a system of autonomous nonlinear ODEs, and that general orbits of these ODEs resemble a (chaotic) cascade of heteroclinic orbits called "BKL bounces". In this talk, we present recent work verifying the validity of BKL's heuristics in a large class of symmetric, but spatially inhomogeneous, spacetimes which exhibit (up to one) BKL bounce on causal curves reaching the singularity. In particular, we prove AVTD behavior (i.e. decoupling) even in the presence of inhomogeneous BKL bounces. The proof uses nonlinear ODE analysis coupled to hyperbolic energy estimates, and one hopes our methods may be applied more generally.

Jonathan Luk Stanford University, USA

Late time tail of waves on dynamic asymptotically flat spacetimes, part I

We will present a general method for understanding the precise late time asymptotic behavior of solutions to linear and nonlinear wave equations in odd spatial dimensions. In the setting of stationary linear equations, we recover and generalize the Price law decay rates. However, in the presence of a nonlinearity and/or a dynamical background, we prove that the late time tails are in general different(!) from the better-understood case of linear equations on stationary backgrounds.

Siyuan Ma *Academy of Mathematics and Systems Science,CAS, China*

Energy-Morawetz estimates for wave equations in perturbations of Kerr

I will talk about proving energy and Morawetz estimates for solutions to wave equations in spacetimes with metrics that are perturbations, compatible with nonlinear applications, of Kerr metrics in the full subextremal range. Central to our approach is the proof of a global in time energy-Morawetz estimate conditional on a low frequency control of the solution using microlocal multipliers adapted to the r-foliation of the spacetime. This result constitutes a first step towards extending the current existed proof of Kerr stability, valid in the slowly rotating case, to a complete resolution of the black hole stability conjecture, i.e., the statement that the Kerr family of spacetimes is nonlinearly stable for all subextremal angular momenta.

Sung-Jin Oh University of California, Berkeley, USA

Late time tail of waves on dynamic asymptotically flat spacetimes, part 2

We will present a general method for understanding the precise late time asymptotic behavior of solutions to linear and nonlinear wave equations in odd spatial dimensions. In the setting of stationary linear equations, we recover and generalize the Price law decay rates. However, in the presence of a nonlinearity and/or a dynamical background, we prove that the late time tails are in general different(!) from the better-understood case of linear equations on stationary backgrounds.

Todd Oliynyk *Monash University, Australia*

Localised Big Bang Stability

Since the 1920s, it has been known that the spatially homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) spacetimes generically develop curvature singularities in the contracting time direction along spacelike hypersurfaces. known as Big Bang singularities, both in vacuum and across a wide range of matter models. For many years, it remained unclear whether these big bang singularities were physically relevant. A partial resolution to this question came in 1967 when Hawking established his singularity theorem, which guarantees that a cosmological spacetime will be geodesically incomplete for a large class of matter models and initial data sets, including highly anisotropic ones. While Hawking's singularity theorem ensures that cosmological spacetimes are geodesically incomplete for many initial data sets, it is silent on the cause of the geodesic incompleteness. It has been widely anticipated that the geodesic incompleteness is due to the formation of curvature singularities, and it remains an outstanding problem in mathematical cosmology to rigorously establish the conditions under which this expectation is true and to understand the dynamical behaviour of cosmological solutions near singularities.

In this talk, I will begin by introducing the FLRW and Kasner solutions of the Einsteinscalar field equations, which are exact, spatially homogeneous solutions that play a distinguished role in the analysis of Big Bang singularities. After briefly providing historical context for the FLRW and Kasner solutions in the development of cosmology, I will define what it means for an FLRW/Kasner big bang singularity to be stable. With this notion in hand, I will then discuss the recent influential FLRW and Kasner Big Bang stability proofs by Rodnianski-Speck and Fournodavlos-Rodnianski-Speck. One aspect of these stability results to which I will pay particular attention is their global nature. I will then discuss recent work done in collaborationwith Florian Beyer and Weihang Zheng, where we improve upon the Fournodavlos-Rodnianski-Speck big bang stability result by establishing the localized stability of Kasner big bang singularities in 4 spacetime dimensions over the full range of Kasner exponents where quiescent behaviour is expected. Localized stability is a stronger notion of stability that has interesting physical consequences, which I will briefly discuss at the end of my talk, time permitting.

Jan Sbierski *The University of Edinburgh, UK*

TBA Singularity structure of FLRW spacetimes at low regularities

This talk investigates the structure of the Big Bang singularity in a variety of FLRW spacetimes. It is straightforward to compute scalar curvature invariants to determine whether a curvature singularity is present which excludes a continuation as a strong solution to the Einstein equations. In this talk the focus is on capturing the singularity structure at the level of the connection and the metric itself, determining which geometric quantities blow up and in which regularity class the solution breaks down.

Dawei Shen *Columbia University, USA*

Global stability of Minkowski spacetime with minimal and borderline decay

The global stability of Minkowski spacetime has been proven in the celebrated work of Christodoulou-Klainerman in 1993. In 2007, Bieri extended the result of Christodoulou-Klainerman under lower decay and regularity assumptions on the initial data. In this talk, I will introduce my recent work, which extends the result of Bieri to minimal decay assumptions. Moreover, I will also discuss another recent work, which proves that the exterior stability of Minkowski holds with decay which is borderline.

Jeremie Szeftel *Sorbonne University, France*

Blow up for supercritical defocusing NLS and compressible fluids

I will present results concerning finite time blow up for the defocusing supercritical NLS equation and for compressible fluids.

<u>*Colloquium Talk</u> On the black hole stability problem*</u>

I will introduce the celebrated black hole stability conjecture according to which the Kerr family of metrics are stable as solutions to the Einstein vacuum equations of general relativity. I will then discuss the history of this problem, including a recent work on the resolution of the black hole stability conjecture for small angular momentum.

Martin Taylor *Imperial College London, UK*

Radiative properties of collisionless matter in isolated charged systems

The Vlasov--Poisson system describes the evolution of an ensemble of either:

- 1. Electrically charged particles, interacting via an electrostatic Coulomb force;
- 2. Self-gravitating particles, interacting via a Newtonian gravitational force.

In 3 space dimensions, for isolated systems, dispersive solutions asymptotically exhibit logarithmically corrected linear behaviour, i.e. such solutions ``scatter'' in a modified sense (in contrast to 4 space dimensions and higher, where such solutions asymptotically behave linearly).

I will discuss a new proof of well posedness of the inverse modified scattering problem: for every suitable scattering profile, there exists a solution of Vlasov--Poisson which disperses and scatters, in a modified sense, to this profile. Further, as a consequence of the proof, the solutions are shown to admit a ``polyhomogeneous expansion'', to any finite but arbitrarily high order, with coefficients given explicitly in terms of the scattering profile.

I will then discuss a generalisation to the study of the electromagnetic radiation created by a collection of infalling particles in the context of the Vlasov--Maxwell system. This is joint work with Volker Schlue (Melbourne).

Zoe Wyatt *University of Cambridge, UK*

A new phase transition in cosmological fluid dynamics

On a background Minkowski spacetime, the Euler equations (both relativistic and not) are known to admit unstable homogeneous solutions with finite-time shock formation. Such shock formation can be suppressed on cosmological spacetimes whose spatial slices expand at an accelerated rate. However, situations with decelerated expansion, which are relevant in our early universe, are not as well understood. I will present some recent joint work in this direction, based on collaborations with David Fajman, Maciej Maliborski, Todd Oliynyk and Max Ofner.