Abstracts

Contents

Fabrizio Bianchi
Università di Pisa, Italy

Parabolic implosion in two complex variables

The theory of parabolic implosion - i.e., the study of perturbations of parabolic fixed points - has been at the centre of the research in one-dimensional holomorphic dynamics in the last several decades. I will discuss recent progress in the generalization of this theory to two complex variables, as well as some applications. In particular, I will describe a recent result obtained with Matthieu Astorg on the dynamics of perturbations of semi-parabolic Hénon maps.

Back to Contents Page →

Jean-Michel Bismut
Université Paris-Saclay, France

IMS Distinguished Lecture Series

The hypoelliptic Laplacian in real and complex geometry

The purpose of the talk is to explain the construction of the hypoelliptic Laplacian in real and complex geometry, and to review some applications that have been obtained so far. The hypoelliptic Laplacian is a deformation of the Laplacian by a family of operators acting on a bigger space (which is often the total space of the tangent bundle) that interpolates between the Laplacian and the generator of the geodesic flow. These new operators are geometric Fokker-Planck operators. The geometric content of the deformation is specific to the geometry that is considered. There are are different motivations for this construction: either because of the interpolation property itself, and also because moving away from the original space allows for the lifting of obvious geometric obstructions, like the existence of a K¨ahler metric. In the talk, we will illustrate the construction in the context of de Rham theory and of Dolbeault theory.

Back to Contents Page →

Slawomir Dinew
Jagiellonian University, Poland

Geodesics in function spaces

We discuss the construction of geodesics in the space of K\"ahler potentials and the corresponding ideas in various function spaces. We shall survey the problem of existence of classical geodesics focusing on the regularity issues.

Back to Contents Page →

Siarhei Finski
École Polytechnique, France

Wess-Zumino-Witten equation and asymptotic semistabilty of direct images

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We will prove a partial generalisation of Kobayashi-Hitchin correspondence where instead of holomorphic vector bundles, we consider arbitrary polarized fibrations. More precisely, for a polarized family of complex projective manifolds, we consider a version of Wess-Zumino- Witten equation (which can be seen as a generalisation of Hermite-Einstein equation) and show that if there is a solution to this equation, then the direct image sheaves associated with high tensor powers of the polarising line bundle have to be asymptotically semistable. This will be established by providing lower bounds on a fibered version of Yang-Mills functionals in terms of Harder-Narasimhan slopes of the direct images. We discuss the optimality of these lower bounds and, as an application, provide an analytic characterisation of a fibered version of generic nefness.

Back to Contents Page →

Chin-Yu Hsiao
Academia Sinica, Taipei

Semi-classical analysis in several complex variables

Semi-classical analysis plays an important role in geometry, mathematical physics. In this talk, I will explain how to use semi-classical analysis to study some classical problems in several complex variables. I will also introduce my works about Semi-classical Bergman kernel asymptotic for pseudoconvex domains. This talk is based on joint works with Hendrik Herrmann, George Marinescu, Xiaoshan Li and Wei-Chuan Shen.

Back to Contents Page →

Lucas Kaufmann
Université d'Orléans, France

Localization of characteristic forms of coherent sheaves and holomorphic foliations

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Chern classes of coherent sheaves are typically defined through global resolutions by vector bundles, provided they exist. These classes are represented by smooth forms with compact support. In scenarios where certain vanishing theorems apply — such as for normal sheaves of holomorphic foliations or for sheaves supported by proper analytic subsets — it is well known that such Chern classes localize to a neighbourhood of an analytic set. It has been shown by Lärkang-Wulcan and Kaufmann-Lärkang-Wulcan that this localization admits well defined limits as currents of residue type. In this talk I will overview these results and present a work in progress that extends these constructions to the setting where no global resolutions are available.

Back to Contents Page →

Slawomir Kolodziej
Jagiellonian University, Poland

Hölder continuous solution to complex Monge-Ampère equations

In this talk we review the assumptions on the right hand side of the complex Monge-Ampère equation which yield Hölder continuous solutions. This is done for the following settings: strictly pseudoconvex domains, compact Kähler manifolds, compact Hermitian manifolds, and Hermitian manifolds with boundaries. Such solutions do appear in several geometric contexts.

Back to Contents Page →

George Marinescu
Universität zu Köln, Germany

Toeplitz operators and zeros of square-integrable random holomorphic sections

We employ the theory of abstract Wiener space to construct a probabilistic model for Berezin-Toepltiz quantization on a complete Hermitian complex manifold equipped with a positive line bundle.

We consider the asymptotic distributions of their zeros in the semi-classical limit, in particular, we prove equidistribution results, large deviation estimates, central limit theorem of the random zeros on the support of the given function. This is a joint work with Alexander Drewitz and Bingxiao Liu.

Back to Contents Page →

Ngaiming Mok
The University of Hong Kong, Hong Kong

From holomorphic isometries to functional transcendence on quotients of bounded symmetric domains by arbitrary cocompact lattices

Consider a Kähler manifold (X, g). When g can be expanded in power series, in his seminal work on holomorphic isometries Eugenio Calabi introduced the notion of the diastasis and proved powerful extension theorems on holomorphic isometries from Kähler manifolds into space forms such as the projective space equipped with the Fubini-Study metric. On a bounded domain $U \Subset \mathbb{C}^n$ we denote by ds^2_U the Bergman metric on U (which is Kähler). Among bounded domains there are the bounded symmetric domains Ω classified by Élie Cartan such that (Ω, ds^2_Ω) are symmetric in the sense of Riemannian geometry, $\Omega = G/K$ in standard notation. Here $\Omega \Subset \mathbb{C}^n$ in their standard realizations are semi-algebraic, i.e., defined by algebraic inequalities in the $2n$ real Euclidean coordinates underlying $\mathbb{C}^n \cong \mathbb{R}^{2n}$. By an irreducible algebraic subvariety of Ω we mean an irreducible component of the intersection $V \cap \Omega$ of an affine algebraic subvariety $V \subset \mathbb{C}^n$ with the bounded symmetric domain $\Omega \Subset \mathbb{C}^n$. In this lecture I will explain: *(1) why germs of holomorphic isometries between bounded domains with respect to the Bergman metric extends algebraically under a rationality assumption on the Bergman kernel, (2) how the study of the asymptotic behaviour of holomorphic isometries of the Poincaré disk led to a uniformization theorem for projective varieties covered by algebraic subvarieties of Ω, and (3) how the latter serves as a starting point for research in functional transcendence theory concerning* $X_\Gamma = \Omega/\Gamma$, *where* $\Gamma \subset G$ *is an* <u>*arbitrary*</u> *lattice.*

In the special case of arithmetic lattices, (3) has been settled yielding the *Ax-Schanuel theorem on Shimura varieties* (with extensive generalizations by now) by Mok-Pila-Tsimerman (2019), using techniques involving in particular model theory from mathematical logic, techniques which are no longer available for arbitrary lattices. In the case where $X = \Omega/\Gamma$ and $\Gamma \subset \mathrm{Aut}(\Omega)$ is an arbitrary cocompact lattice, I will explain how functional transcendence results on X can be proven using analytic techniques, starting with the <u>rescaling</u> method related to *partial Cayley* transforms on subvarieties of a bounded symmetric domain exiting $\partial\Omega$

Back to Contents Page →

Ngoc-Cuong Nguyen
Korea Advanced Institute of Science & Technology, Korea

Regularity of the Siciak-Zaharjuta extremal function on compact Kähler manifolds

We prove that the regularity of the extremal function of a compact subset of a compact Kähler manifold is a local property, and that the continuity and Hölder continuity are equivalent to classical notions of the local L-regularity and the locally Hölder continuous property in pluripolential theory. As a consequence we give an effective characterization of the $(C_\alpha, C_{\alpha'})$-regularity of compact sets, the notion introduced by Dinh, Ma and Nguyen. Using this criterion all compact fat subanalytic subsets in $\mathbb{R}^n$ are shown to be regular in this sense.

Back to Contents Page →

Junjiro Noguchi
The University of Tokyo, Japan

On Lang's conjecture, number of rational sections and related topics

A conjecture of S. Lagn claims the finiteness of rational points on algebraic varieties defined over a number field which is Kobayashi hyperbolic. After recalling my work on the analogue over function fields, we propose a problem to count the number of sections “modulo constant”. In the case of curve family, we show some estimate of those number by making use of Kobayashi hyperbolic distance and the Imyoshi-Shiga rigidity theorem.

Back to Contents Page →

Vamsi Pritham Pingali
Indian Institute of Science, India

Gravitating vortices and cosmic strings

The gravitating vortex (GV) equations on a compact Riemann surface arise as a dimensional reduction of the Kaehler-Yang-Mills equations. A special case of the GV equations includes the equations governing the (as of now, hypothetical) cosmic strings. I shall describe existence, uniqueness, and algebro-geometric obstructions to existence to these equations. Towards the end, I shall also make a small remark on two about speculations on the "quantisation" of these equations using balanced metrics and Bergman kernels. This talk is based on joint work with M. Garcia-Fernandez, L. Alvarez-Consul, O. Garcia-Prada, and Chengjian Yao.

Back to Contents Page →

Karim Rakhimov
Uzbekistan

Dynamics of Hénon-like maps

Hénon maps are among the most studied dynamical systems. Hénon-like maps are invertible holomorphic maps, defined on some convex bounded domain of $\mathbb{C}^k$, that have an expanding behaviour in p directions and contracting behaviour in the remaining $k-p$ directions. They form a large class of dynamical systems in any dimension, that contain Hénon maps in dimension 2. In this talk, we show that the sequence of their dynamical degrees is non-decreasing until the main dynamical degree, and non-increasing after that. As an application, we also show that their Green currents are woven. This is a joint work with Fabrizio Bianchi and Tien-Cuong Dinh.

Back to Contents Page →

Min Ru
University of Houston, USA

Non-integrated defect relation for holomorphic curves into algebraic varieties

In 1983, inspired by his investigation into the value distribution of the Gauss map of minimal surfaces, H. Fujimoto introduced the concept of non-integrated defect for holomorphic maps from a complete Riemann surface (with a Kahler metric) into complex projective space. When the Riemann surface is the complex plane, the non-integrated defect serves as a refinement of the classical Nevanlinna's defect notion.

In recent years, significant advancements have been made in the study of complex hyperbolicity and the general Second Main Theorem. Notable examples include the general Second Main Theorem (through the jet-differentials) established by K. Yamanoi and the resolution of the Kobayashi conjecture by D. Brotbek, along with its generalization concerning the Second Main Theorem for generic hypersurfaces in P^n by D. Brotbek and Y. Deng. In this talk, I will discuss how to extend these results to non-integrated defect relations. This is a joint work with Q. Cai and C. J. Yang.

Back to Contents Page →

Nikhil Savale
Universität zu Köln, Germany

The eta invariant of a circle bundle on a Fano manifold

We consider the spin-c Dirac operator on the unit circle bundle of a positive line bundle over a Fano manifold of even complex dimension. We compute the corresponding eta invariant in terms of Zhang's value of its adiabatic limit. This extends the earlier computation of the author from small to arbitrary values of the adiabatic parameter.

Back to Contents Page →

Laurent Stolovitch
Université Côte d'Azur, France

Local rigidity of actions of isometries on compact real analytic Riemannian manifolds

In this work in collaboration with Z. Zhao (Nice), we consider analytic perturbations of isometries of an analytic Riemannian manifold M. We prove that, under some conditions, a finitely presented group of such small enough perturbations is analytically conjugate on M to the same group of isometry it is a perturbation of. Our result relies on a ``Diophantine-like" condition, relating the actions of the isometry group and the eigenvalues of the Laplace-Beltrami operator. Our result generalizes Arnold-Herman's theorem about diffeomorphisms of the circle that are small perturbations of rotations.

Back to Contents Page →

Do Duc Thai
Hanoi National University of Education, Vietnam

Volume of Components of Lelong Upper-Level Sets

We prove an upper bound for the volume of maximal analytic sets on which the generic Lelong number of a closed positive current is positive. As a particular case, we give a uniform upper bound on the volume of the singular locus of an analytic set in terms of its volume on a compact Kähler manifold. This work is a joint work with Dr. Duc-Viet Vu.

Back to Contents Page →

Gabriel Vigny
Université de Picardie Jules Verne, France

The Central Limit Theorem for generic birational maps

Very recently, in the case of the measure of maximal entropy of Hénon maps, Bianchi and Dinh established the exponential mixing of all orders and as a consequence of a work of Björklund and Gorodnik, the Central Limit Theorem (CLT) for Hölder observables.

With De Thélin, we extended their results to the larger class of generic birational maps of P^k. Because of the indeterminacy set, H\"older maps are not stable under iteration, so we need to work with a suitable space of test functions.

Back to Contents Page →

Duc-Viet Vu
Universität zu Köln, Germany

Lebesgue points of functions in complex Sobolev spaces

The complex Sobolev space was introduced by Dinh and Sibony in complex dynamics. We show that the complement of the set of Lebesgue points for functions in complex Sobolev spaces is pluripolar. This in particular says that there is a canonical representation of functions in the complex Sobolev spaces. This is a joint-work with Gabriel Vigny.

Back to Contents Page →

Julie Tzu-Yueh Wang
Academia Sinica, Taipei

GCD, abc and quasi-hyperbolicity

In a recent joint work with Guo, Nguyen, and Sun, we derived Vojta's generalized abc conjecture for algebraic tori over function fields with explicitly determinable exceptional sets via a GCD theorem for multivariable polynomials evaluating at S-units arguments. As an application, we investigate the Lang-Vojta Conjecture for varieties of log general type that are ramified covers of algebraic tori over function fields.

The aforementioned methods also apply to the complex situation, enabling the identification of exceptional sets for the corresponding complex version of Vojta's general abc conjecture and the Green-Griffith-Lang conjecture for varieties of log general type that are ramified covers of algebraic tori. In this talk, a GCD theorem with moving targets established by Levin and the speaker will be introduced first. We then discuss how this theorem is employed to study complex cases of Vojta's abc conjecture and Campana's orbifold curves, leading to conclusions on some cases of the Green Griffith-Lang conjecture with explicit exceptional sets.

Back to Contents Page →

Xiangyu Zhou
Chinese Academy of Sciences, China

Multiplier ideal/submodule sheaves and converse L^2 theory

We recall some recent results on multiplier ideal sheaves associated to pseudoeffective line bundles, including the solution of Demailly's strong openness conjecture (by Guan Zhou), and explain some new results on multiplier submodule sheaves associated to singular hermitian metric on holomorphic vector bundles. We also present our recent study on converse L^2 theory (including converses of L^2 existence theorems and L^2 extensions) by a series of joint works with Deng, Ning, Wang, Zhang, in particular, we give a criteria of hermitian holomorphic vector bundles being Nakano semipositivity (established by Deng-Ning-Wang-Zhou) and as an application of the criteria a positive solution of Lempert's problem on Nakano semipositivity , which asked whether the limit metric of an increasing sequence of hermitian metrics with Nakano semi positive curvature on holomorphic vector bundle s is still Nakano semi positivepositive.

Back to Contents Page →