# <u>Abstracts</u>

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Longyun Ding Nankai University, China

*On equivalence relations induced by Polish groups* 

In this talk, we recall Borel reducibility among equivalence relations first. Then we introduce equivalence relations E(G) induced by Polish groups G. Main part of this talk is to precent many rigid results concerning various kinds of Polish groups: non-archimedean, TSI, CLI,  $\alpha$ -unbalanced, abelian, locally compact, Lie groups, and Banach spaces (as additive groups) and so on.

Rod Downey Victoria University of Wellington, New Zealand

The Interplay of Genericity and Minimality

We examine developments concerning the relationships of reducibilities, *n*-genericity, and minimality. In particular, I'll look at the some old work of Chong and myself characterizing when a minimal degree can be computable from a 1-generic degree; and more recent developments.

Su Gao *Nankai University, China* 

Borel Rectangulations of Bernoulli Shifts

A rectangulation or a regulated partition of  $\mathbb{R}^n$  is a dissection of  $\mathbb{R}^n$  into rectangles with disjoint interiors whose side lengths form a discrete set of positive numbers. The regulation number of a rectangulation is the largest number of rectangles containing a given point. These concepts can be generalized to the Borel context for the free Bernoulli shifts on  $\mathbb{Z}^n$ . We show that when  $n \ge 3$ , there is no Borel rectangulation of the free Bernoulli shifts on  $\mathbb{Z}^n$  with the minimum possible regulation number n + 1. This is joint work with Steve Jackson.

Conference on Limits on Computability, Definability, Provability– Celebrating the Mathematical and Professional Contributions of Chong Chi Tat

Junle Goh National University of Singapore, Singapore

Prof Chong's impact on education in Singapore

Over his 50-year career, Prof Chong has made immeasurable contributions to education in Singapore, at both the tertiary and secondary levels. We will mention various instances of Prof Chong's impact, including the many systemic reforms and his role in mentoring students (such as myself).

## Andre Nies *University of Auckland, New Zealand*

Randomness via effective descriptive set theory

In the algorithmic theory of randomness, one imposes effectiveness conditions on null sets; a real is random in a specific sense if it avoids all null sets satis-fying a particular type of condition. This leads to notions such as Martin-Loef randomness, where the null sets to be avoided are intersections of uniformly  $\Sigma_1^0$  sets that have a computable upper bound on the measure.

A little-known 1970 paper by Martin-Loef studies  $\Delta_1^1$ -randomness. Hjorth and the speaker, in an eponymous 2008 paper, started the systematic study of randomness notions based on conditions on null sets from effective descriptive set theory, including being (lightface)  $\Pi_1^1$ .

The talk will trace the development of this area, which includes a second 2008 paper of Chitat Chong, Yu Liang and the speaker entitled "Lowness for higher randomness". Work up to 2017 of Greenberg, Monin, Yu and others have resolved several long-standing questions. For instance, the only oracles that are low for  $\Pi_1^1$  randomness are the hyperarithmetic ones.

Conference on Limits on Computability, Definability, Provability– Celebrating the Mathematical and Professional Contributions of Chong Chi Tat

Theodore Slaman *University of California, Berkeley, USA* 

Travels with CT

We will discuss some of the topics and motivational themes found in Chong ChiTat's mathematical research. The areas in which he has worked include alpha-recursion theory, subsystems of first and second order arithmetics, priority methods, and the Turing degrees. The questions that he has investigated are instances of his abiding curiosity about whether the ingredients of familiar/fundamental constructions are intrinsic to the properties they are used to produce.

#### Chieu Minh Tran National University of Singapore, Singapore

From Morley's Categoricity to Nonabelian Brunn-Minkowski

Morley's Categoricity Theorem, taught in model theory first courses, tells us that if a theory *T* in a countable language has a single model (up to isomorphism) when the cardinality of the underlying set is some fixed uncountable  $\kappa$ , then *T* does so for all uncountable  $\kappa$ . Brunn-Minkowski Theorem, in convex geometry, says that for any two compact subsets *A* and *B* of *R*<sup>*d*</sup>, we have  $\mu$  (*AB*)<sup>1/*d*</sup>  $\geq \mu$ (*A*)<sup>1/*d*</sup> +  $\mu$ (*B*)<sup>1/*d*</sup> and the equality holds when *A* and *B* are homothetic convex sets. In this talk, I will explain how ideas from the proof of the former is relevant in generalizing the latter to nonabelian locally compact groups.

## Wei Wang Sun Yat-Sen University, China

Definable Combinatorial Principles in Fragments of Arithmetic

In fragments of arithmetic, pigeonhole principle may fail for definable partitions of finite sets. Dimitracoupolous and Paris proved that over  $I\Sigma_1$  the ordinary pigeonhole principle for  $\Sigma_{n+1}$  partition is equivalent to  $B\Sigma_{n+1}$  (n > 0). Later Kaye formulated several second order pigeonhole principles which are used to axiomatise  $\kappa$ -like models of arithmetic. A first order fragment derived from one of Kaye's pigeonhole principles, known as  $\Sigma_{n}$ -Cardinality scheme or  $C\Sigma_n$ , has interesting independence property proved by Kaye himself and also proved useful in reverse mathematics. In a joint work published in 2011, Prof. Chong and his colleagues (including the speaker) introduced a new variant of pigeonhole principle (so-called WPHP) and showed that it plays an interesting role in reverse mathematics. Inspired by these previous works, we study the so-called Generalised Pigeonhole Principle (GPHP) introduced by Kaye. We show that GPHP restricted to  $\Sigma_{n+1}$ -definable functions is strictly between the corresponding fragments of the Cardinality Scheme and WPHP. Moreover, we extend the research to include similar variants of finite Ramsey theorem and obtain some interesting results.

Conference on Limits on Computability, Definability, Provability– Celebrating the Mathematical and Professional Contributions of Chong Chi Tat

Tin Lok Wong National University of Singapore, Singapore

*Defining standardness in nonstandard arithmetic* 

A definition of the natural numbers connects a nonstandard model of arithmetic to the standard world. Such connections are exploited in the model theory of arithmetic as well as in reverse mathematics. So far, the definability of the natural numbers has mostly remained a tool. I will explore this as a topic of study on its own instead, and see what we get and not get.

#### Hugh Woodin Harvard University, USA

Definable Determinacy in Second Order Number Theory.

We show that the following are equiconsistent and here, definable is 'lightface".

Second Order Number Theory + Definable Determinacy.
Second Order Number Theory + Definable Turing Determinacy.

This we prove by showing that in Second Order Number Theory that the following are actually equivalent.

- 1) Definable Turing Determinacy holds after adding a Cohen real.
- 2) Definable Determinacy holds after adding a Cohen real.

Curiously, the proof does not show that in Second Order Number Theory, that these determinacy hypotheses are equivalent. This leaves open an interesting question which is open even in ZFC.

Question: Suppose  $\Sigma_2^1$ -Determinacy holds in V[c] where c is a V-generic Cohen real. Must  $\Sigma_2^1$ -Determinacy hold in V?

## Yue Yang National University of Singapore, Singapore

Halpern-Läuchli Theorem and Σ<sub>2</sub>-Induction

This is an ongoing joint project with Chitat Chong, Wei Li from National University of Singapore and Lu Liu from Central South University, China.

Let *T* denote the full binary tree. For each  $n \ge 1$ , let T(n) be the set of level *n* nodes in *T*. An subtree  $S \subseteq T$  is strong if (i) for all  $m, S(m) \subseteq T(n)$  for some  $n \ge m$ , and (ii) if  $\sigma \in S(m)$  then every immediate successor of  $\sigma$  in *T* has one and exactly one extension in S(m + 1). The Halpern-Läuchli Theorem (HL) states that for all d,  $k \ge 1$ , for all

$$f: \bigcup_n (T_0(n) \mathbf{x} \dots \mathbf{x} T_{d-1}(n)) \to k,$$

there exist strong subtrees  $Si \subseteq T_i$  such that  $f \upharpoonright \bigcup_n \prod_{i \le d-1} Si(n)$  is a constant. It is known that with sufficient induction, Halpern-Läuchli Theorem holds recursively, that is, the homogeneous strong subtrees can be computed recursively from the coloring f. However, we will demonstrate that without  $\Sigma_2$ -induction, the picture is different. Moreover, We show that over the base system  $RCA_0 + \Sigma_2^0$ -bounding,  $RCA_0 + HL$  does not imply  $\Sigma_2^0$ -induction. I will also take this opportunity to highlight some of Chitat Chong's research in reverse mathematics and recursion theory in nonstandard models.

Liang Yu *Nanjing University, China* 

*The theory of higher randomness: progress and open questions* 

I shall survey the current status of higher randomness theory and present some open questions.