## <u>Abstracts</u>

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## Abstracts for Short Talks

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#### Ken Abe *Osaka Metropolitan University, Japan*

*Stationary self-similar profiles for the two-dimensional inviscid Boussinesq* 

We consider homogeneous (stationary self-similar) solutions (of degree - $\alpha$ ) for the twodimensional inviscid Boussinesq equations in a half-plane. We show their nonexistence and existence with both regular and singular profile functions. More specifically, we demonstrate: (i) Non-existence of rotational homogeneous solutions with regular profiles for  $-1 \le \alpha \le 1$ (ii) Existence of rotational homogeneous solutions with regular profiles for  $\alpha > 1$  and  $\alpha < -2$  (ii) Existence of rotational homogeneous solutions with  $x_1$ symmetric singular profiles for  $-1 \le \alpha \le 1$ . The homogeneous solutions (iii) provide theoretical examples of self-similar weak solutions for the scaling exponent  $\alpha \approx -0.617$ at which Wang et al. (2023) numerically discovered backward self-similar solutions.

This talk is based on a joint work with Daniel Ginsberg (Brooklyn College of CUNY) and In-Jee Jeong (Seoul National University).

Dallas Albritton *University of Wisconsin-Madison, USA* 

Kinetic shock profiles for the Landau equation

Compressible Euler solutions develop jump discontinuities known as shocks. However, physical shocks are not, strictly speaking, discontinuous. Rather, they exhibit an internal structure which, in certain regimes, can be represented by a smooth function, the shock profile. We demonstrate the existence of weak shock profiles to the kinetic Landau equation.

Joint work with Matthew Novack (Purdue University) and Jacob Bedrossian (UCLA).

#### Maria Colombo *École Polytechnique Fédérale de Lausanne, Switzerland*

Flexibility of Two-Dimensional Euler Flows

For the two dimensional Euler equations, a classical result by Yudovich states that solutions are unique in the class of bounded vorticity; it is a celebrated open problem whether this uniqueness result can be extended to the class of Lp-vorticities. In recent years, many contributions lead to significant progress: just to mention a few, there were results based on convex integration, on instability in self-similar variables, the study of certain point vortices, numerical evidences. The talk will highlight new results obtained thanks to a new version of the convex integration method.

#### Diego Cordoba Institute of Mathematical Sciences (ICMAT), Spain

*Finite time singularities for incompressible fluids.* 

We begin by reviewing recent constructions of finite time singularities in the 3D incompressible Euler equations and the hypodissipative Navier-Stokes equations. Additionally, we present a mechanism for blow-up in the 2D Boussinesq equation, achieved through a multi-layer degenerate pendula with a uniform  $C^{1,\lambda}$  forcing term.

These are joint works with Luis Martínez-Zoroa, Fan Zheng and Andrés Laín Sanclemente.

#### Gianluca Crippa *University of Basel, Switzerland*

Nonlocal conservation laws with BV kernel

I will present a recent result in collaboration with Maria Colombo and Laura Spinolo ensuring local-in-time existence and uniqueness results for nonlocal conservation laws in several space dimensions under weak (that is, Sobolev or BV) differentiability assumptions on the convolution kernel. In contrast to the case of a smooth kernel, in general the solution experiences finite-time blow-up. I will describe an explicit example showing that solutions corresponding to different smooth approximations of the convolution kernel in general converge to different measures after the blow-up time. This rules out a fairly natural strategy for extending the notion of solution of the nonlocal conservation law after the blow-up time.

Tarek Elgindi *Duke University, USA* 

Aspects of the long-time behavior of 2d Euler flows

We will discuss some issues related to the long-time behavior of solutions to the 2d incompressible Euler equation. In particular, we will discuss some results of rigidity and flexibility for steady Euler flows. We will then move to discuss their stability properties and how they can be used to establish some non-trivial dynamical behavior, such as filamentation, for unsteady solutions. In particular, we will discuss the concept of twisting, its stability, and various consequences thereof.

This is based on joint works with several co-authors, including M. Coti-Zelati, T. Drivas, Y. Huang, I. Jeong, A. Said, K. Widmayer, and C. Xie.

Eduard Feireisl Institute of Mathematics CAS, Czech Republic

Long time behaviour of open fluid systems

We consider the long time behavior of the Navier-Stokes-Fourier system describing the motion of a compressible viscous and heat conducting fluid. Having introduced and discussed the crucial concepts of dissipativity and asymptotic compactness, we address the questions of attractors, stationary statistical solutions and the convergence of ergodic averages.

Javier Gomez-Serrano Brown University, USA

*Existence of non convex V-states* 

V-states are uniformly rotating vortex patches of the 2D Euler equation. The only known explicit examples are circles and ellipses: the rest of positive existence results use local or global bifurcation arguments and don't give any quantitative information of the solutions. In this talk I will prove the existence of solutions far from the perturbative regime, being able to extract nontrivial features of them and a precise quantitative description. The proof uses a combination of analysis and computer-assisted techniques.

#### Thomas Hou *California Institute of Technology, USA*

#### Potentially singular behavior of 3D incompressible Navier-Stokes equations

Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data is one of the most challenging problems in onlinear PDEs. In this talk, I will present some numerical evidence that the 3D Navier-Stokes quations develop nearly self-similar singular scaling properties with maximum vorticity increased by a factor of 107. This potentially singular behavior is induced by a potential finite time singularity of the 3D Eu-ler equations. Unlike the Hou-Luo blowup scenario, the potential singularity of the 3D Euler and Navier-Stokes equations occurs at the origin. We have applied several blowup criteria to study the potentially singular behavior of the Navier-Stokes equations. The Beale-Kato-Majda blow-up criterion, the blowup criteriabased on the growth of enstrophy and negative pressure, the Ladyzhenskaya-Prodi-Serrin regularity criteria all seem to imply that the Navier-Stokes equations develop potentially singular behavior. Finally, we present some new numerical evidence that a generalized axisymmetric Navier-Stokes equation with time dependent fractional dimension develops nearly self-similar blowup with maximum vorticity increased by a factor of  $10^{30}$ .

Sameer Iyer *University of California, Davis, USA* 

Reversal in the Stationary Prandtl Equations

We discuss a result in which we investigate reversal and recirculation for the stationary Prandtl equations. Reversal describes the solution after the Goldstein singularity, and is characterized by spatio-temporal regions in which u>0 and u<0. The classical point of view of regarding the Prandtl equations as an evolution x completely breaks down. Instead, we view the problem as a quasilinear, mixed-type, free-boundary problem.

Joint work with Nader Masmoudi.

Juhi Jang *University of Southern California, USA* 

*Vacuum free boundary problems in gas dynamics* 

We will discuss recent progress on the vacuum free boundary problems arising in the dynamics of isolated gases with or without gravity. We give an overview of the well-posedness and stability theory, and present some new results on waiting time solutions.

#### In-Jee Jeong Seoul National University, Korea

# *On the rate of vortex stretching for axisymmetric Euler flows without swirl*

On the rate of vortex stretching for axisymmetric Euler flows without swirl Abstract: For axisymmetric flows without swirl, we prove the optimal upper bound of  $t^{4/3}$  for the growth of the vorticity maximum, confirming a prediction by Childress in [Phys. D, 237:1921–1925, 2008].

This is based on a joint work with Deokwoo Lim.

#### Moon-Jin Kang *KAIST, Korea*

*From Brenner-Navier-Stokes-Fourier to Euler : stability of a Riemann shock* 

In this talk, we answer the open question on stability of a Riemann shock (as an entropy solution to Euler) in a class of inviscid limits from the physical system.

For the physical system, we consider the so-called Brenner-Navier-Stokes-Fourier system.

This system was proposed by Howard Brenner as a improved continuum model of Navier-Stokes-Fourier, based on 'bi-velocity theory' which indicates the existence of two different velocities: one is the mass velocity that appears in the convection term of the classical compressible fluid model, while the other is the volume velocity is introduced to define momentum, work, energy and viscous stress.

The two velocities should be different if the density is not uniform as in compressible flow.

Our proof is based on the method of a-contraction with shifts.

#### Yasunori Maekawa *Kyoto University, Japan*

Local Rigidity of the Couette flow for the stationary triple-deck equations

The Triple-Deck equations are a classical boundary layer model which describes the asymptotics of a viscous flow near the separation point, and the Couette flow is an exact stationary solution to the Triple-Deck equations. In this talk we prove the local rigidity of the Couette flow in the sense that there are no other stationary solutions near the Couette flow in a scale invariant space. This provides a stark contrast to the well-studied stationary Prandtl counterpart, and in particular offers a first result towards the rigidity question raised by R. E. Meyer in 1983.

This talk is based on a joint work with Sameer Iyer (University of California, Davis).

Luis Martinez-Zoroa *University of Basel, Switzerland* 

*Singularity formation for IPM with a smooth source* 

Singularity formation in incompressible fluids is a topic that has received a lot of attention in the recent years. In this talk, I will present a recent result regarding finite time singularities: The construction of a smooth solution to the IPM equation (incompressible porous media) with a smooth source that blows up in finite time.

#### Laurel Ohm University of Wisconsin–Madison, USA

*Free boundary dynamics of an elastic filament in 3D Stokes flow* 

Motivated by biophysical applications, we consider a free boundary problem for a thin elastic filament immersed in 3D Stokes flow. The 3D fluid is coupled to the quasi-1D filament dynamics via a novel type of angle-averaged Neumann-to-Dirichlet operator. Much of the difficulty in the analysis lies in understanding this operator. We show that the principal part of this NtD map is the corresponding operator about a straight, periodic filament, for which we derive an explicit symbol. It is then possible to establish local well-posedness for an immersed filament evolving via a simple elasticity law. This establishes a mathematical foundation for the myriad computational results based on slender body approximations for thin immersed elastic structures.

#### Jaemin Park *Yonsei University, Korea*

*Stability of stratified density under incompressible flows* 

In this talk, I will discuss asymptotic stability in the incompressible porous media equation in a periodic channel. It is well known that a stratified density, which strictly decreases in the vertical direction, is asymptotically stable under sufficiently small, smooth perturba- tions. We achieve improvements in the regularity assumptions on the perturbation and in the convergence rate. We apply a similar idea to the Stokes transport system. Instead of relying on the linearized equations, we directly address the nonlinear problem, and the decay of solutions will be obtained from the gradient flow structure of the equation.

Matthew Schrecker University of Bath, UK

Stability of gravitational collapse

In the Newtonian setting, a star is modelled as a spherically symmetric gas obeying the compressible Euler-Poisson system. In certain regimes, smooth initial data may give rise to blow-up solutions, corresponding to the collapse of a star under its own gravity, and such solutions have been rigorously constructed In recent years. In this talk, I will present the nonlinear stability of the simplest of these blow-up profiles, the Larson- Penston solution to the Euler-Poisson equations.

This is based on joint works with Yan Guo, Mahir Hadzic, and Juhi Jang.

### Jia Shi *Massachusetts Institute of Technology, USA*

Non-radial implosion for compressible Euler, Navier-Stokes and defocusing NLS in T<sup>d</sup> and R<sup>d</sup>

We will discuss the smooth, non-radial solutions of the compressible Euler, Navier-Stokes equation and defocusing nonlinear Schrodinger equation that develop an imploding finite time singularity. The construction is motivated by the radial imploding solutions from Merle--Raphael--Rodnianski--Szeftel, and Cao-Labora--Buckmaster--Gomez-Serrano but is flexible enough to handle both periodic and non-radial initial data.

This is a joint work with Gonzalo Cao-Labora, Javier Gomez-Serrano, and Gigliola Staffilani.

#### Zhouping Xin *The Chinese University of Hong Kong, Hong Kong SAR*

On the Prandtl's Boundary Layer Theory for Steady Sink-Type Flows

In this talk, I will present some results on the large Reynolds number limits and asymptotic behaviors of solutions to the steady incompressible Navier-Stokes equations in two-dimensional infinitely long convergent nozzles. The main results show that the Prandtl's laminar boundary layer theory can be rigorously established and the sink-type Euler flow superposed with a self-similar Prandtl's boundary layer flow is shown to be uniformly structurally stable as long as the viscous flow has a given negative mass flus and the boundaries of the nozzle satisfy a curvature decreasing condition. Furthermore, the asymptotic behaviors of the solutions at both the vertex and infinity can be determined uniquely which plays a key role in the stability analysis.

Some of key ideas in the theory will be discussed. This talk is based on a joint work with Dr. Chen Gao.

#### Tsuyoshi Yoneda *Hitotsubashi University, Japan*

*Effectiveness of Littlewood-Paley theory in the study of turbulence and machine learning* 

In this talk, I will explain recent works on turbulence and machine learning in terms of scale decomposition.

Goto-Saito-Kawahara (2017) employed the Littlewood-Paley decomposition applying to 3D Navier-Stokes turbulence. By direct numerical simulations, they discovered that each scale vortices transfer energy to the adjacent smaller scale vortices through stretching process. In this talk, I will explain that the vortex stretching spills over into the dissipation range and mathematically show that the vortex stretching truly enhances the dissipation rate. Next, I will explain that such scale decomposition method is also useful for machine learning (Reservoir computing). More specifically, I will present a future forecasting study of the El Niño phenomenon.

Andrej Zlatos *University of California, San Diego, USA* 

Stable regime singularity for the Muskat problem

The Muskat problem on the half-plane models motion of an interface between two fluids of distinct densities in a porous medium that sits atop an impermeable layer, such as oil and water in an aquifer above bedrock. We develop a local well-posedness theory for this model in the stable regime (lighter fluid above the heavier one), which includes more general fluid interface geometries than even existing whole plane results and allows the interface to touch the bottom. The latter applies to the important scenario of the heavier fluid invading a region occupied by the lighter fluid along the impermeable layer. We also show that finite time singularities do arise in this setting, including from arbitrarily small smooth initial data, by obtaining maximum principles for the height, slope, and potential energy of the fluid interface.

#### Haoyang Chen National University of Singapore, Singapore

*Low regularity ill-posedness for elastic waves and for MHD system in 3D and 2D* 

We construct counterexamples to the local existence of low-regularity solutions to elastic wave equations and to the ideal compressible magnetohydrodynam- ics (MHD) system in three and two spatial dimensions (3D and 2D). For 3D, inspired by the recent works of Christodoulou, we generalize Lindblad's classic results on the scalar wave equation by showing that the Cauchy problems for 3D elastic waves and for 3D MHD system are illposed in  $H^3$  and  $H^2$ , respectively. Both elastic waves and MHD are physical systems with multiple wave-speeds. We further prove that the ill-posedness is caused by instantaneous shock forma-tion, which is characterized by the vanishing of the inverse foliation density. In particular, when the magnetic field is absent in MHD, we also provide a desired lowregularity ill-posedness result for the 3D compressible Euler equations, and it is sharp with respect to the regularity of the fluid velocity. Our proofs for elastic waves and for MHD are based on a coalition of a carefully designed al-gebraic approach and a geometric approach. In 2D, we prove the  $H^1 1/4$  and  $H^7 /4$  ill-posedness for the elastic wave equations and ideal MHD system (also for Euler equations). Compared with the 3D case, the construction of ill-posed profile in 2D is more delicate. While in 3D, the shock formation argument is more involved due to the more complicated structures of the systems.

This talk is based on joint works with Xinliang An and Silu Yin.

Min Jun Jo *Duke Universtiy, USA* 

*Cusp formation of vortex patches* 

We prove instantaneous cusp formation of any vortex patches with acute corners, which was conjectured to occur in the numerical literature.

This is a joint work with Tarek Elgindi.

Junha Kim *Ajou University, Korea* 

On the wellposedness of alpha-SQG equation in a half-plane

In this talk, we consider the  $\alpha$ -SQG equation in a half-plane, where  $\alpha = 0$  and  $\alpha = 1$  correspond to the 2D Euler and SQG equations respectively. We prove the local wellposedness of  $\alpha$ -SQG in an anisotropic Lipschitz space and the instantaneous blow-up of solutions in H<sup>°</sup>older spaces when initial data does not vanish at the boundary. Then, we briefly discuss the case where initial data vanishes at the boundary.

This talk is based on joint work with In-Jee Jeong(Seoul National University), Yao Yao(National University of Singapore), and Hideyuki Miura(Institute of Science Tokyo).

Shi Zhuo Looi *California Institute of Technology, USA* 

Onsager's conjecture for the SQG equation

We construct solutions to the SQG equation that fail to conserve the Hamiltonian while having the maximal allowable regularity for this property to hold. This result solves the generalized Onsager conjecture on the threshold regularity for Hamiltonian conservation for SQG. Our proof builds on the recent solution of the two-dimensional Onsager conjecture for the Euler equations by Giri and Radu, incorporating additional harmonic analysis techniques to address the specific challenges posed by the SQG equation.

This is joint work with P. Isett.

### Sifan Yu *National University of Singapore, Singapore*

Low-Regularity Local Well-Posedness for the Elastic Wave System

We study the elastic wave system in three spatial dimensions. For admissible harmonic elastic materials, we prove a desired low-regularity local well-posedness result for the corresponding elastic wave equations. For such materials, we can split the dynamics into the divergence-part and the curl-part, and each part satisfies a distinct coupled quasilinear wave system with respect to different acoustical metrics. Our main result is that the Sobolev norm  $H^{3+}$  of the divergence-part (the faster-wave part) and the  $H^{4+}$  of the curl-part (the slower-wave part) can be controlled in terms of initial data for short times. We note that the Sobolev norm assumption  $H^{3+}$  is optimal for the divergence-part.

This marks the first favorable low-regularity local well-posedness result for a wave system with multiple wave speeds.

#### Junyan Zhang National University of Singapore, Singapore

*Low Mach number limit of non-isentropic ideal MHD with a perfectly conducting boundary* 

We prove the incompressible limit for non-isentropic ideal MHD with a perfectly conducting boundary for general initial data. The key observation is based on a hidden structure contributed by Lorentz force in vorticity analysis, which motivates us to establish uniform estimates in certain anisotropic Sobolev norms.