Abstracts

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Djamel Eddine Amir
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Some results in computable topology

The topological properties of a set have a strong impact on its computability properties. A striking illustration of this idea is given by spheres, closed manifolds and finite graphs without endpoints : if a set X is homeomorphic to a sphere, a closed manifold or a such graph, then any algorithm that semicomputes X in some sense can be converted into an algorithm that fully computes X. In other words, the topological properties of X enable one to derive full information about X from partial information about X. In that case, we say that X has computable type. Those results have been obtained by Miller and Iljazović and others in the recent years. We give a characterization of finite simplicial complexes having computable type using a local property of stars of vertices. We argue that the stronger, relativized version of computable type, is better behaved. Consequently, we obtain characterizations of strong computable type, related to the descriptive complexity of topological invariants. We study two families of topological invariants of low descriptive complexity, expressing the extensibility and the null-homotopy of continuous functions. We apply the theory to revisit previous results and obtain new ones. This leads us to investigate the expressive power of low complexity invariants in the Borel hierarchy and their ability to differentiate between spaces. Notably, we show that they are sufficient for distinguishing finite topological graphs.

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Berenice Boveland
University of Vienna, Austria

Higher Baire Spaces Cardinal Characteristics

In this talk we will give a survey of results regarding the higher Baire spaces cardinal characteristics, many of which have been only recently obtained, emphasizing the distinction between the infinitary combinatorics captured by these characteristics in the classical (countable) and general (uncountable) settings. Of particular interest for us will be the bounding, dominating, splitting, refining, almost disjoint and independent families.

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Natthajak Kamkru
Chulalongkorn University, Thailand

Cardinalities of the set of functions on a set

We investigate relationships between the cardinalities of the sets of functions to the various sets in the context of ZF without the Axiom of Choice.

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Thibaut Kouptchinsky
Technische Universität Wien, Austria

Determinacy in high-order arithmetic

This talk is about the foundations of mathematics, studying determinacy axioms derived from game theory, with a reverse mathematics point of view. Reverse Mathematics is an endeavour to compare theorems according to the "strength" of the axioms a mathematician needs to prove them.

We study a refined case of the proof from Martin of Borel determinacy, which showed how to use the existence of high-order objects when one wants to show that infinite games with increasingly difficult w inning conditions in the Borel hierarchy have winning strategies. The use of such principles had already been shown to be necessary by Friedmann. In the terms that will be ours, Martin provided the final p roof that (2+γ)th-order arithmetic (Z_2+γ) is the first to witness the determinacy of $\Pi^0_{1+\gamma+2}$ Gale-Stewart games ($\gamma < \omega_1^C K$).

For our part, we will look in more detail at the situation in second-order and higher-order arithmetic, examining a paper by Montalbán and Shore. We present a generalisation of their results, in some natural interpretation of third-order arithmetic about infinite games which winning conditions are located in the difference hierarchy of Π^0_4 sets.

Along the way, we underline a shift compared to the analogous situation in the countable case (about differences of Π^0_3 sets), which is the object of the paper of Montalbán and Shore. Namely, we need less of the separation scheme than in the countable case. Finally, we use these generalisations to answer a question of Pacheco and Yokoyama about reflection principles in higher-order arithmetics.

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Gian Marco Osso
University of Udine, Italy

The Galvin-Prikry theorem in the Weihrauch lattice

I will address the classification of different fragments of the Galvin-Prikry theorem, an infinite dimensional generalization of Ramsey's theorem, in terms of their uniform computational content (Weihrauch degree).

This can be seen as a continuation of previous work by Marcone and Valenti, which focused on the Weihrauch classification of functions related to the Nash-Williams theorem, i.e. the restriction of the Galvin-Prikry theorem to open sets.

We have shown that most of the functions related to the Galvin-Prikry theorem for Borel sets of rank n are strictly between the (n+1)-th and n-th iterate of the hyperjump operator. To establish this classification we obtain the following computability theoretic result (along the lines of the results of Jockush for Ramsey's theorem): a Turing jump ideal containing homogeneous sets for all $\Delta^0_{n+1}(X)$ sets must also contain $HJ^n(X)$.

Similar results also hold for Borel sets of transfinite rank.

This is joint work with Alberto Marcone.

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Karthik Ravishankar
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Spectra of structures

After introducing the notion of spectrum of a structure, we present some known and some original negative results about structure spectra.

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Michel Smykalla
Universidade de São Paulo, Brazil

Forcing and Category Theory

In 1931, Kurt Gödel proved his incompleteness theorems, showing that any logical system with the minimum desirable properties to formalize mathematics is incomplete. In other words, there exist formulas in which we can neither prove nor prove their negation. However, it was only in 1964 that one mathematical conjecture, known as the continuum hypothesis (CH), was proved to be formally undecidable from the axiomatic system most used in modern mathematics, the Zermelo-Fränkel set theory. To show this independence, Paul Cohen created a technique called forcing. Since then, forcing has been applied to several areas of mathematics, including algebra, analysis, and topology. In topos theory, one can interpret a topos as a generalized universe of sets. Moreover, it is possible to give an alternative proof of the independence of CH, constructing a category called Cohen topos where CH fails. In this talk, we will present this alternative proof of the independence of CH and discuss the connections with the classical version of forcing.

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Yuto Takeda
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The relationship between the Tree Theorem and induction axioms

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Ramsey's theorem has long been a major theme in the study of reverse mathematics and computability theory. As a result of this research, analysis of theorems in infinite combinatorics, termed Ramsey-type theorems, has also seen recent progress. In this presentation, I will discuss the analysis of Ramsey-type theorems, specifically focusing on the Tree Theorem, from the perspectives of reverse mathematics. Specifically, I will talk about the relationship between the Tree Theorem and induction axioms.

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Yunsong Wang
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General Topological Frames for Polymodal Provability Logic

The polymodal provability logic GLP has significant applications in proof theory and arithmetic, however, GLP is Kripke incomplete. GLP is complete with respect to topological semantics, yet the relevant class of spaces is rather involved. The question of completeness of GLP w.r.t. natural topologies on ordinals turns out to be dependent on large cardinal axioms and is still open. So, we are lacking a usable class of models for which GLP is complete.

In this joint work (with Lev Beklemishev), we de fine a natural class of countable general topological frames on ordinals for which GLP is sound and complete. The associated topologies happen to be the same as the ordinal topologies introduced by Thomas Icard [2]. However, the key point is to consider a suitable algebra of subsets of an ordinal closed under the boolean and topological derivative operations. The algebras we defi ne are based on the notion of a periodic set of ordinals generalizing that of an ultimately periodic binary omega-word.

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Jing Yu
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Large-scale geometry of Borel graphs of polynomial growth

Krauthgamer and Lee showed that every connected graph of polynomial growth admits an injective contraction mapping to $(\mathbb{Z}^n, \|\cdot\|_\infty)$ for some $n \in \mathbb{N}$. We strengthen and generalize this result in a number of ways. In particular, answering a question of Papasoglu, we construct coarse embeddings from graphs of polynomial growth to $\mathbb{Z}^n$. Furthermore, we extend these results to Borel graphs. Namely, we show that graphs generated by free Borel actions of $\mathbb{Z}^n$ are in a certain sense universal for the class of Borel graphs of polynomial growth. This provides a general method for extending results about $\mathbb{Z}^n$-actions to all Borel graphs of polynomial growth. For example, an immediate consequence of our main result is that all Borel graphs of polynomial growth are hyperfinite, which answers a well-known question in the area. An important tool in our arguments is the notion of Borel asymptotic dimension. This is joint work with Anton Bernshteyn.

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Xiaoyan Zhang
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Dimensionality and randomness

The deficiency of a string is the maximum number of bits it can be compressed. We study sets of strings with finite (bounded) deficiency, which are called incompressible sets. Among them, observation shows that "thin" ones cannot be effectively transformed into "fat" ones unless they are complete. For example, a random real cannot compute an incompressible tree with infinitely many paths unless it is complete. In this talk, we provide several theorems in this form and quantify what conditions are needed on the fat set to prevent it from being computed from a thin one.

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