Abstracts

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Anjie Dong,
Peking University, China

An AI-based approach to improving interactive theorem proving environment

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Practical insights serve as the foundation of our journey with interactive proof assistants, as both Lean4 prover users and educators. In this presentation, we will discuss some of the common pain points we have explored within the Lean4 ecosystem and strategies for mitigating them. Our proposed strategies underscore the necessity of an integrated proving environment designed for interactive proof assistants for enhancing efficiency and productivity. Additionally, we reflect on our past Lean4 seminars and short courses covering various areas of mathematics and Lean4 itself. This reflection helps us identify opportunities for enhancing the quality of future teaching plans. Furthermore, we provide an overview of the development journey of the integrated proving environment, detailing accomplished milestones thus far and outlining our future objectives.

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Ashvni Narayanan,
University of Sydney, Australia

The knotted pizza

Knots have gained a lot of attention in the mathematical world for a while. They are known for their connection to various fields such as topology, geometry, and number theory. There is still a lot of work going on to characterize knots or determining their invariants. Some knot invariants are known to be connected to the Burau representation defined on braids. I will talk about my attempt at formalizing knots and their relationship to the Burau representation in Lean.

A part of the talk will also be used to introduce the audience to a workshop based on the formalization of a special case of the pizza problem. We show that, given an even number of slices, a pizza can be eaten by 2 players in such a way that the first player gets at least 4/9th of the pizza.

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Bhavik Mehta,
University of Cambridge, UK

Formalisation of Combinatorics

I will discuss the recent history of formalisation in combinatorics, focusing on the Lean theorem prover, aiming to demonstrate that Lean's knowledge of mathematics is - in some fields - rapidly approaching the human limit. I will detail some of the unique techniques and particular challenges that arise in this particular area of mathematics, and discuss how these can be useful to any formalisation efforts.

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Bichang Lei & Yichang Tao,
Renmin University of China, China

Learning Mathematics with Lean

In this talk, we will discuss recent progress in developing a semantic search engine for mathlib4. We will explain our method of fine-tuning a text embedding model to enhance the engine's performance and how this retrieval system can be used for premise selection and automated theorem proving.

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Bin Dong, Guoxiong Gao, Haocheng Ju,
Peking University, China

AI for Mathematics: Goals, Plans and Tools

In recent years, the integration of Artificial Intelligence (AI) with mathematical research has shown promising progress. This talk will begin by delving into the motivation behind employing AI to facilitate mathematical research, followed by a review of some selected recent works in this field. Subsequently, we will outline our future plans to develop an AI assistant specifically tailored for mathematical research. This assistant will be equipped with a range of tools, including a semantic search tool for efficient theorem retrieval, an autoformalizer to translate informal mathematical text into formal language, and an automated theorem prover, among others. In the latter half of the talk, we will introduce a semantic search tool designed for mathlib4. We will describe the implementation of this tool, along with a benchmark for assessing the performance of various search engines for mathlib4.

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Clarence Chew, Jingquan Chong, Mengzhou Sun, Yutong Wang,
National University of Singapore, Singapore

Towards the formalisation of Coxeter combinatorics

Coxeter groups form a class of important groups. It has elegant geometric and combinatorial properties, while maintaining deep connection with foundations of group theory, making its way in various advanced topics in modern mathematics.

In this talk, we will show how we utilize Lean interactive prover to formalize Coxeter groups, as well as its underlying interesting combinatorics. We will provide an overview of current project, outline the challenges encountered, and discuss prospective directions.

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Filippo Nuccio,
Université Jean Monnet, France

Formalizing local fields in Lean

Although some basic results concerning general valuation rings and normed field have been in mathlib for a while, this work introduces for the first time the notion of discrete valuation and local fields, and connects them to the main results about discrete valuation rings. We rely crucially on recent works about completion of Dedekind Domains at prime ideals and, as an application, we formalize the proof that Laurent series are the completion of the field of the rational functions and we propose an equivalent definition of the p-adic numbers working directly with the p-adic valuation rather than starting from the R-valued p-adic metric. This is a joint work with M. I. de Frutos Fernández.

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Haocheng Ju & Guoxiong Gao,
Peking University, China

Recent Progress on Mathlib4 Semantic Search

In this talk, we will discuss recent progress in developing a semantic search engine for mathlib4. We will explain our method of fine-tuning a text embedding model to enhance the engine's performance and how this retrieval system can be used for premise selection and automated theorem proving.

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Ilya Sergey,
National University of Singapore, Singapore

Proving as Programming

In this introductory talk, we will explore the foundational connection between programming languages and mathematical logics. We will see how proving a theorem is equivalent to writing a program, and how stating a theorem is equivalent to specifying a program---a beautiful connection known as the Curry-Howard correspondence. Finally, we will see how this correspondence is exploited by the modern proof assistants to allow for machine-checked formal proofs.

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Jiedong Jiang,
Peking University, China

Formalizing Ramification Groups in Lean

Ramification groups of an extension of a local field play an important role in both local class field theory and p-adic Hodge theory. In this talk, after a brief recall of the ramification theory of local fields, we will discuss the formalization of ramification groups of local fields in Lean. This is joint work with Junjie Bai and Bichang Lei, built on the previous work of Maria Indes de Frutos Fernandez and Filippo A. E. Nuccio on local fields.

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Kelin Xia,
Nanyang Technological University, Singapore

Mathematical AI for Molecular Sciences

Artificial intelligence (AI) based Molecular Sciences have begun to gain momentum due to the great advancement in experimental data, computational power and learning models. However, a major issue that remains for all these AI-based learning models is the efficient molecular representations and featurization. Here we propose advanced mathematics-based molecular representations and featurization. Molecular structures and their interactions are represented by high-order topological and algebraic models (including Rips complex, Alpha complex, Neighborhood complex, Dowker complex, Hom-complex, Tor-algebra, etc). Mathematical invariants (from persistent homology, persistent Ricci curvature, persistent spectral, etc) are used as molecular descriptors for learning models. Further, we develop geometric and topological deep learning models to systematically incorporate molecular high-order and multiscale information, and use them for analysing molecular data from chemistry, biology, and materials.

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Kevin Buzzard,
Imperial College London, UK

Colloquium Talk
Why formalise mathematics?

Computer programs which understand the axioms of mathematics have existed for decades. However, it is only in the last few years that they have captured the imagination of mathematicians. In this talk, I will give an overview of where we are in 2024, what these systems can currently do, and what they cannot yet do. I will discuss how these systems have helped mathematicians, and ways in which they may help mathematicians in the future. I will be careful to distinguish facts from science fiction. The talk will be suitable for a general mathematical audience and will not assume any expertise in AI, computer science, or interactive theorem provers.

Formalising Fermat

The proof by Wiles and Taylor—Wiles of Fermat's Last Theorem was undoubtedly one of the highlights of 20th century mathematics. In this talk I will discuss a strategy to teach the computer proof system Lean a modern proof of this theorem.

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Qianxiao Li,
National University of Singapore, Singapore

Constructing custom thermodynamics using deep learning

We discuss some recent work on constructing stable and interpretable macroscopic thermodynamics from trajectory data using deep learning. We develop a modelling approach: instead of generic neural networks as functional approximators, we use a model-based ansatz for the dynamics following a suitable generalisation of the classical Onsager principle for non-equilibrium systems. This allows the construction of macroscopic dynamics that are physically motivated and can be readily used for subsequent analysis and control. We discuss applications in the analysis of polymer stretching in elongational flow.

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Siddhartha Gadgil,
IISc India, India

Programs with Proofs and Meta-Programming in Lean

The Lean Theorem prover is a programming language with a type system rich enough to encode arbitrary mathematical statements and proofs. It also provides powerful techniques for proving and has a large mathematical library.

I will illustrate how we can implement algorithms and prove their correctness in Lean. I will mostly focus on simple examples but will briefly show some non-trivial cases.

Lean also has a seamless integration of Metaprogramming with programs and proofs. I will illustrate this, especially in the context of using this for Artificial Intelligence based tools.

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Vladimir Gladshtein,
National University of Singapore, Singapore

Small Scale Reflection for the Working Lean User

We present the design and implementation of the Small Scale Reflection proof methodology and tactic language (a.k.a. SSR) for the Lean 4 proof assistant. Like its Coq predecessor SSReflect, our Lean 4 implementation, dubbed LeanSSR, provides powerful rewriting principles and means for effective management of hypotheses in the proof context. Unlike SSReflect for Coq, LeanSSR does not require explicit switching between the logical and symbolic representation of a goal, allowing for even more concise proof scripts that seamlessly combine deduction steps with proofs by computation. In this paper, we first provide a gentle introduction to the principles of structuring mechanised proofs using LeanSSR. Next, we show how the native support for metaprogramming in Lean 4 makes it possible to develop LeanSSR entirely within the proof assistant, greatly improving the overall experience of both tactic implementers and proof engineers. Finally, we demonstrate the utility of LeanSSR by conducting two case studies: (a) porting a collection of Coq lemmas about sequences from the widely used Mathematical Components library and (b) reimplementing proofs in the finite set library of Lean's mathlib4. Both case studies show significant reduction in proof sizes.

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Zaiwen Wen,
Peking University, China

Mathematical Formalization for Applied Mathematics

Theoretical understanding represents the cornerstone of applied mathematics, embodying its intrinsic value and guiding its practical applications. In this talk, we will present our preliminary strategies aimed at formalizing the core disciplines within applied mathematics. We will report our recent advancements in mathematical optimization, alongside the development of a framework employing Large Language Models to transition informal proofs into their formal equivalents using Lean. Furthermore, we are excited to announce "Reaslab", an innovative tool designed for machine-assisted reasoning. Developed by the team at Peking University, Reaslab harnesses the power of artificial intelligence to facilitate and enhance the process of mathematical reasoning and proof verification.

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