Abstracts

Contents

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An Introduction to Intertwining via Strong Stationary Duality

Strong stationary times are a 'pure probability' way of proving rates of convergence of Markov chains to their stationary distribution--just by thinking. You can see the randomness emerging. Hidden behind is a duality between two Markov processes, one with a stationary distribution, the other absorbing. When the second is absorbed, the first is mixed! It sometimes seems like a magic trick. This course is an introduction with many examples so that listeners can appreciate the other developments at the conference.

Lecture 1: First Examples and Basic Setup

Markov chains, mixing times, various measures of mixing--total variation, separation and their relation. Shuffling cards--top to random, riffle shuffling and their analysis. The Diaconis-Chung-Graham process. Existence and first properties of Strong stationary times.

Lecture 2: Duality and Intertwining

ALL of the examples we know have the following features: A natural Markov chain is studied by finding a second process embedded in it. When the embedded process hit's a certain point, the original process is in stationarity. Can this always be done? What good is this 'duality'. I will introduce basic constructions in a 'hands on' elementary manner, introducing the basic intertwining relation 'Pi P = Lambda P'. This links to other dualities;time reversal and Siegmund duality used before. various extensions, set and measure valued duals will be described.

Back to Contents Page →

Lecture Three: Applications of Duality

Dual processes can be used to give 'mechanical' constructions of Strong stationary times (no brilliance needed). They offer a stochastic interpretation of the eigenvalues of a Markov chain. I will show how this gave a solution of the Peres conjecture for Birth and Death chains (joint work with Salof-Coste). Duality offers refinements of the basic bounds via early stopping. All of this illustrated by examples.

Lecture Four: Developments and Open Problems

Many of the examples where Dual chains can be usefully constructed are for random walk on groups where stationary distributions are uniform. Applications in statistics call for constructions with less symmetry. I will illustrate this using Nestoridi's construction of stationary times for random walks on the chambers of a hyperplane arrangement and through Igor Pak's many examples of non-uniform duals.

Back to Contents Page →

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Duality of Markov Processes: a Lie Algebraic Approach

Duality is a way to relate two Markov processes, allowing knowledge transfer from one process to another. Often, beyond a duality of two Markov processes, there is a duality between two algebras. The main aim of this mini-course is to present an algebraic approach where duality emerges as a change of representation of a Lie algebra. The existence of a group of symmetry is thus at the root of a Markov duality and can be used to produce novel dualities constructively.

We will illustrate the algebraic approach using several examples. In particular, we will consider interacting particle systems introduced as models of transport in non-equilibrium statistical physics. In this context, the dual process is absorbing, as it was found for the first time in 1982 in the Kipnis-Marchioro-Presutti (KMP) process. The absorbing duality of the KMP process can be formulated as an intertwining relation between the KMP model and a random version of the Aldous averaging model. This intertwining relation amounts in turn to the propagation of certain mixture measures. We will discuss a distinguished transport model (the "harmonic model", recently introduced and related to integrable spin chains) where the evolution of mixture measure can be followed for large times yielding a closed formula for the stationary measure, i.e. the non-equilibrium steady state.

Lecture 1: Introduction and basic ideas

1.1) Duality definition.
1.2) Algebraic approach.
1.3) Intertwining.
1.4) Simple examples: independent walkers, Wright-Fisher diffusion

Back to Contents Page →

Lecture 2: The su(1,1) Heisenberg spin chain

2.1) An algebraic procedure to construct processes with symmetries.
2.2) The example of the su(1,1) Lie algebra.
2.3) Symmetric inclusion process, Brownian energy process, and their intertwining.
2.4) Thermalized models and the KMP process.

Lecture 3: Absorbing duality and propagation of mixtures

3.1) The non-equilibrium setup: models of transport.
3.2) Absorbing duality and intertwining.
3.3) Propagation of certain mixtures in the KMP process.
3.4) Algebraic perspective on the absorbing duality.

Lecture 4: The integrable su(1,1) Heisenberg spin chain

4.1) Harmonic processes: the discrete version and the continuous version.
4.2) Propagation of certain mixtures in the harmonic processes.
4.3) Solution of the non-equilibrium steady state.
4.4) Large deviations of the empirical density profile and agreement with the predictions of the Macroscopic Fluctuation Theory.

REFERENCES:
will be provided during the lectures. A good starting point is the recent preprint arXiv:2406.01160.

Back to Contents Page →

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On finite interweaving relations

A faithful bi-interweaving relation is a Markovian similarity-type relation between two Markov chains, strengthening the Markovian intertwining relation and introducing warming-up times after which the time-marginal distributions of the chains can be tightly compared (for any initial distributions).

For irreducible transition kernels on the same finite state space, these relations are shown to be equivalent to the generalised isospectrality relation, but this is no longer true for non-transient transition kernels, contrary to the faithful bi-intertwining relations. Some bounds are deduced on corresponding warming-up times, when the eigenvalues are furthermore assumed to be real (but still allowing for Jordan blocks).

When the eigenvalues are non-negative, the same approach enables us to construct strong stationary times for irreducible Markov chains through interweaving relations with model absorbed Markov chains, thus extending a result due to Matthews in the reversible situation.

Back to Contents Page →