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Integral packings and Number Theory

The course will use Apollonian circle packings as a central example for connections between number theory and thin groups. The symmetries of such a packing are governed by a thin group called the Apollonian group, and the curvatures form an orbit of that group. Our goal is to study such orbits, particularly in the case that the orbit consists entirely of integers. Some of the topics that are entwined with the study of these packings include quadratic forms, hyperbolic geometry in 2 and 3 dimensions, arithmetic geometry, continued fractions, spectral theory of graphs and strong approximation. I will give a tour of the area, with the goal of introducing the number theory perspective on such problems, highlighting the tools at hand, and finishing by considering the wider class of problems that can be phrased as questions about the arithmetic of thin orbits.