

Abstracts

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Abstracts for Short Talks

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Xinliang An

National University of Singapore, Singapore

*Tutorial 1: Introduction to mathematical general relativity: physical
heuristics*

In this tutorial, we will discuss the physical intuitions toward several important topics in mathematical general relativity, such as the big bang singularity, the Penrose singularity theorem and the formation of trapped surfaces.

Tutorial 2: Trapped surface formation

In this tutorial, we will explain the short-pulse hierarchy and will outline a proof toward black hole formation.

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Mahir Hadzic

University College London, UK

Stability, oscillations, and damping in galactic dynamics

The gravitational Vlasov-Poisson system is a classical astrophysics model describing the dynamics of galaxies. It allows for a vast amount of steady states, whose stability is a question of central importance in the field. While a lot is known about their orbital stability, such results do not tell us much about the asymptotic stability of the solutions.

After a brief review of the problem, I will focus on recent progress on the asymptotic stability question, which is also known as the problem of gravitational relaxation. This topic was initiated in the physics literature in a series of seminal papers by Lynden-Bell from 1960s, however the first rigorous results are very recent. We shall highlight the use of spectral theoretic tools, including the Birman-Schwinger principle, gravitational phase-mixing, and ideas from scattering theory.

Amusingly, different aspects of steady state stability theory beg analogies to both incompressible and compressible phenomena.

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Taoran He
National University of Singapore, Singapore

Tutorial 1: Local well-posedness of Einstein vacuum equations

In this tutorial, we will discuss the local well-posedness of Einstein vacuum equations in harmonic gauge.

Tutorial 2: Spacetime decomposition and double null formalism

In this tutorial, we will first discuss the characteristic local existence result of Einstein vacuum equations established by Rendall. Then we will introduce the framework of the double null formalism.

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In-Jee Jeong
Seoul National University, Korea

Desingularization and vortex confinement for incompressible Euler equations

Volume preserving and inviscid flows are governed by the incompressible Euler equations. In two dimensions, the vorticity of the fluid is transported by the flow while conserving all the L^p norms. Atmospheric observations, experiments and numerical simulations reveal creation and persistence of concentrated vortex structures. To analyze this phenomenon, one can consider the desingularization problem, which studies persistence of vorticities which are sharply concentrated near a few points. Alternatively, one may study the vortex confinement problem, which asks how quickly a large-scale vortex can disperse in space. We review classical works in both directions, and then discuss corresponding problems for the three dimensional incompressible fluids, under the axisymmetric and swirl free assumptions. While the vorticity equation looks similar as in the two dimensional case, the vorticity can exhibit very different dynamical behavior. We focus on these differences and remaining challenges.

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Dawei Shen
Colombia University, USA

Tutorial 1: Stability of Minkowski spacetime

The global stability of Minkowski spacetime has been proven in the celebrated work of Christodoulou-Klainerman in 1993. In this tutorial, I will first introduce the basic set-up of Minkowski stability. I will then report a recent work, which extends the results of Minkowski stability to minimal decay assumptions.

Tutorial 2: Stability of Kerr black holes

Kerr stability for small angular momentum has been proved in a series of works of Klainerman-Szeftel, Giorgi-Klainerman-Szeftel and I in 2022. In the tutorial, I will briefly introduce the basic set-up of the Kerr black hole stability problem and the main ideas used in the proof of this project

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Zhongtian Hu
Duke University, USA

*Suppression of chemotactic singularity by Navier-Stokes Flow with
Large Buoyancy*

Chemotactic singularity formation in the context of the Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that the presence of fluid advection can arrest the singularity formation given that the fluid flow possesses mixing or diffusion enhancing properties -- this effect is conjectured to hold for more general classes of nonlinear PDEs. In this talk, I will introduce Keller-Segel equation coupled with a Navier-Stokes flow via large buoyancy. We prove that, in contrast to previous results driven by passive mixing flows, such active fluid coupling is capable of suppressing singularity formation via an interesting dynamical and nonlinear damping mechanism

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Woojae Lee
Yonsei University, Korea

Long time behaviour of open fluid systems

We consider the long time behavior of the Navier-Stokes-Fourier system describing the motion of a compressible viscous and heat conducting fluid. Having introduced and discussed the crucial concepts of dissipativity and asymptotic compactness, we address the questions of attractors, stationary statistical solutions and the convergence of ergodic averages.

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Wenze Su

National University of Singapore, Singapore

*Shock formation for compressible Euler equations and related systems
via self-similar approach*

This talk presents the construction of shock solutions for the 2D isentropic compressible Euler equations. A set of smooth initial data is identified that leads to finite-time shock formation at a single point near planar symmetry. These solutions are associated with non-zero vorticity at the shock and have uniform- in-time $1/3$ -Hölder bound. Moreover, these point shocks are asymptotically self-similar; that is, the solutions converge in the self-similar coordinates to a blow-up profile, which solves the 2D self-similar Burgers equation.

Additionally, shock solutions for a simplified shallow water system are constructed using similar techniques. If time permits, we will also present a shock formation result for compressible Euler equations on the two-dimensional sphere.

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Tao Zhou
National University of Singapore, Singapore

Finite-time blowup for Keller-Segel-Navier-Stokes system in three dimensions

While finite-time blowup solutions have been studied in depth for the Keller-Segel equation, a fundamental model describing chemotaxis, the existence of finite-time blowup solutions to chemotaxis-fluid models remains largely unexplored. To fill this gap in the literature, we use a quantitative method to directly construct a smooth finite-time blowup solution for the Keller-Segel-Navier-Stokes system with buoyancy in 3D. The heart of the proof is to establish the non-radial finite-codimensional stability of an explicit self-similar blowup solution to 3D Keller-Segel equation with the abstract semigroup tool from [Merle-Raphaël-Rodnianski-Szeftel, 2022], which partially generalizes the radial stability result [Glogić-Schörkhuber, 2024] to the non-radial setting. Additionally, we introduce a robust localization argument to find blowup solutions with non-negative density and finite mass. This talk is based on the joint work with Zexing Li.

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