DISTINGUISHED LECTURE SERIES

The hypoelliptic Laplacian in real and complex geometry

The purpose of the talk is to explain the construction of the hypoelliptic Laplacian in real and complex geometry, and to review some applications that have been obtained so far.

The hypoelliptic Laplacian is a deformation of the Laplacian by a family of operators acting on a bigger space (which is often the total space of the tangent bundle) that interpolates between the Laplacian and the generator of the geodesic flow. These new operators are geometric Fokker-Planck operators. The geometric content of the deformation is specific to the geometry that is considered.

There are are different motivations for this construction: either because of the interpolation property itself, and also because moving away from the original space allows for the lifting of obvious geometric obstructions, like the existence of a Kähler metric.

In the talk, we will illustrate the construction in the context of de Rham theory and of Dolbeault theory.

Date : Friday, 3 May 2024
Time : 9 — 9.50am
Venue : IMS Auditorium
3 Prince George's Park
Singapore 118402

Hypoelliptic Laplacian and the trace formula

The heat equation method in index theory gives an explicit local formula for the index of a Dirac operator. Similar methods can be developed to give a geometric formula for semisimple orbital integrals associated with the Casimir operator of a reductive group, this computation being related to Selberg's trace formula. The analogue of the heat equation method is a suitable deformation of the Laplacian by a family of Fokker-Planck operators that interpolates between the Casimir operator and the geodesic flow. Finally, we will explain results obtained by Shu SHEN and ourselves when the Casimir operator is replaced by an arbitrary element of the center of the Lie algebra.

Date : Monday, 6 May 2024
Time : 9:30—10.30am
Venue : IMS Auditorium
3 Prince George's Park
Singapore 118402

Professor Jean-Michel Bismut
Université Paris-Saclay, France

Jean-Michel Bismut received his PhD from Université Paris VI. He joined the department of Mathematics at Université Paris-Sud (now Université Paris-Saclay) in 1976. He is a member of the Académie des Sciences, of Academia Europaea, of the German National Academy Leopoldina, and an international member of the National Academy of Sciences (USA). Prof. Bismut is a 2021 Shaw Prize recipient in Mathematical Sciences. His research interests include the Atiyah Singer index theory and its local refinements, and the interplay between probability theory, analysis and differential geometry. His work includes the stochastic calculus of variations, refined local versions of the Atiyah-Singer index theorem, eta invariants, analytic torsion, and Quillen metrics. His recent work includes the development of the hypoelliptic Laplacian, a family of operators that interpolates between the standard Laplacian and the geodesic flow.

The talk is part of the program on
Index Theory and Complex Geometry Part 2
(29 Apr —10 May 2024)

Registration
https://tinyurl.com/IMS2024Index2Reg

Program webpage
https://ims.nus.edu.sg/events/index-theory-and-complex-geometry2