Hints for the problems

1. By considering a composition series of $\mathbb{F}^n$ as an $\langle G\rangle_{\mathbb{F}}$-module, show that there exists a block upper triangular subgroup of $\mathrm{GL}(n,\mathbb{F})$ conjugate to G, with irreducible main diagonal blocks (write the elements of G with respect to a basis obtained from the composition series). Then look at the kernel of a homomorphism defined on this block triangular group.

2. For $x \in E$ and $v \in \mathbb{F}^n \setminus \{\mathbf{0}\}$, consider the actions of A on $x\mathbb{F}^n$ and on Ev. If A is absolutely irreducible, assume $\mathbb{F}$ algebraically closed and consider the action of A on $(a - \lambda 1_n)\mathbb{F}^n$ where $\lambda \in \mathbb{F}$ is an eigenvalue of $a \in A$.

3. Suppose that G has a unipotent normal subgroup N such that G/N is abelian. Let $U(G)$ be the subgroup in Q.1, i.e., $U(G)$ is the unique normal unipotent subgroup of G that contains all such subgroups of G. Assume that $\mathbb{F}$ is algebraically closed; then $G/U(G)$ is a diagonal matrix group by Q.2.

4. Cf. Q.2. First show that $Z(G)$ consists of scalar matrices.

5. Choose $g \in K$ of prime order p. We have $g = 1_n + mx$ for some non-zero $n \times n$ matrix x, whose entries can be assumed pairwise coprime. Expand the left-hand side of $g^p = 1_n$ using the binomial theorem. Show that $m = p$, and derive a contradiction.

8. G has the congruence subgroup property: each $H \leq G$ of finite index contains a principal congruence subgroup, i.e., the kernel of φ_m for some m coprime to μ. Identify an isomorphism between a subgroup of H and an ideal of $\mathbb{Z}[1/\mu]$.

10. If $G = \varphi_p(H)$ is absolutely irreducible, then the enveloping algebra $\langle G\rangle_{\mathbb{Z}_p}$ has a basis of size n^2 contained in G.

12. Use the following: if X is a closed subset of $\mathrm{SL}(n,\mathbb{C})$, then so too is $gX\ \forall\, g \in \mathrm{SL}(n,\mathbb{C})$; the closure of a subgroup of $\mathrm{SL}(n,\mathbb{C})$ is also a subgroup; $\mathrm{SL}(n,\mathbb{C})$ has no proper closed finite-index subgroups (it is connected).

15. Proofs of the congruence subgroup property for $\mathrm{SL}(n,\mathbb{Z})$ hinge on the fact (with a long proof) that $\Gamma_{n,m}$ is the normal closure of the subgroup generated by $t_{1,2}(m)$.

16. $\mathrm{SL}(n,\mathbb{Z}_k)$ is generated by transvections $t_{i,j}(1)$, and m has a multiplicative inverse mod k.