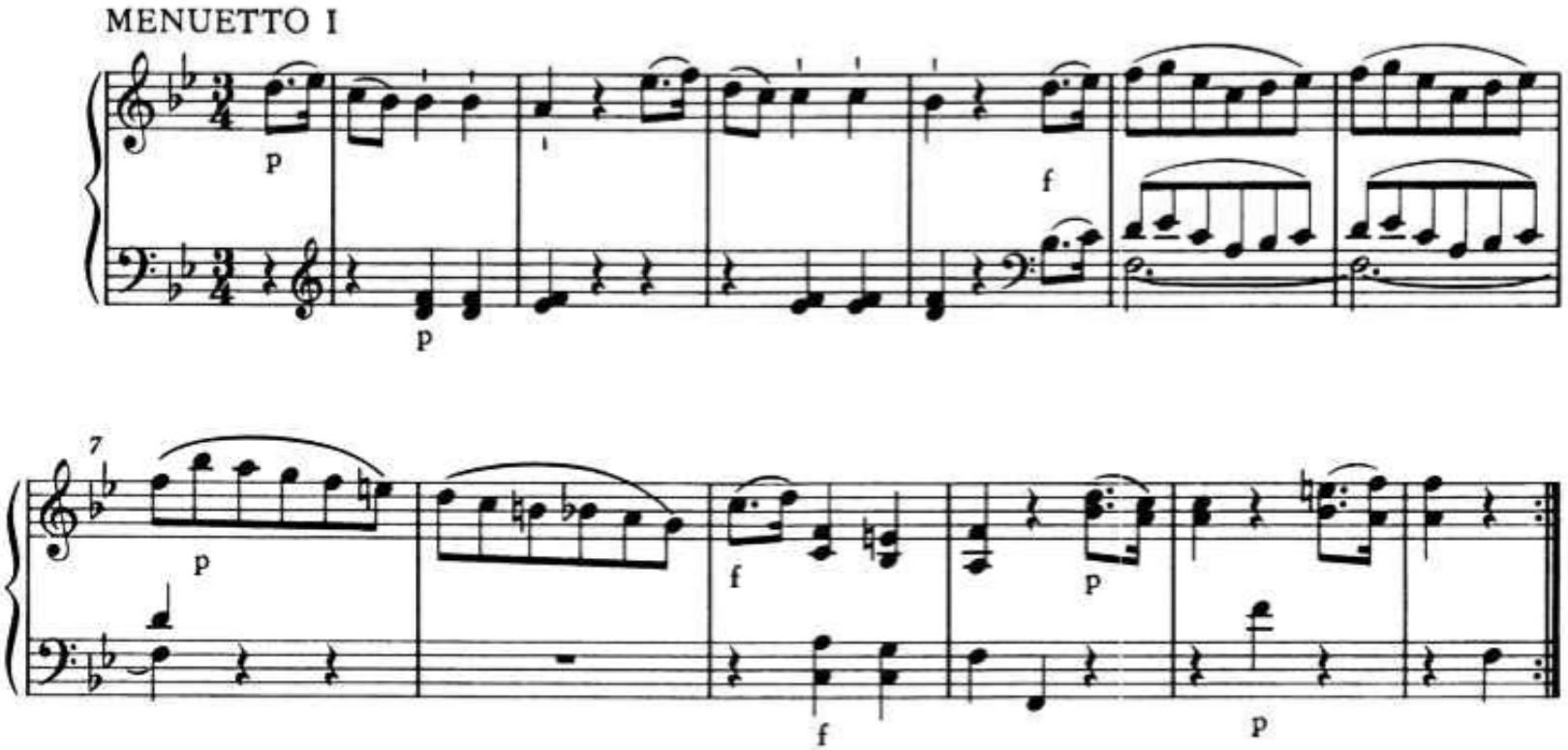
DEPARTMENT of MUSIC AT PRINCETON

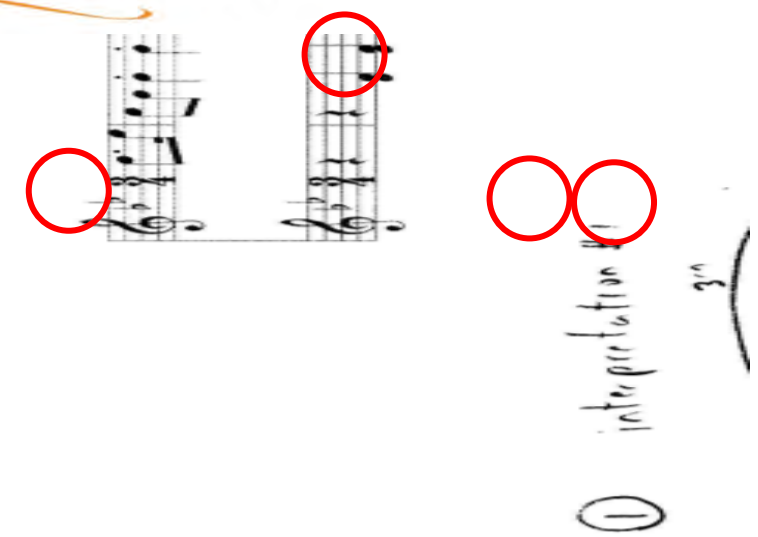
http://dmitri.tymoczko.com

A NEW MODEL OF MUSICAL HIERARCHY

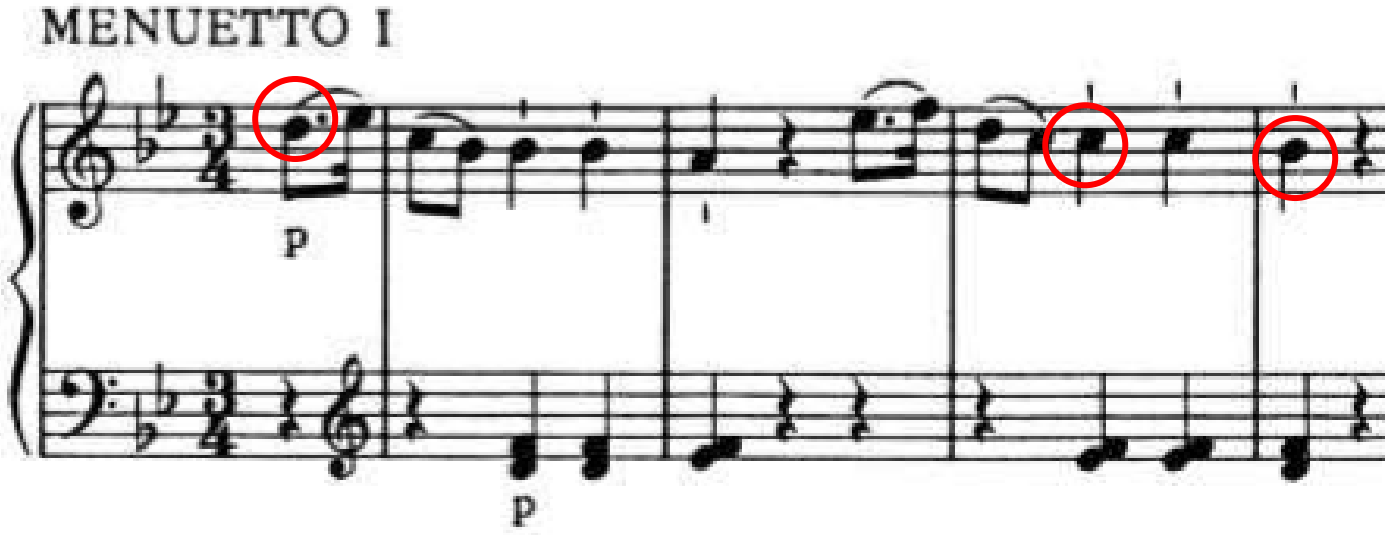
Dmitri Tymoczko
Princeton University



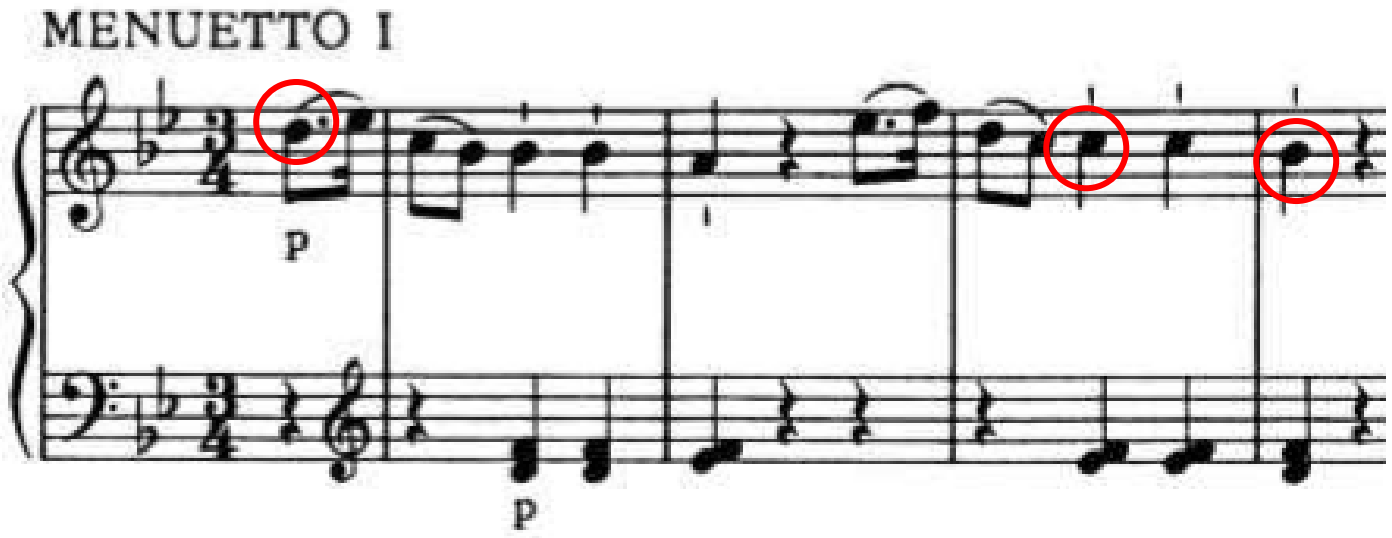
Mozart K. 282, II



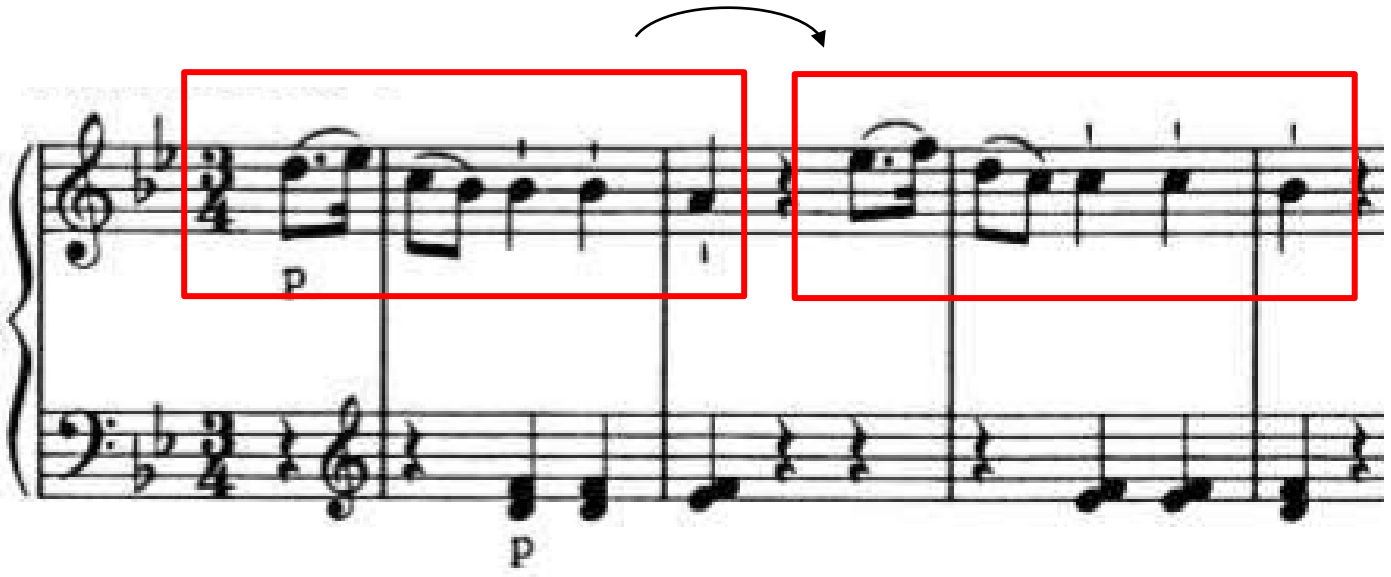
Poundie Burstein (unpublished)



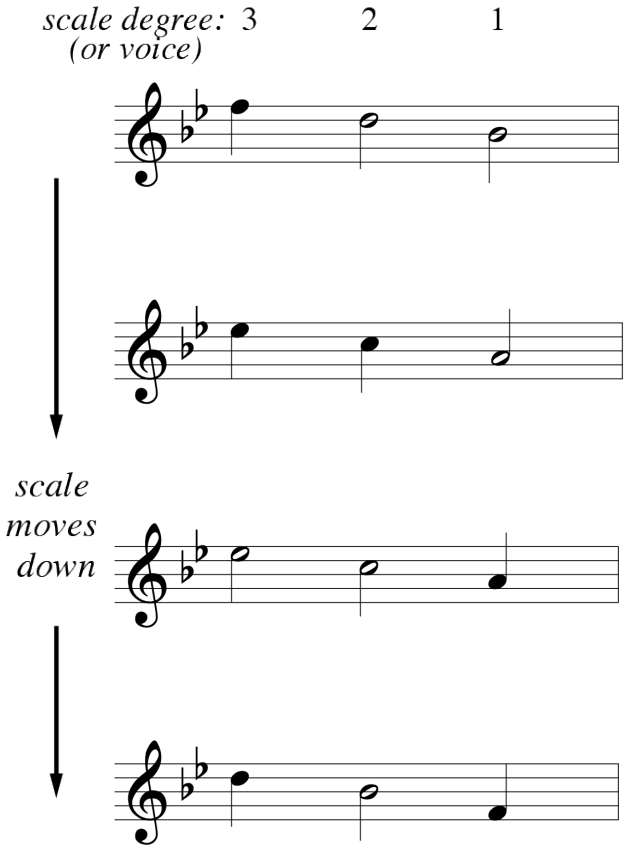
- Analysis is asymmetrical ☹
  - links first part of first motive with last part of the repeat.



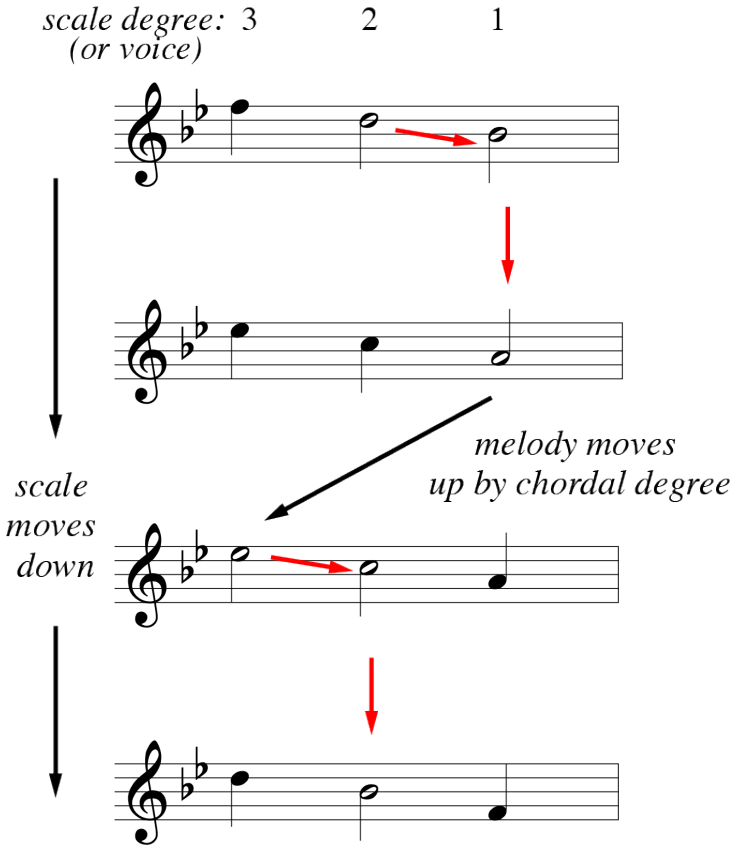
- Phrase is depicted as descending ☹
  - The motive ascends by scale step.



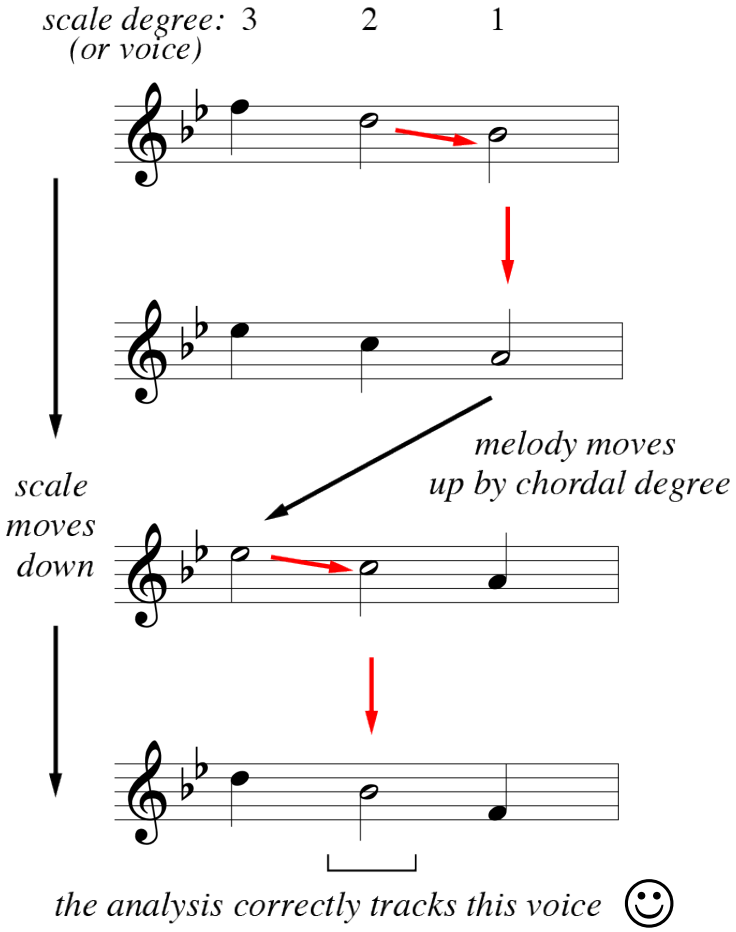
- All of this makes sense if we think of chords as little scales.



- All of this makes sense if we think of chords as little scales.



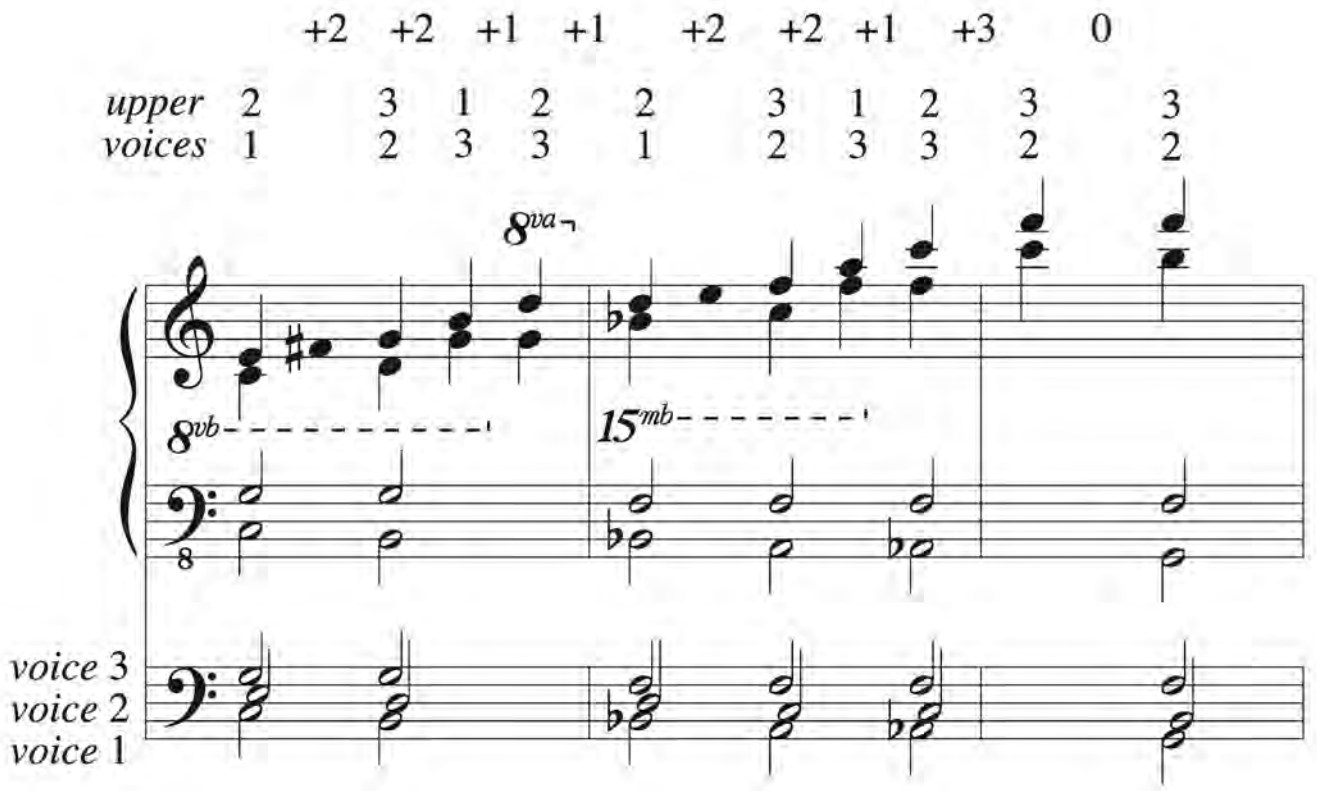
- All of this makes sense if we think of chords as little scales.



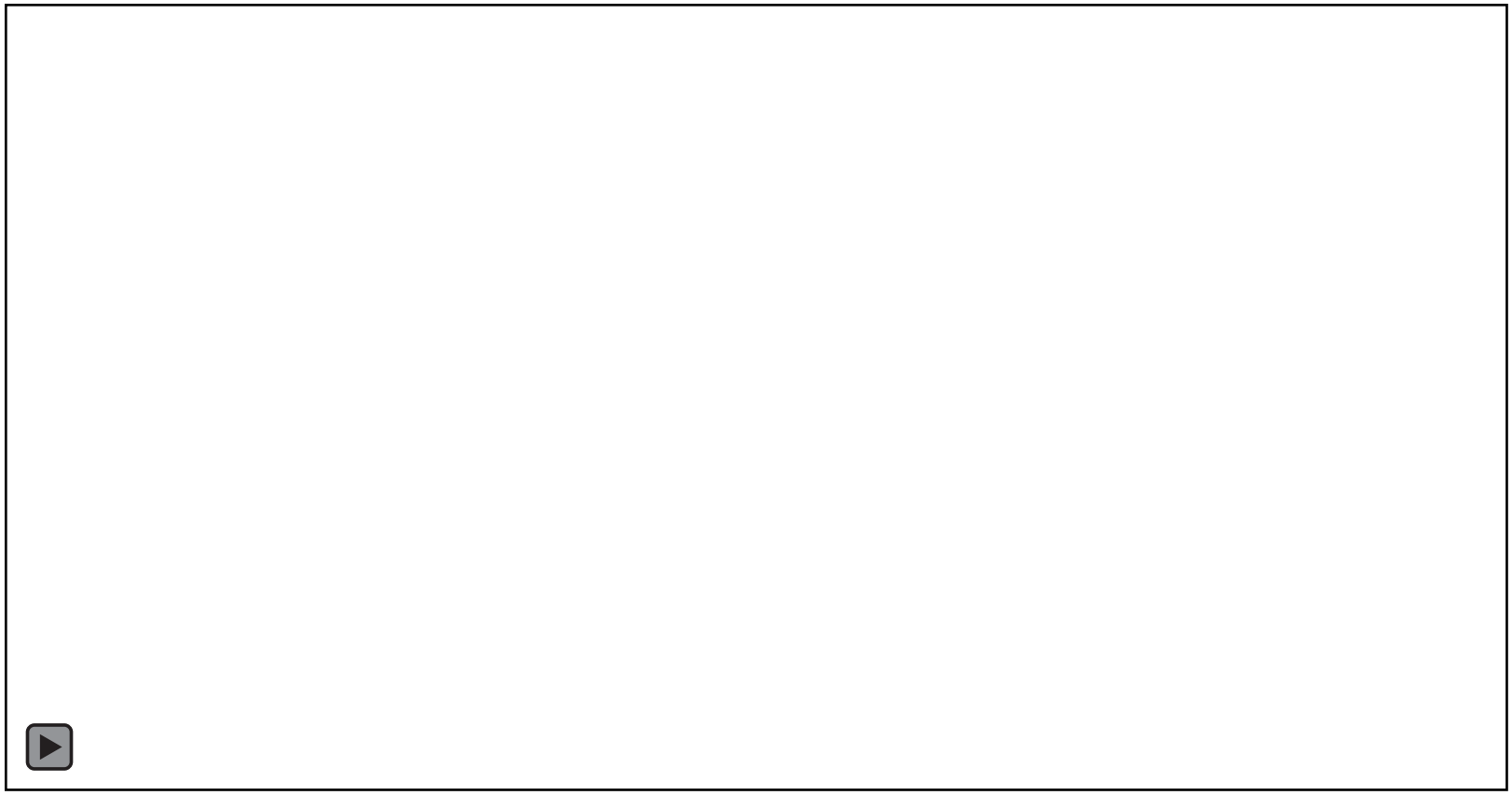
Once you realize chords can be little scales, you start to find all sorts of manifestations of this pattern.

*In particular, you often see voices moving **contrary** to the underlying voice leading.*

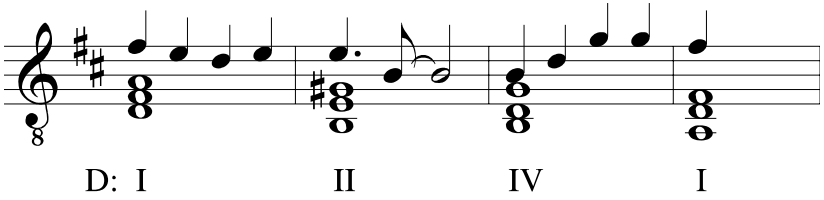
EXAMPLE: Waldstein



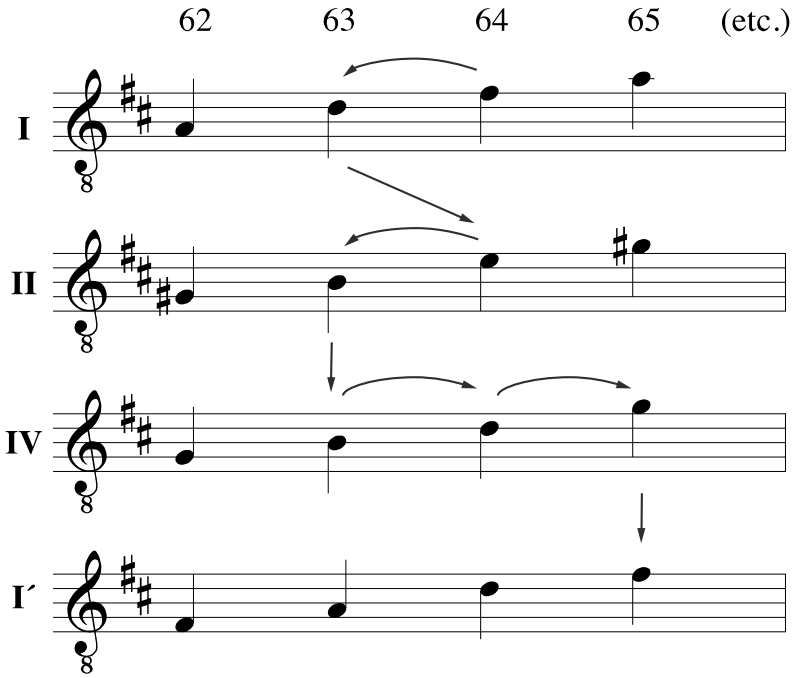
EXAMPLE: Waldstein



"Eight Days a Week" (The Beatles)



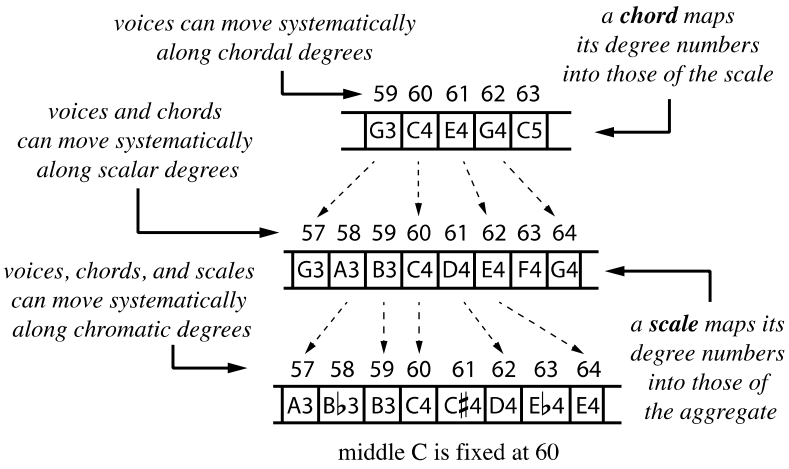
scale degrees:



(F♯4 is scale degree 65 on returning to D major)

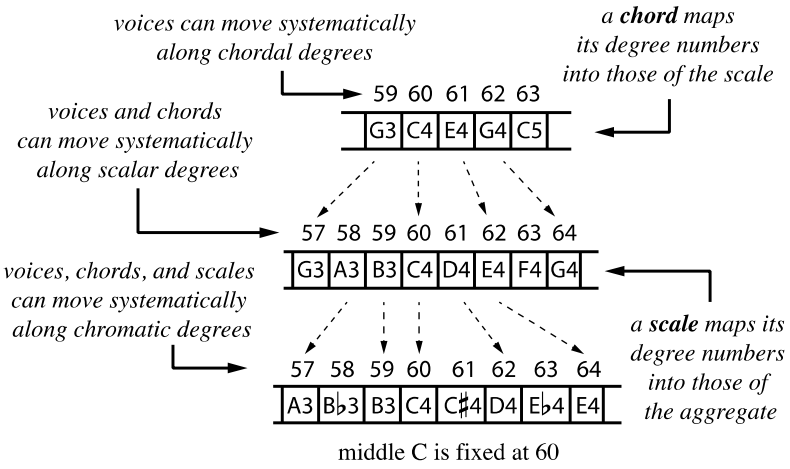
FUNDAMENTAL CLAIM

- Music moves simultaneously along many nested collections:
  - Chord
  - Scale
  - Sometimes chordal or scalar subsets
- This motion is simple when we look at any one level in isolation.
- It becomes quite complex when we look at multiple levels simultaneously.



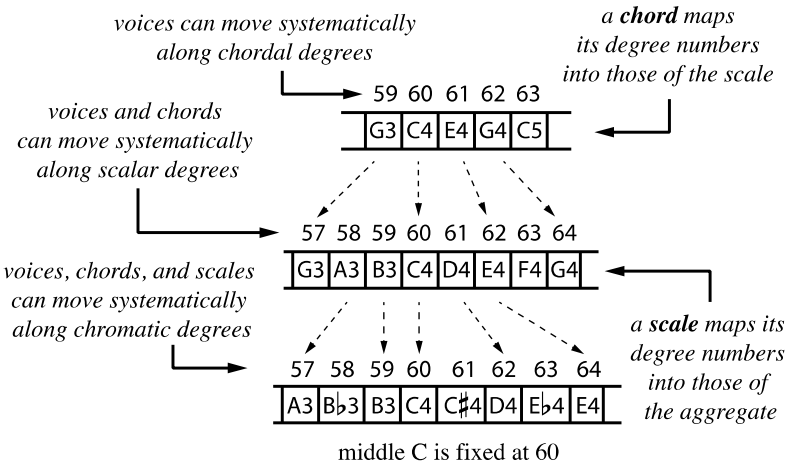
FUNDAMENTAL CLAIM

- This picture originates with the cognitive scientist Herbert Simon.
  - It was developed by Deutsch and Feroe.
  - Later Lerdahl
- What I add:
  - Systematic mathematical tools for describing voice leading on each level.
  - Generalize the picture to arbitrary collections.



FUNDAMENTAL CLAIM

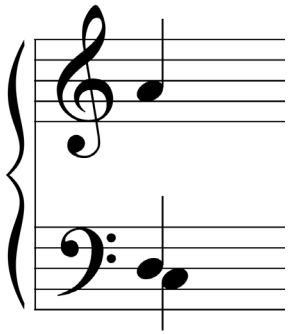
- Conjecture: the reason machine-learning doesn't work so well with music is because there isn't enough data, in any one musical style, to do much more than just learn this structure.
  - Ideas about how to fix this.



I. Applications

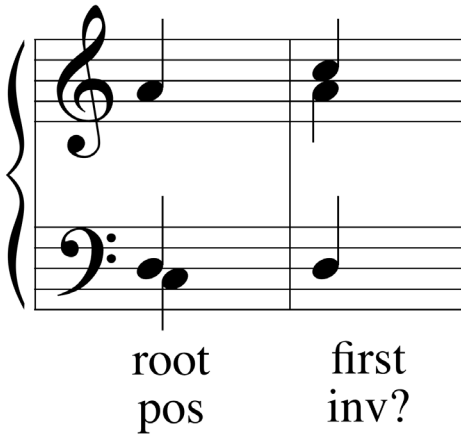
1. REGISTRAL INVERSION

- Suppose I gave you this chord and told you it was in root position. What is the first-inversion form?



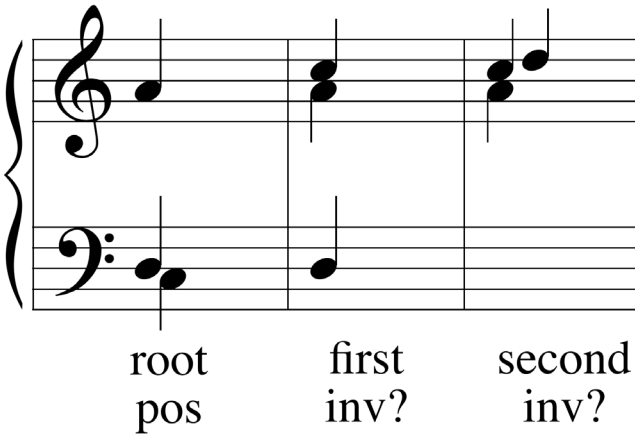
1. REGISTRAL INVERSION

- Suppose I gave you this chord and told you it was in root position. What is the first-inversion form?



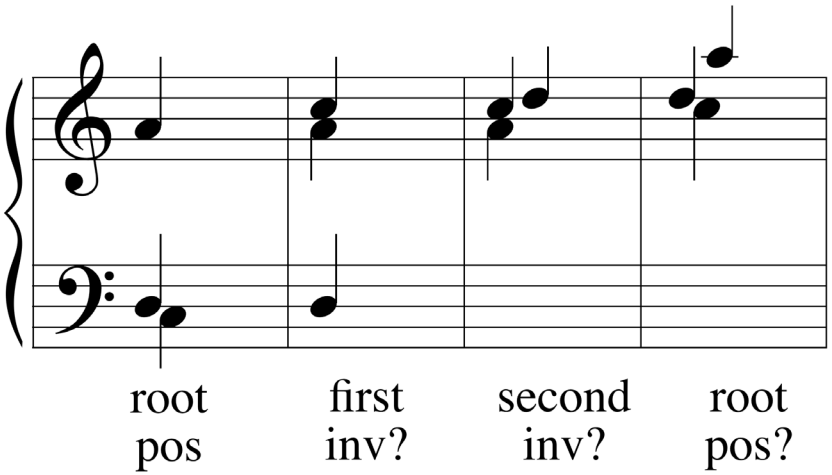
1. REGISTRAL INVERSION

- Suppose I gave you this chord and told you it was in root position. What is the first-inversion form?



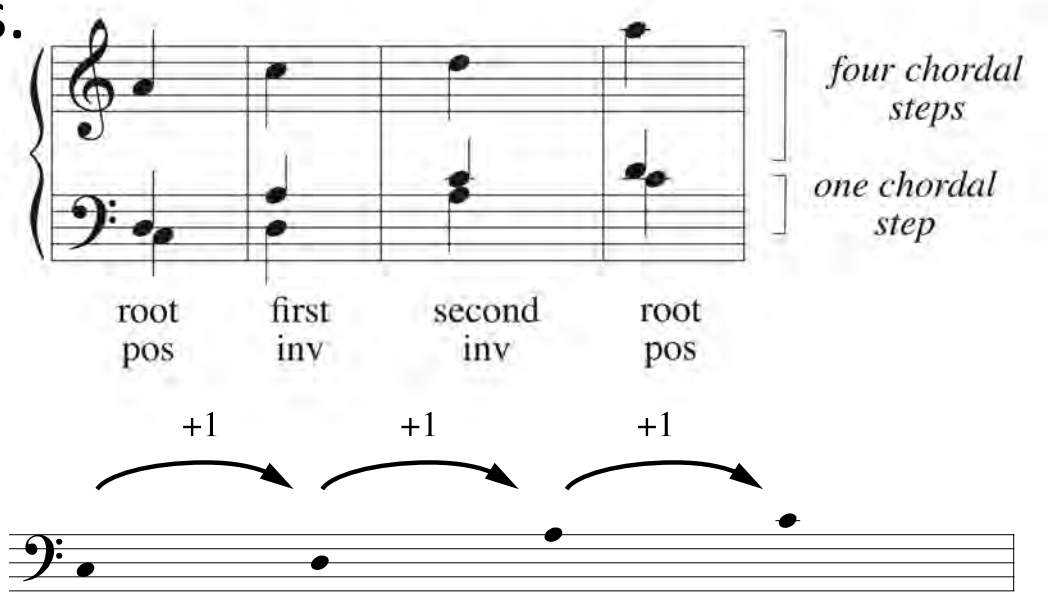
1. REGISTRAL INVERSION

- Suppose I gave you this chord and told you it was in root position. What is the first-inversion form?



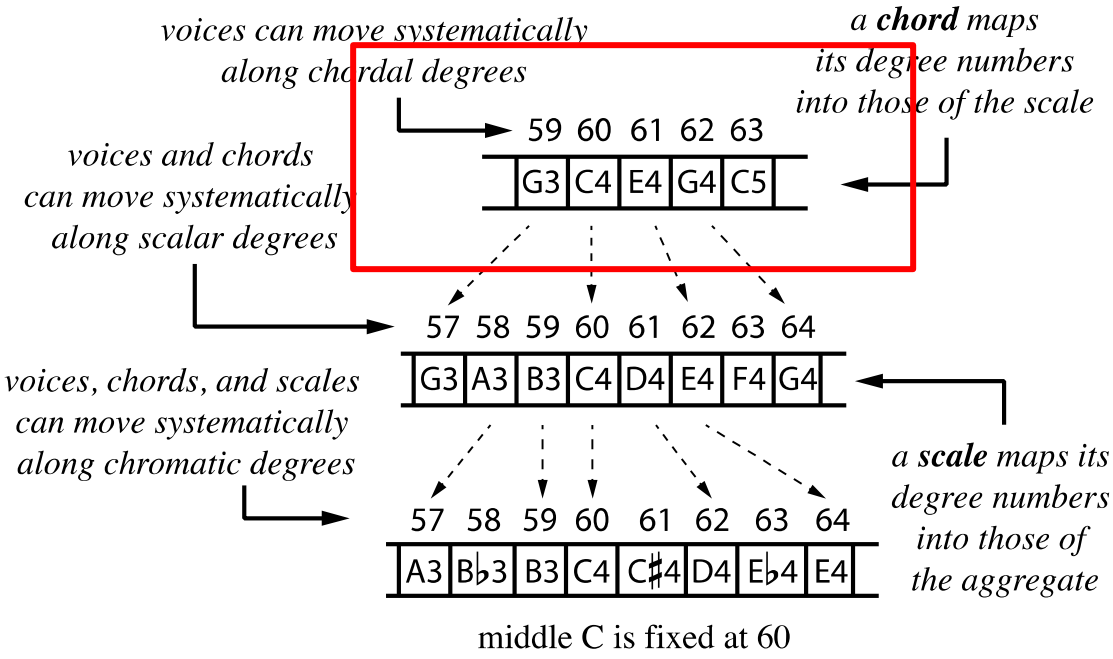
A BETTER ANSWER

- We can form inversions by transposing along the *intrinsic scale* consisting of the chord's own notes.
  - This preserves *voicing,* or spacing in chordal steps.



A BETTER ANSWER

- This is just the top level of my collectional hierarchy.

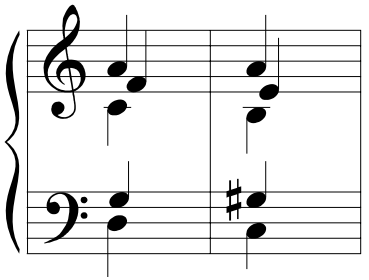


2. VOICING

Every chord presents multiple interval patterns, depending on whether we measure it extrinsically or intrinsically. These give us different ways to transform the chord and different conceptions of chordal identity.

	CHR	DIA	CHORD
F–A	4	2	2
C–F	5	3	2
G–C	5	3	2
D–G	5	3	2

	CHR	DIA	CHORD
E–A	5	3	2
B–E	5	3	2
G♯–B	3	2	2
C–G♯	8	4	2



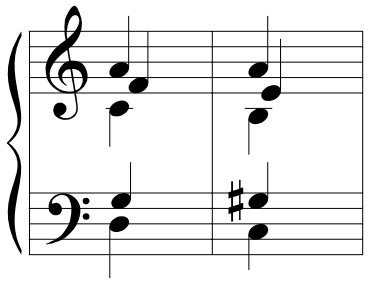
2. VOICING

For a chord of a given size, we can relate voicings to chord quality.

	CHR	DIA	CHORD
F–A	4	2	2
C–F	5	3	2
G–C	5	3	2
D–G	5	3	2

i.e. for a five-note chord, the (2, 2, 2, 2) voicing will be most "quartal," while the (2, 1, 1, 2) voicing will be most "tertian."

	CHR	DIA	CHORD
E–A	5	3	2
B–E	5	3	2
G♯–B	3	2	2
C–G♯	8	4	2



2. VOICING

In a recent paper I argue that *voicing* provides a set of approximate chord categories that simplify the world of musical possibilities. These correspond to the intuitive terms *clustered*, *tertian*, and *quartal*.

Approximate Set Theory

Chord Categories, Voicings, and Interval Cycles

Dmitri Tymoczko

Abstract This article describes an approximate set theory modeling intuitions shared by musicians such as Cowell, Schoenberg, Messiaen, and Persichetti. The author considers five approximation strategies, showing that in each case the result resembles an exact seven-tone set theory. Since most seven-tone sets are interval cycles, approximate twelve-tone sets are approximately cyclic as well. The theory explains how to highlight this cyclic structure using *voicings*, modeled by intervals in the *intrinsic scale* formed from a chord's own notes. This connection to voicing is what gives approximate chord categories much of their significance. The approach is most useful for chords with five or fewer notes and works tolerably for hexachords, but it breaks down with larger collections. This is not a failure of the model but a reflection of the fact that quality space contracts as cardinality increases.

Keywords set theory, voicing, intrinsic scale, analysis

APPROXIMATE INTERVAL CATEGORIES are a staple of informal musical discourse; musicians of all stripes speak of *steps* and *thirds*, or *clustered* and *quartal* harmonies. Approximate terminology has the cognitive advantage of reducing our harmonic taxonomy and the perceptual advantage of reflecting the often imprecise nature of musical experience: rather than requiring (or postulating, or hoping) that listeners maintain an exact tally of all the intervals they hear, generic qualia (e.g., *tertian*, *quartal*) allow for a degree of listener imperfection. Analysts frequently encounter passages saturated with a single type of generic interval, for example, an abundance of major and minor thirds or perfect and augmented fourths.[1] And as we will see, approximate categories highlight compositional affordances that might otherwise go unnoticed.

Yet academic music theory tends to valorize exact relationships. This is most obviously true of musical set theory, which categorizes chords by exact interval content, measured along the diatonic, chromatic, or some other scale. It is also a feature of twelve-tone music, which emphasizes rigid transformations of ordered

1 The distinction between "generic" and "specific" intervals originates with Clough and Myerson 1985, though that work emphasizes scale membership rather than direct categorization of chromatic intervals.

Thanks to Brad Gersh, Daniel Harrison, Andrew Mead, Ian Quinn, Steve Taylor, and Jason Yust for helpful discussions and comments.

Journal of Music Theory 67:1, April 2023
DOI 10.1215/00222909-10232057

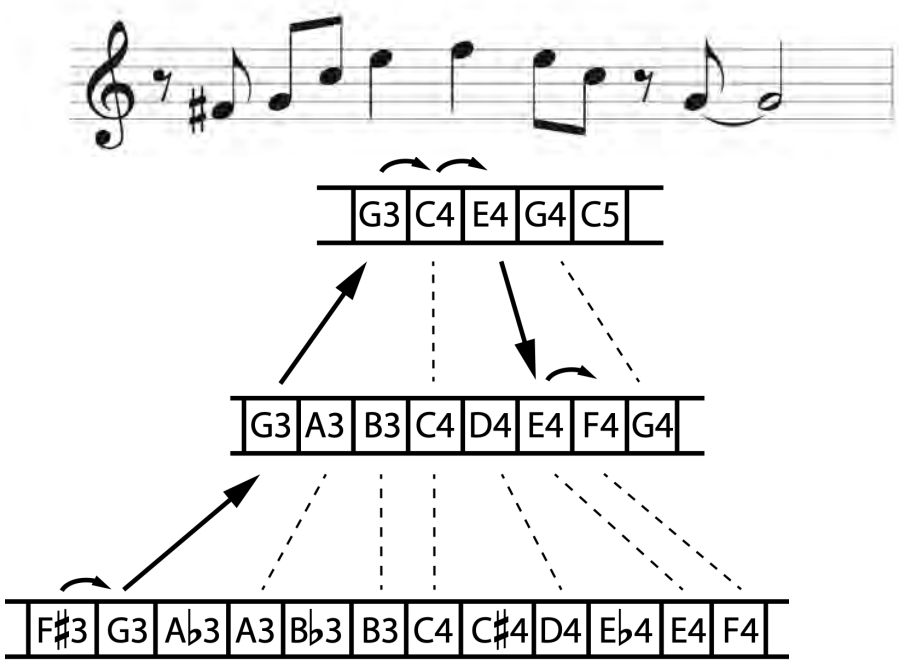
2. VOICING

This is a useful compositional tool.

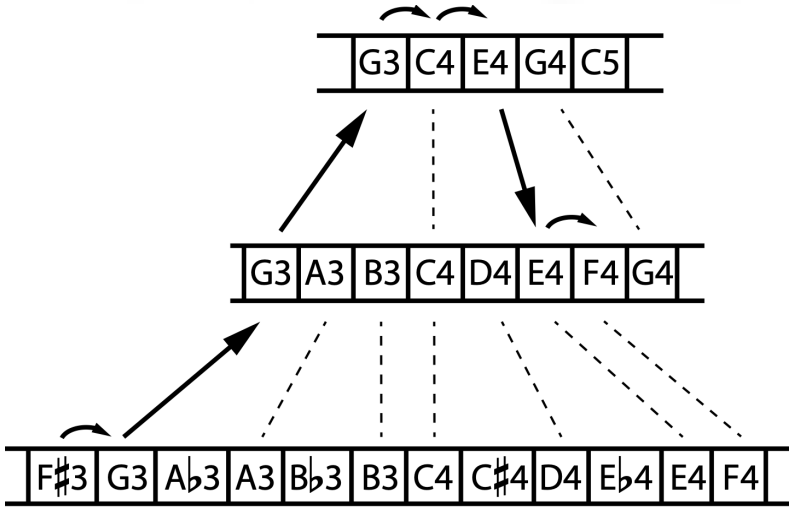
http://www.madmusicalscience.com/nr.html

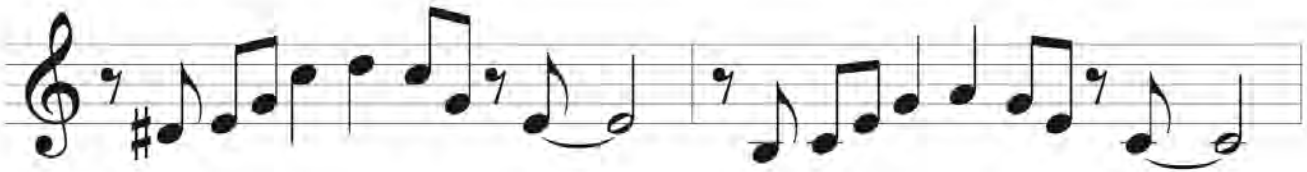
3. MOTIVES

- Motives are voicings augmented with
  - Rhythmic structure
  - Hierarchical pitch structure



3. MOTIVES


G3 C4 E4 G4 C5
G3 A3 B3 C4 D4 E4 F4 G4
F♯3 G3 A♭3 A3 B♭3 B3 C4 C♯4 D4 E♭4 E4 F4



ARCA

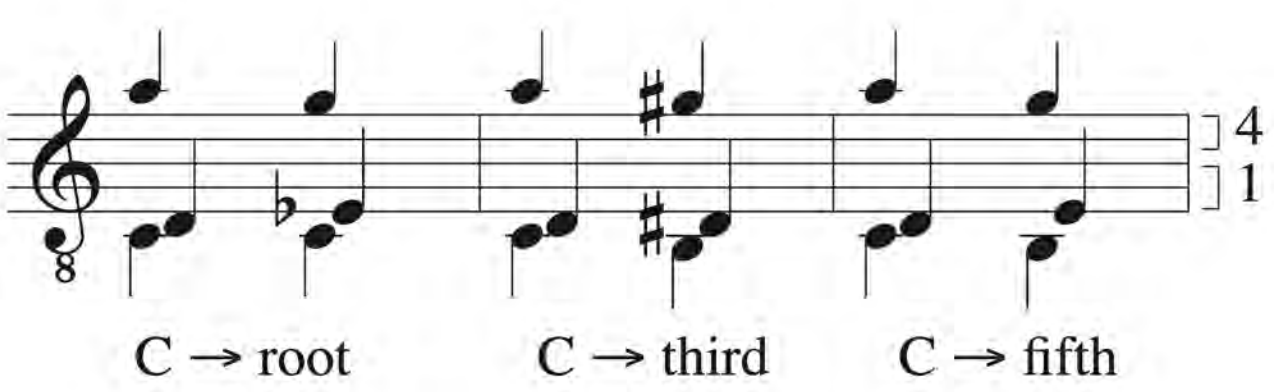
- Because this gets so complex I wrote a syntax for manipulating motives:
  - Website
  - Free python libraries
  - DAW (patent applied for!)

4. VOICE LEADING

- The most efficient voice leading between two chords preserves voicing.
  - Suppose we want to move from this chord to a minor triad.

4. VOICE LEADING

- The most efficient voice leading between two chords preserves voicing.
  - Suppose we want to move from this chord to a minor triad.
  - There are 3 basic possibilities:

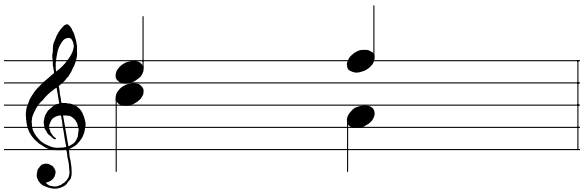


4. VOICE LEADING

- This means that motion-along-a-chord can be explanatorily useful *even if composers are not explicitly thinking about it.*

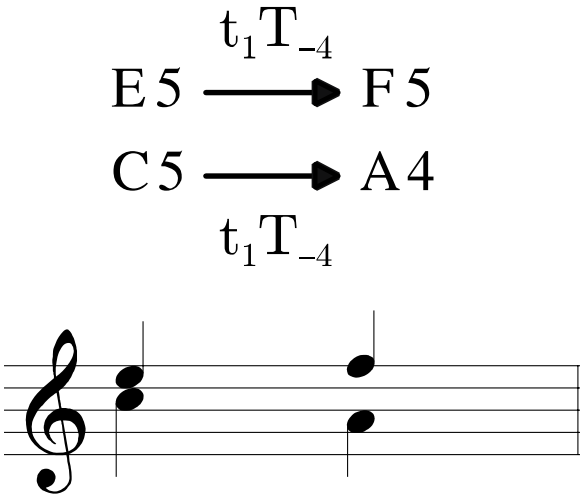
5. REPETITION

- When we describe a voice leading using a combination of intrinsic and extrinsic transposition, we provide a recipe for *repeating* it.



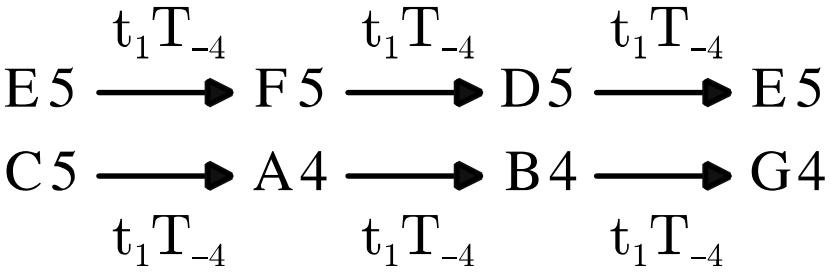
5. REPETITION

- When we describe a voice leading using a combination of intrinsic and extrinsic transposition, we provide a recipe for *repeating* it.

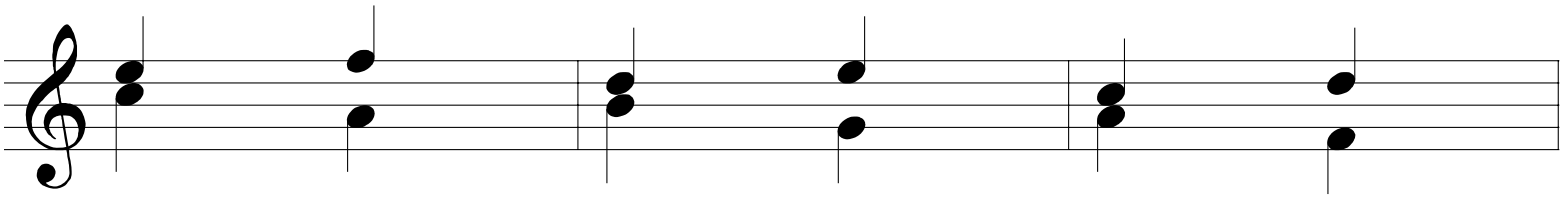


5. REPETITION

- When we describe a voice leading using a combination of intrinsic and extrinsic transposition, we provide a recipe for *repeating* it.

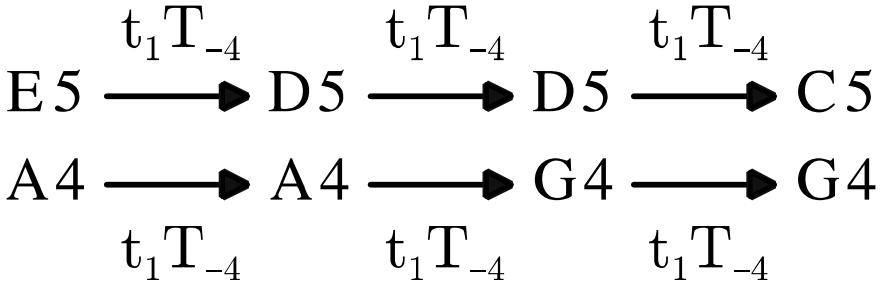


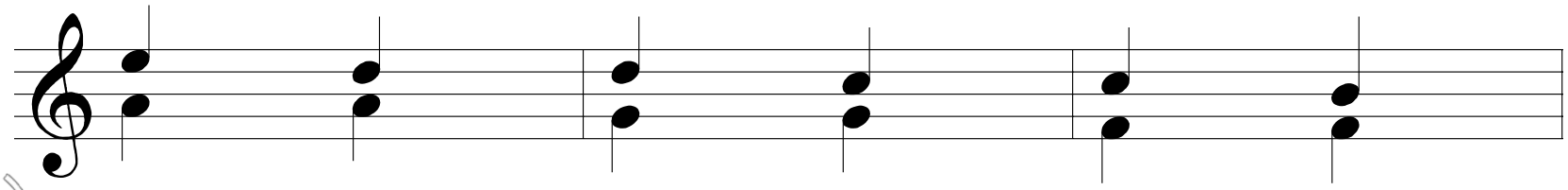
Little t creates canons!



5. REPETITION

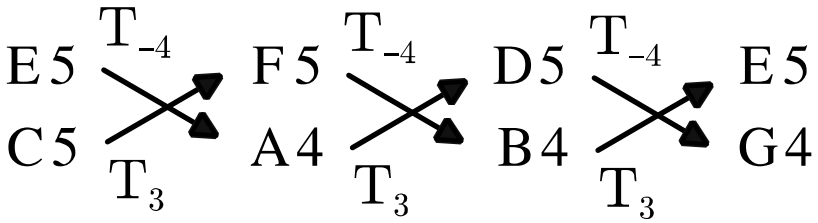
- This description is *structure neutral,* applying to all chords of that size.

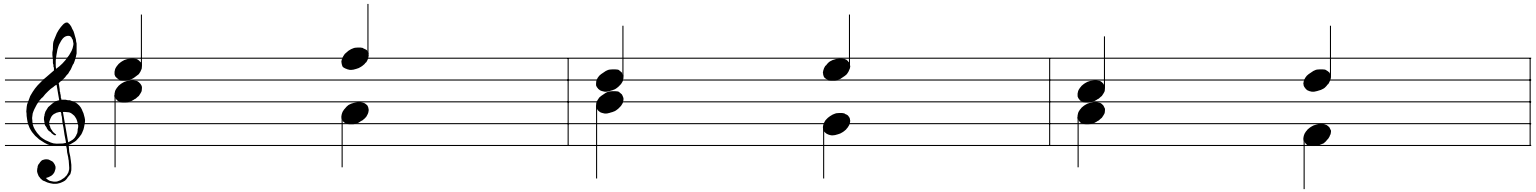




5. REPETITION

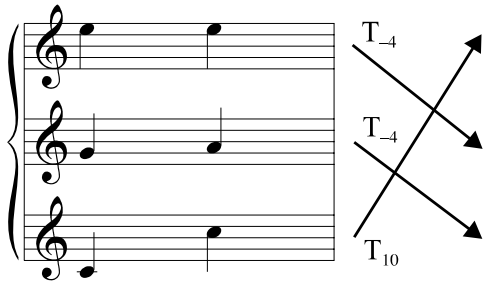
- This description is *equivalent to,* but different from, a description in terms of symmetries.

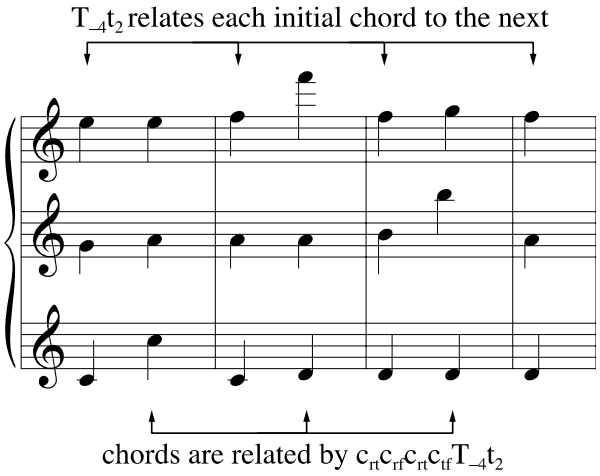




5. SEQUENCES

- The *repetition group* is isomorphic to the *voice leading group.*
  - Every repetition is a voice leading.
  - This gives us two different languages for describing repetition.
  - It is possibly the origin of the concept of a chord.





II. The spiral diagrams

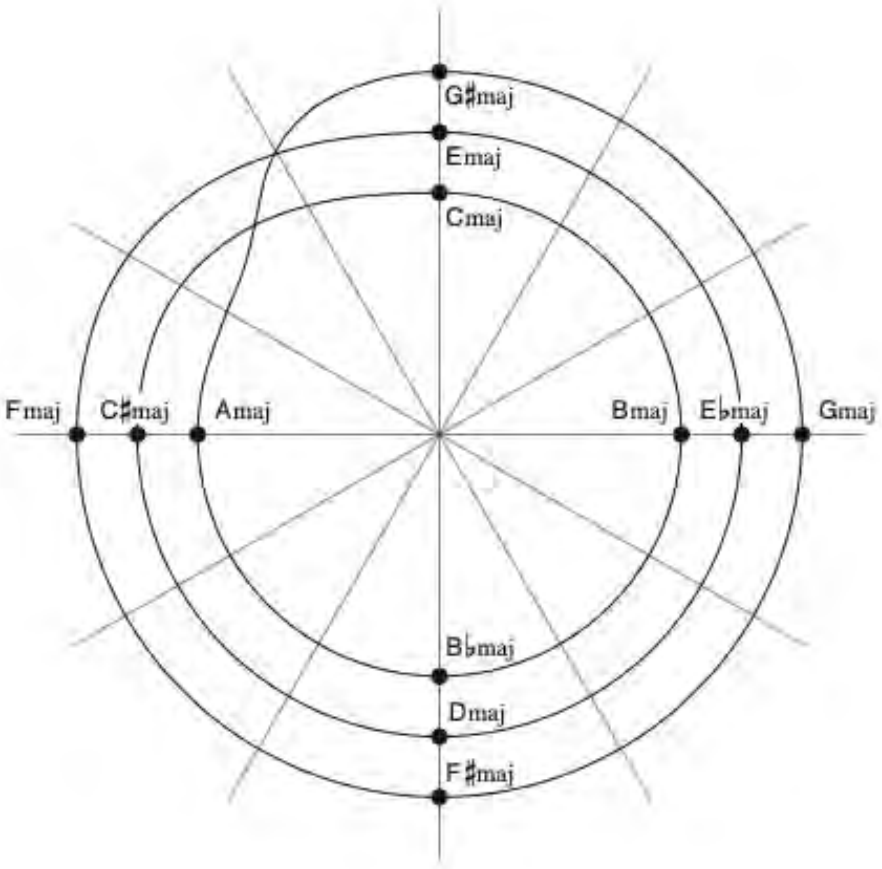
There is a simple two-dimensional geometry of hierarchically nested transposition, intrinsic and extrinsic, for any pair of objects whatsoever.

They represent all the (n-voice) voice leadings linking the transpositions of any chord in any scale, except for the ***voice exchanges***

(which we can add to the diagrams if we want)

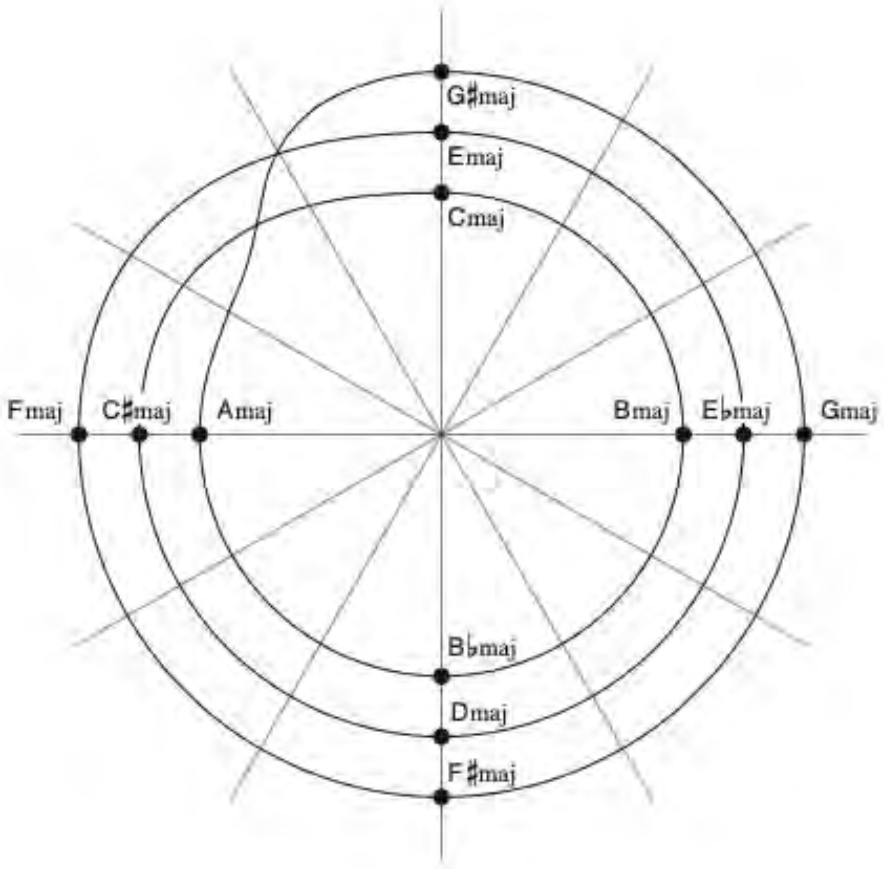
SPIRAL DIAGRAMS

- The combined operation of "transposition along a chord" (intrinsic transposition) and "transposition along a scale" (extrinsic transposition) can be represented, for any chord and any scale, by a spiral.



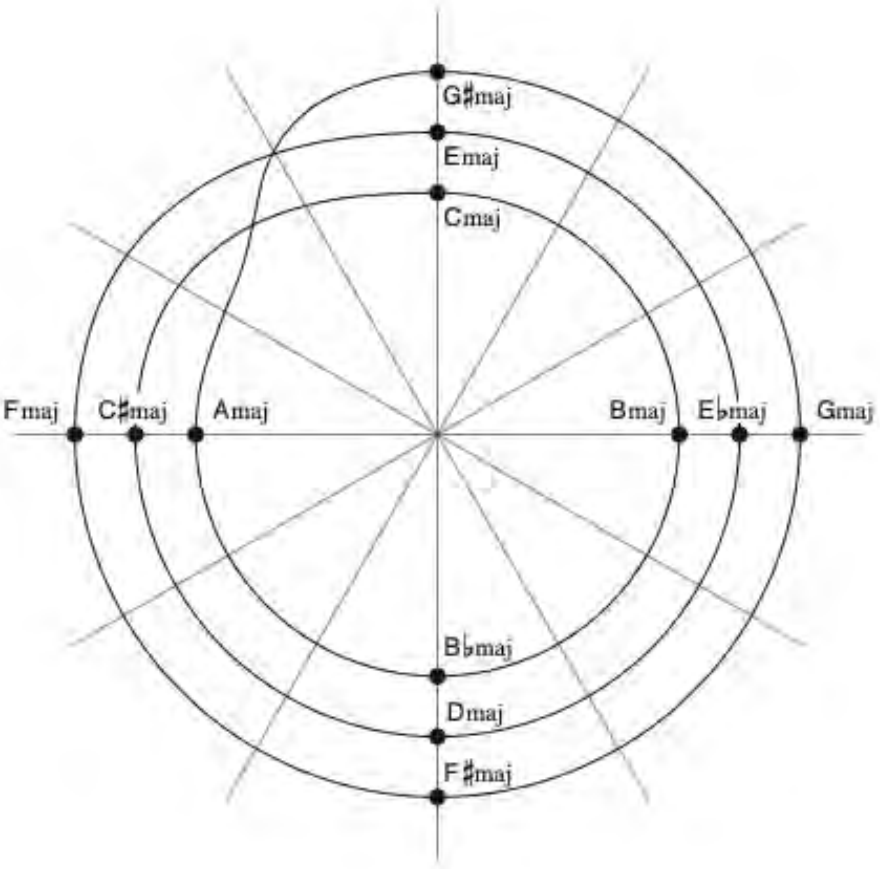
SPIRAL DIAGRAMS

- These diagrams derive from the more complicated orbifolds representing *all* chords of a given size.



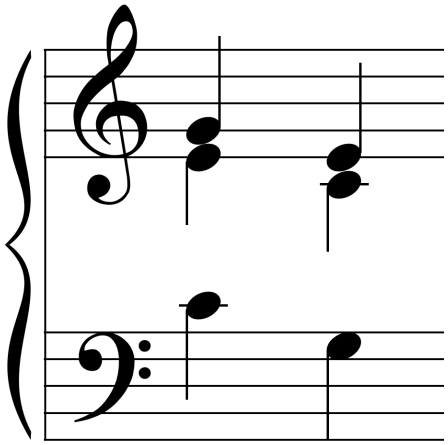
SPIRAL DIAGRAMS

- Clockwise motion descends, counterclockwise motion ascends.

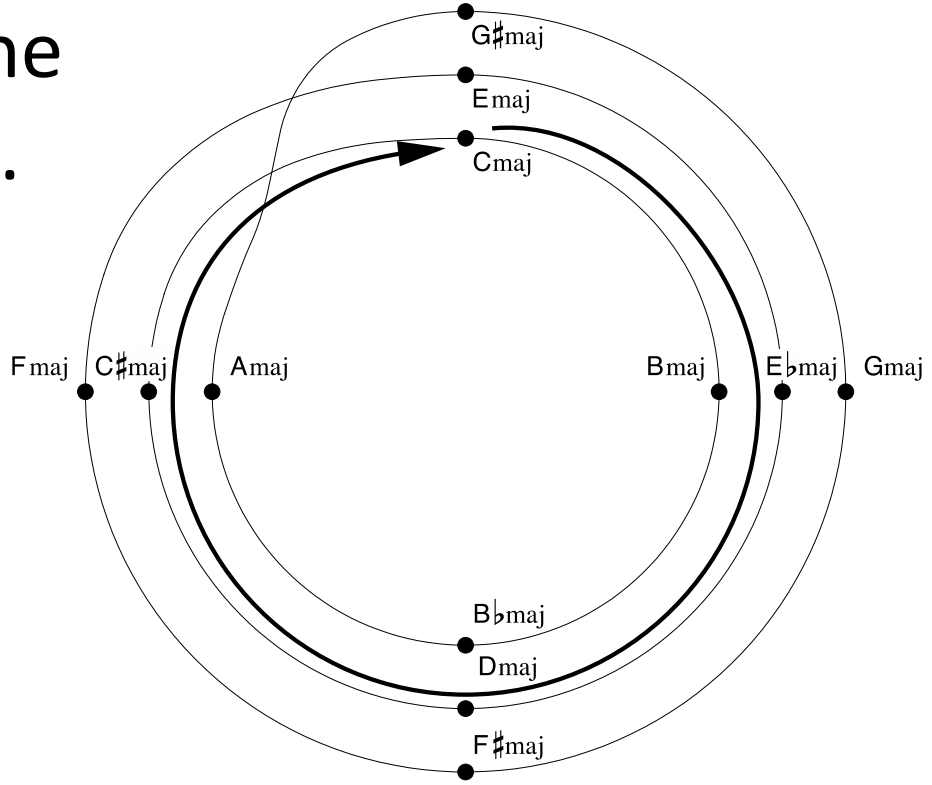


SPIRAL DIAGRAMS

- *Loops* represent transposition along the intrinsic scale (chord).

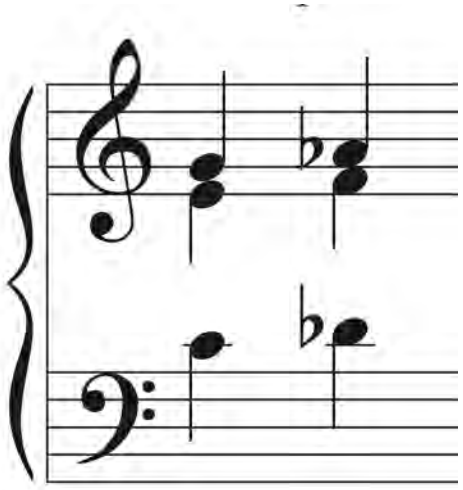


t_{-1}

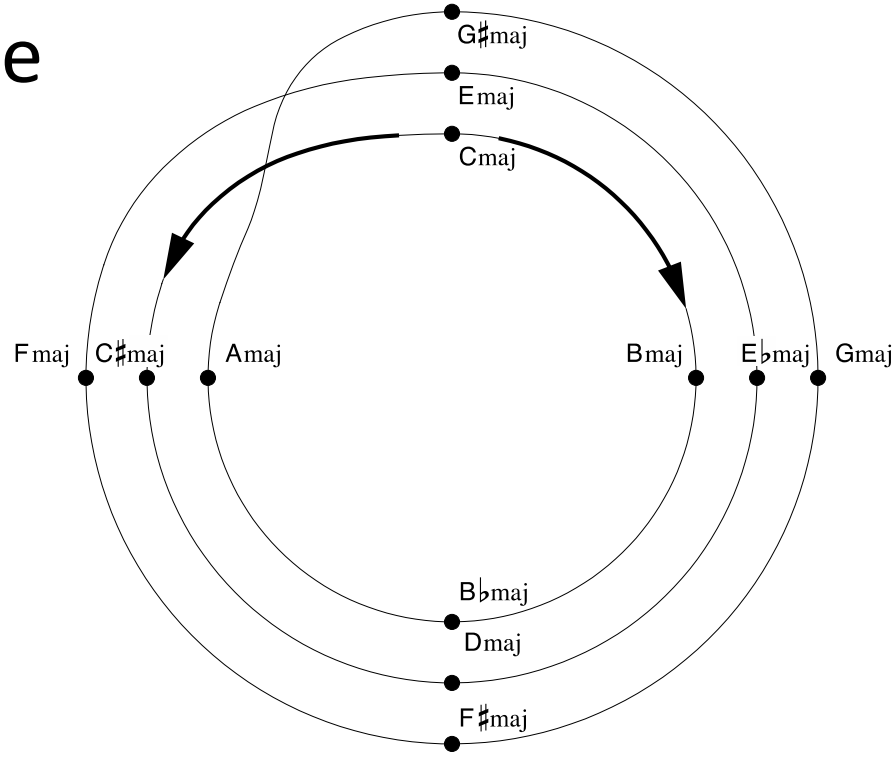


SPIRAL DIAGRAMS

- *Slides* represent transposition along the extrinsic or enclosing collection (scale).

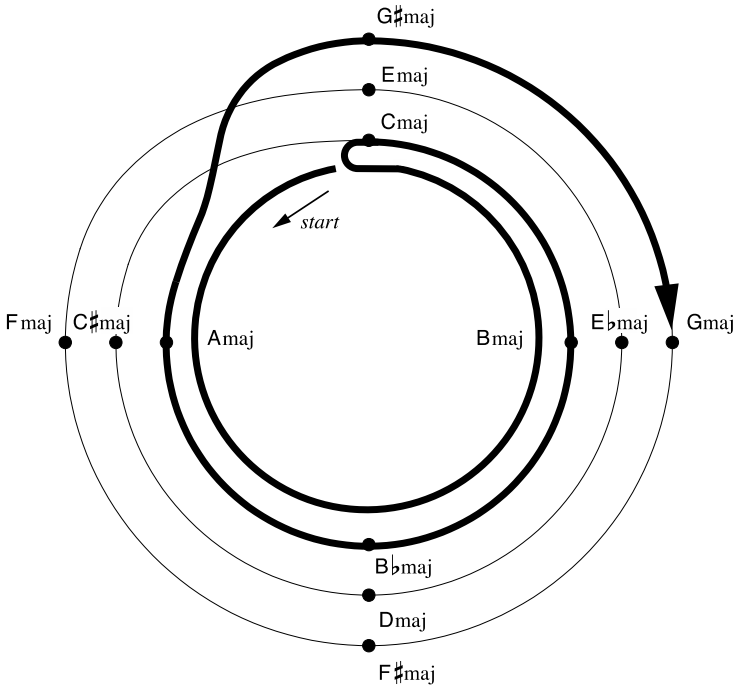


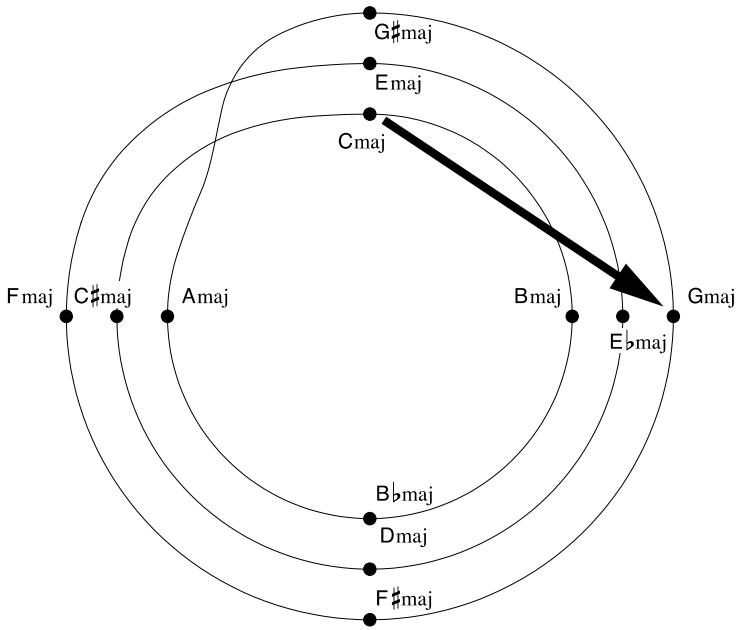
T_1



SPIRAL DIAGRAMS

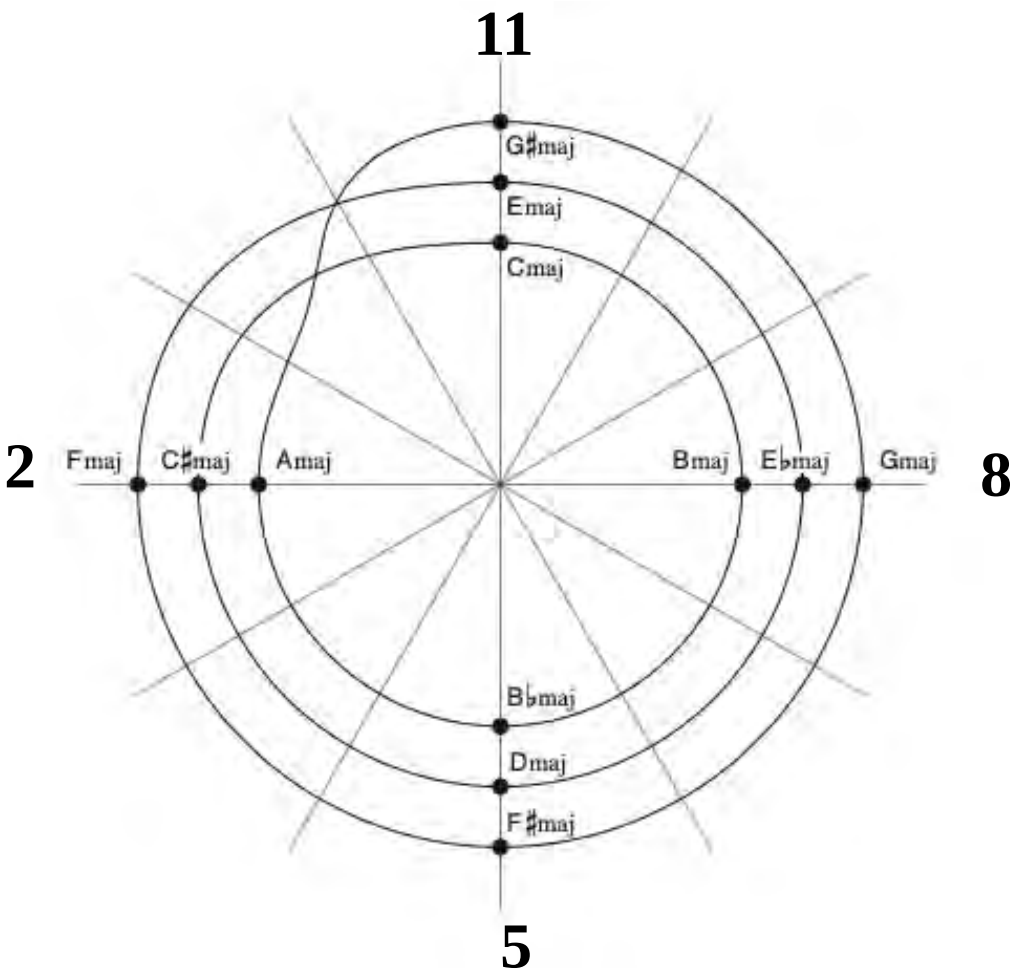
- Two paths represent the same voice leading if they start and end at the same points and involve the same total amount of angular motion.





SPIRAL DIAGRAMS

- Chords' angular position is determined by the sum of their pitch classes (mod **o**).
- The sum of the melodic intervals in a voice leading corresponds to the total amount of angular motion in the corresponding path.



SPIRAL DIAGRAMS

http://www.madmusicalscience.com/cs.html

http://www.madmusicalscience.com/multichord.html

Rock, Beethoven, NR-voice leadings, etc. …

III. The quadruple hierarchy

THE QUADRUPLE HIERARCHY

1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales*

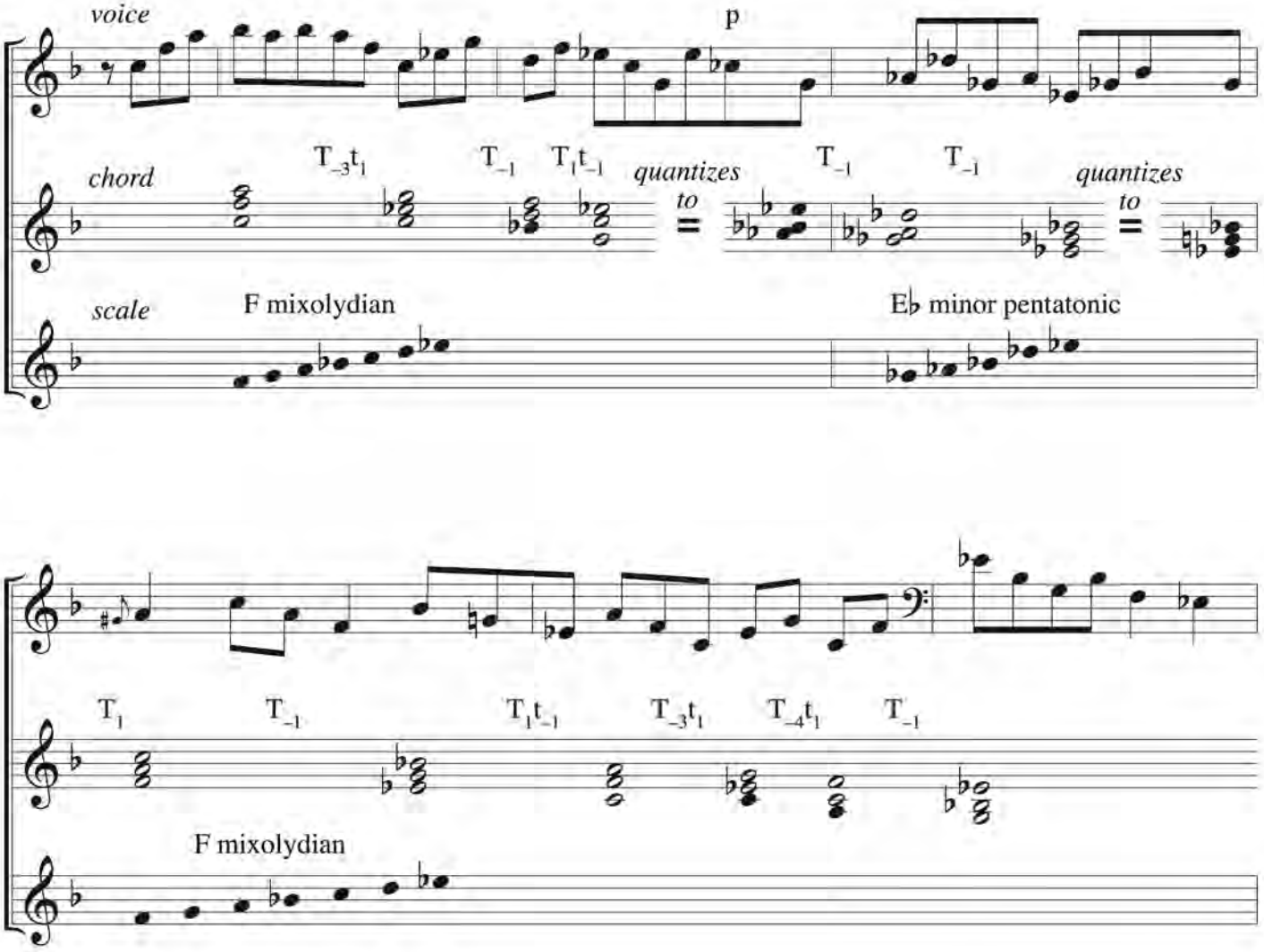
THE QUADRUPLE HIERARCHY

1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales* that move inside
4. A chromatic collection.

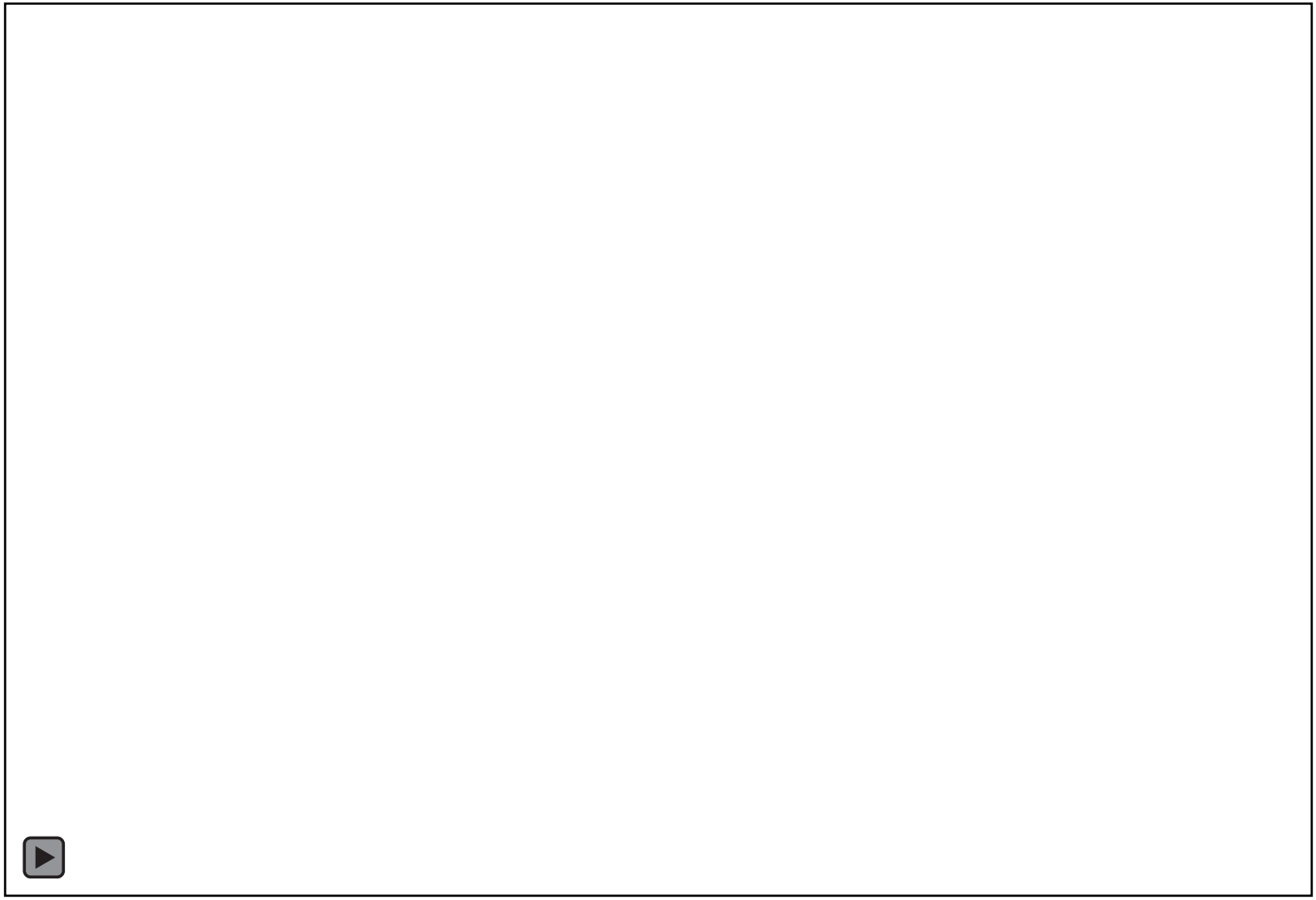
(which could be 12-tone equal temperament,
some unequal but fixed temperament,
or the continuous space of just intonation)

EXAMPLE: Beethoven's Ninth

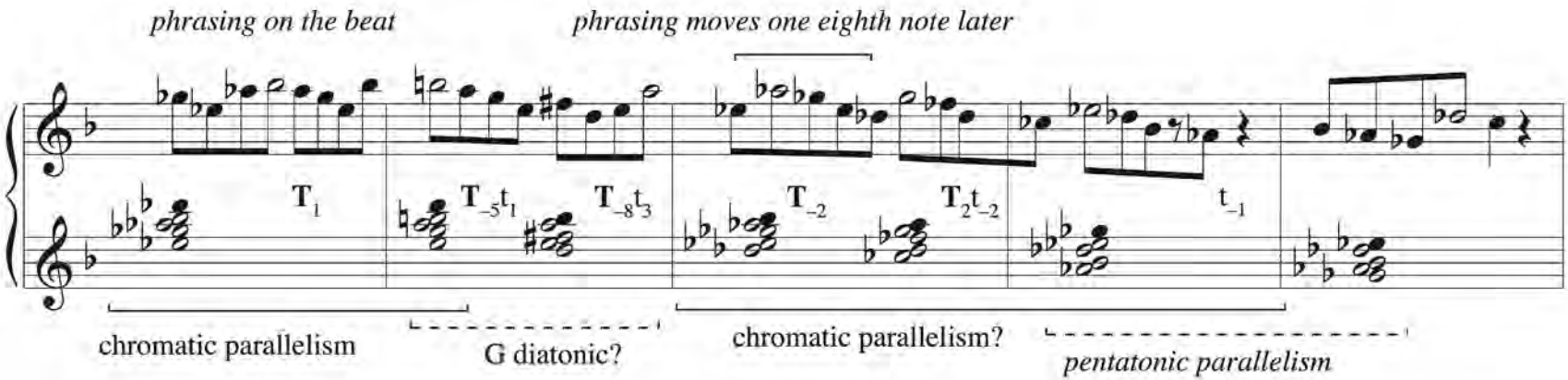
EXAMPLE: McCoy Tyner

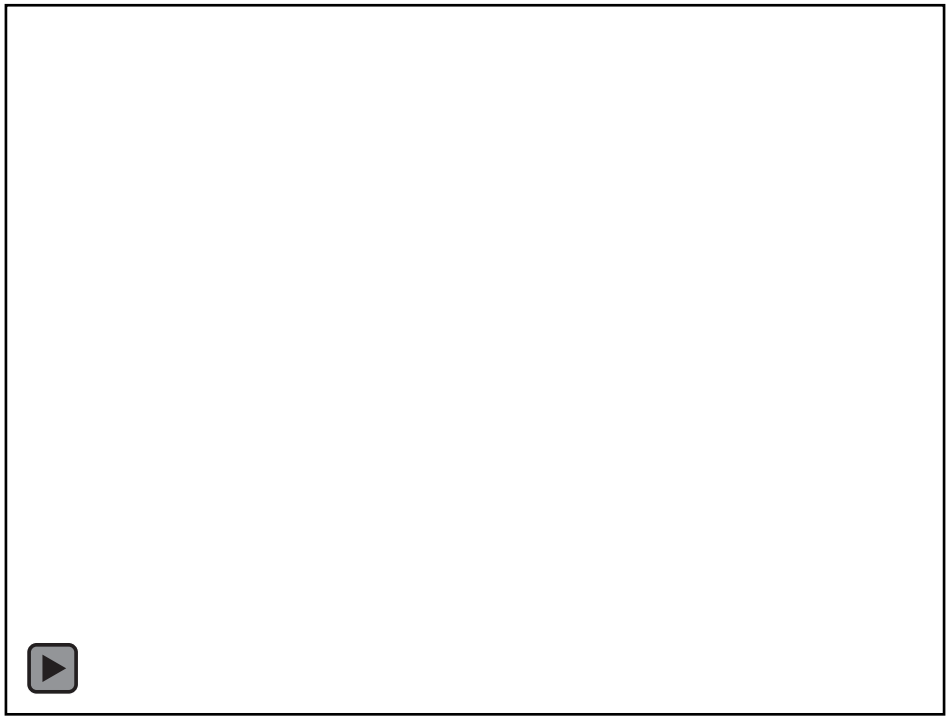


EXAMPLE: McCoy Tyner



EXAMPLE: McCoy Tyner





GEOMETRY AND HIERARCHY

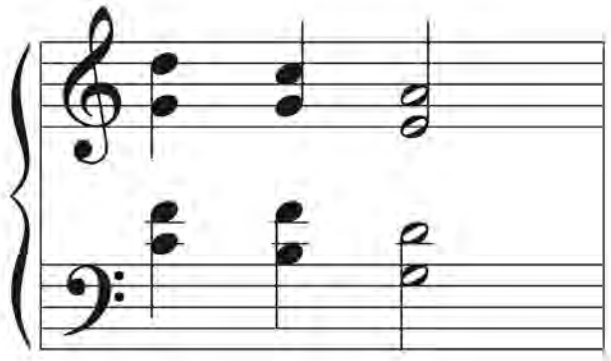
- This perspective links the flat spaces of voice-leading geometry with a more hierarchical perspective reminiscent of Schenker.
  - Allows us to describe contrapuntal processes occurring in the background, decorated on the surface in non-obvious ways.
  - *Chords* can be little *scales*.
  - The spiral models provide unified and intuitable representations of each of three different levels.

IV. Making music

let's generalize the Waldstein opening by writing a progression where voices move contrary to abstract harmonies.

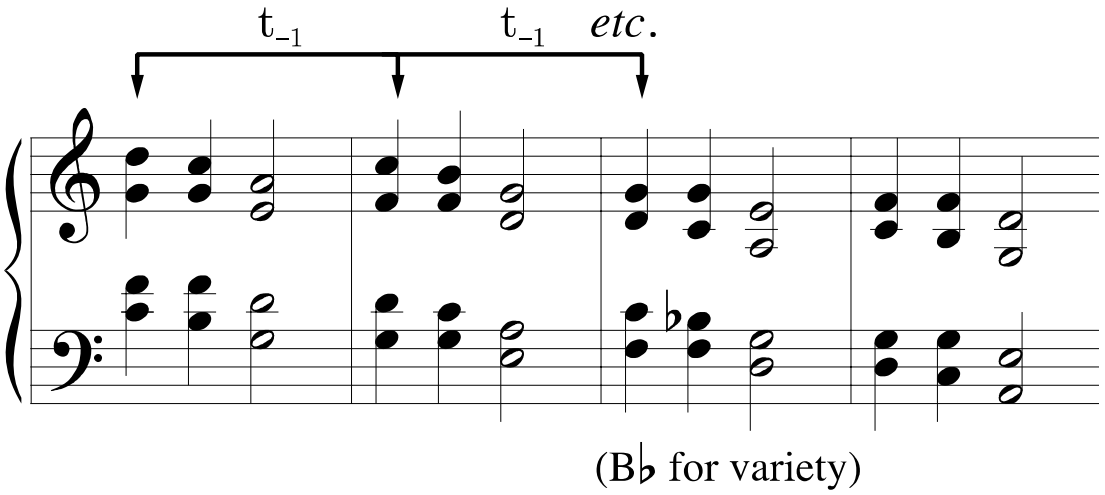
COMPOSITION

- We will start with a progression of diatonic fourth chords.



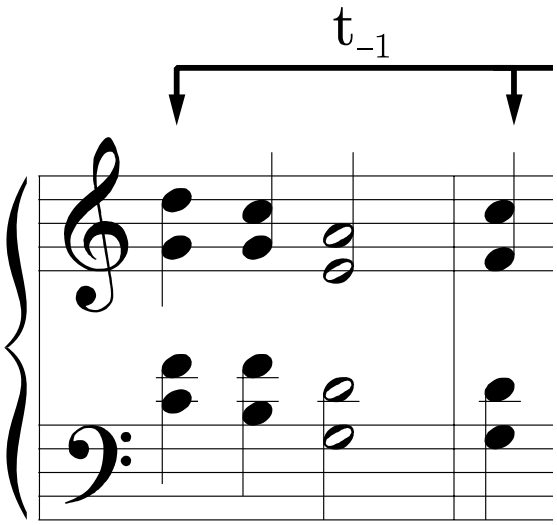
COMPOSITION

- We repeat it, permuting the voices to form a round.

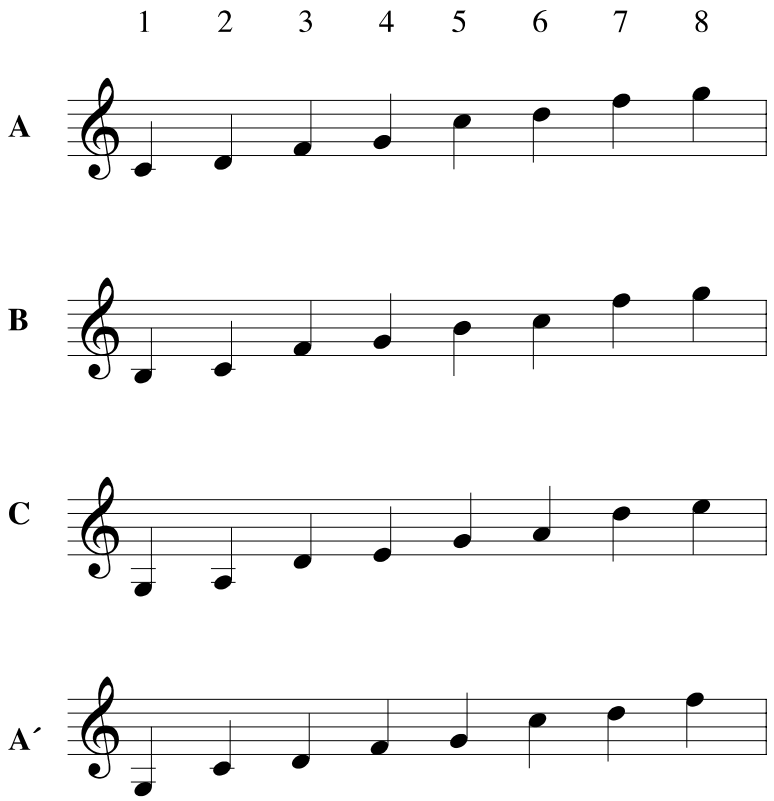


COMPOSITION

- This gives us a sequence of descending scales.



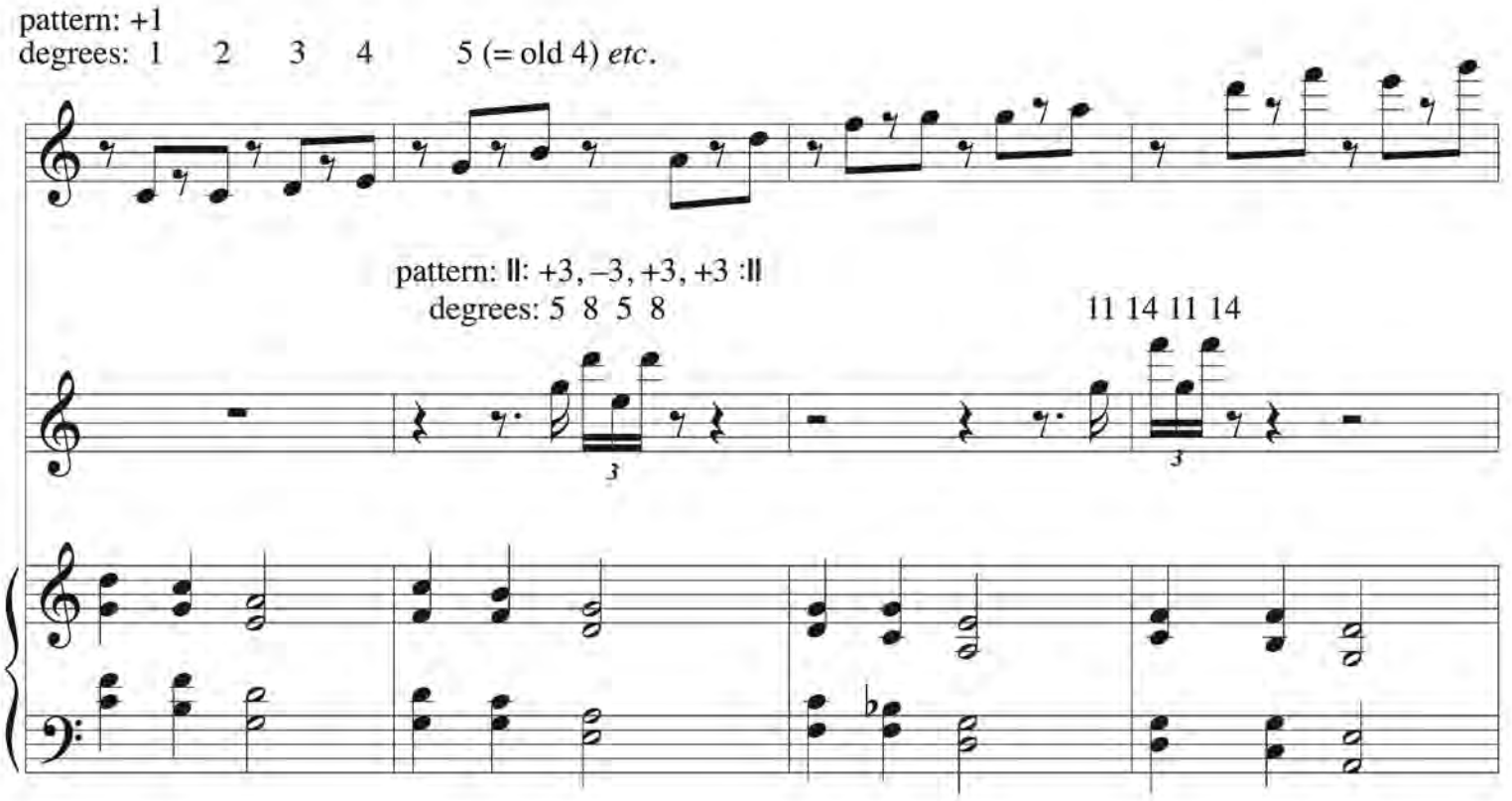
read scale degrees down in columns:



(back to the top, but adding 1 to each scale degree:
middle C is now scale degree 2!!)

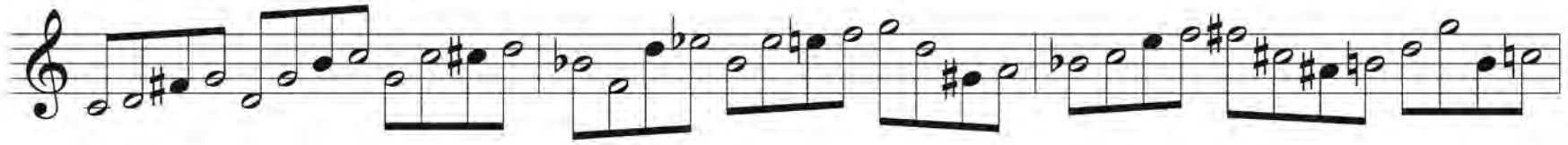
COMPOSITION

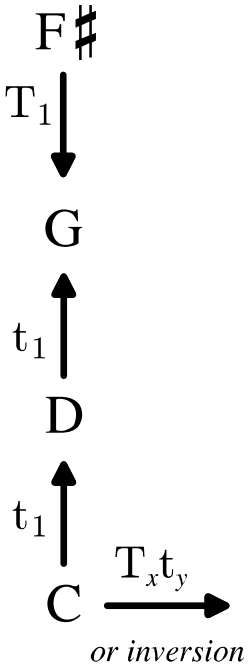
- We can create melodic patterns along these scales.



MOTIVES

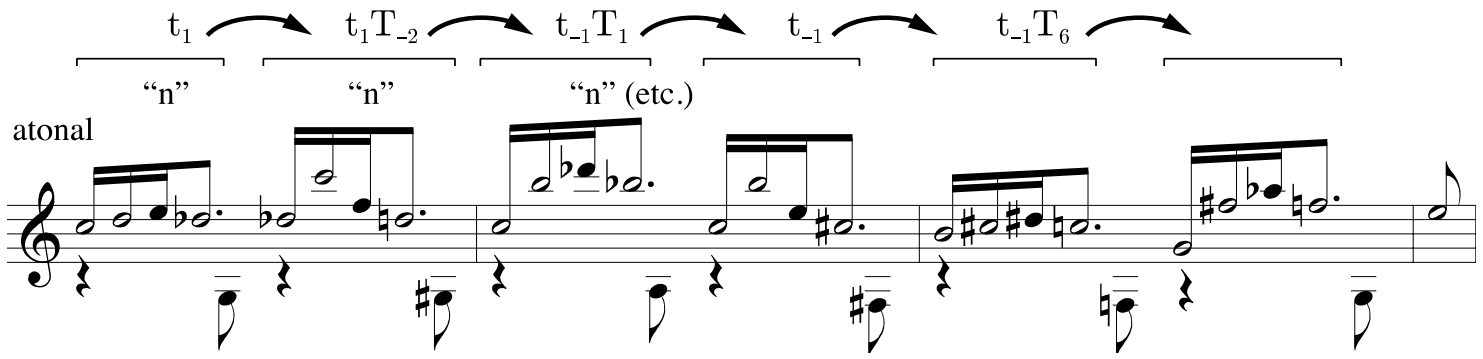
fourth chords with one neighbor note

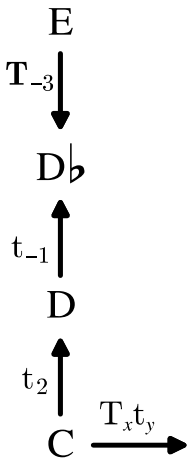




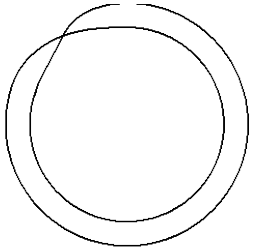
MOTIVES

top voice: 012 chromatic clusters, with a "nonharmonic" tone a minor third above the last note
bottom voice: free





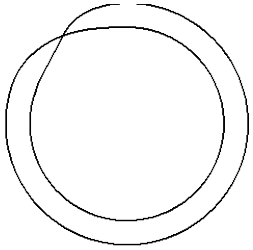
WHEELS WITHIN WHEELS



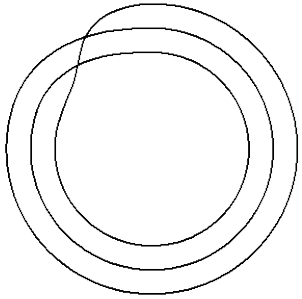
voices in chord

1. *Voices* move inside
2. (Possibly scale-like) *chords*

WHEELS WITHIN WHEELS



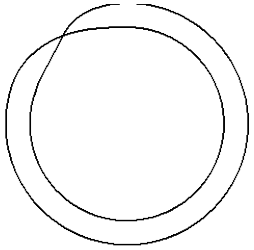
voices in chord



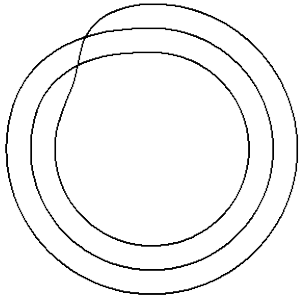
chord in scale

1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales*

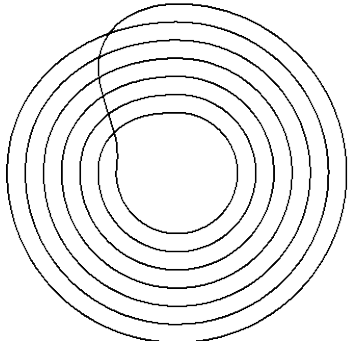
WHEELS WITHIN WHEELS



voices in chord



chord in scale

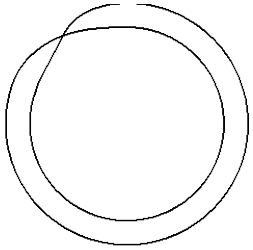


scale in aggregate

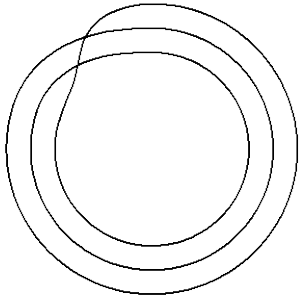
1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales* that move inside
4. A chromatic aggregate.

This is a very simple organizational strategy that generates music of extraordinary psychological complexity.

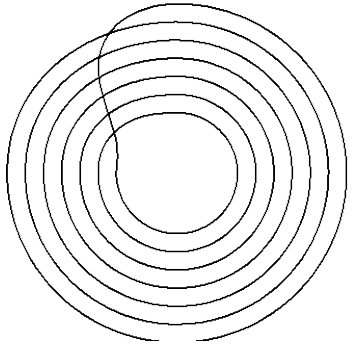
WHEELS WITHIN WHEELS



voices in chord



chord in scale

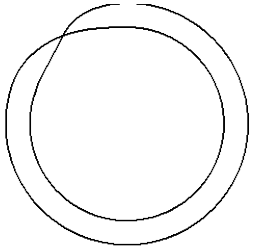


scale in aggregate

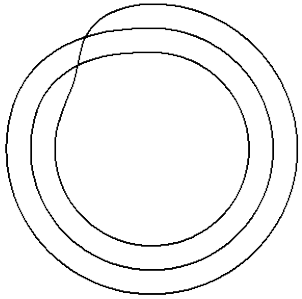
1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales* that move inside
4. A chromatic aggregate.

It is an opportunity for theory! Algorithmic composition, compositional assistants.

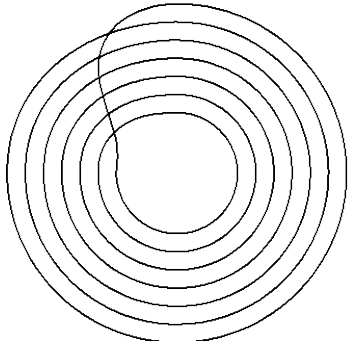
WHEELS WITHIN WHEELS



voices in chord



chord in scale

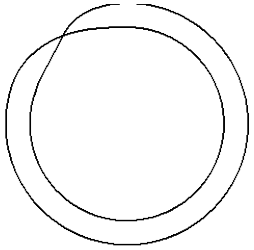


scale in aggregate

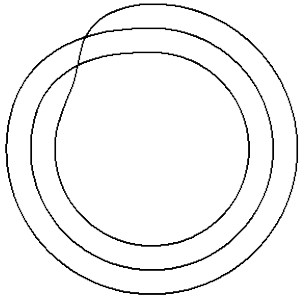
1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales* that move inside
4. A chromatic aggregate.

*It is **really hard** to manage three separate levels of motion-along-a-collection, particularly with unfamiliar chords and scales.*

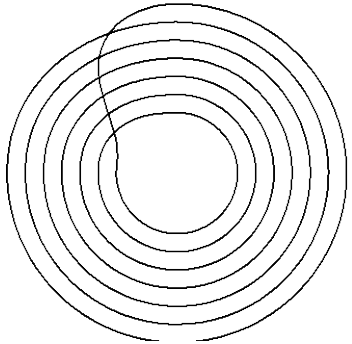
WHEELS WITHIN WHEELS



voices in chord



chord in scale

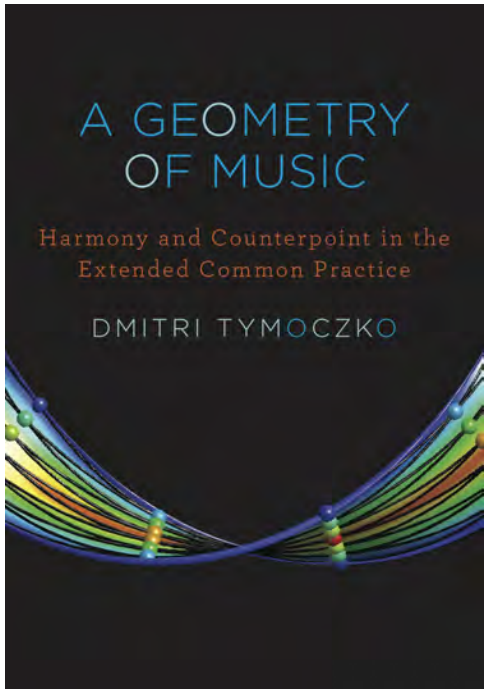


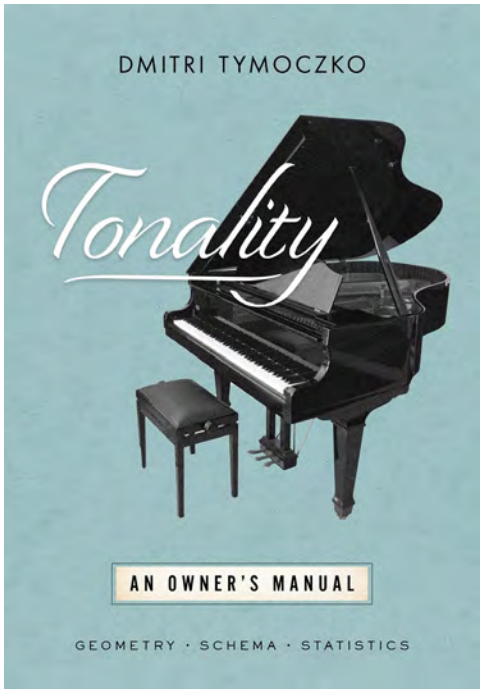
scale in aggregate

1. *Voices* move inside
2. (Possibly scale-like) *chords* that move inside
3. *Scales* that move inside
4. A chromatic aggregate.

*For analysts, it can be **really hard** to recognize these structures in pre-existing music!*

Thank you!





www.madmusicalscience.com

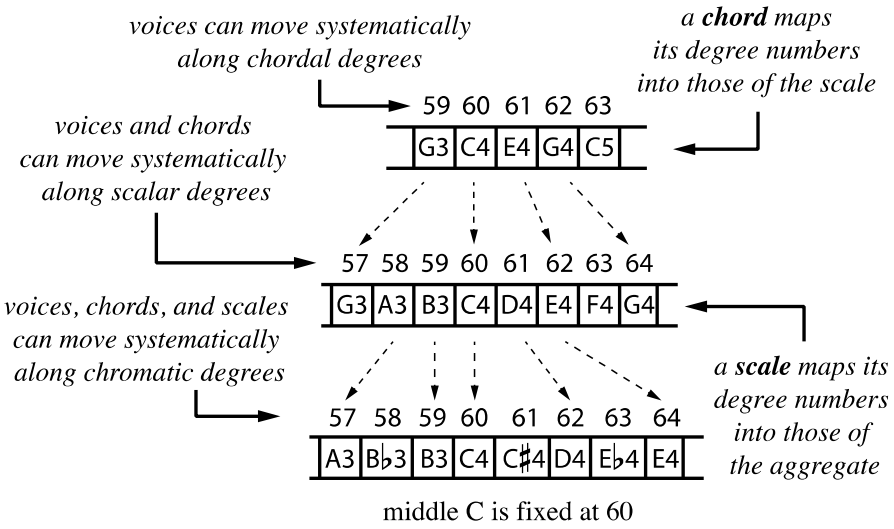
Videos, software, and more

METHODOLOGY

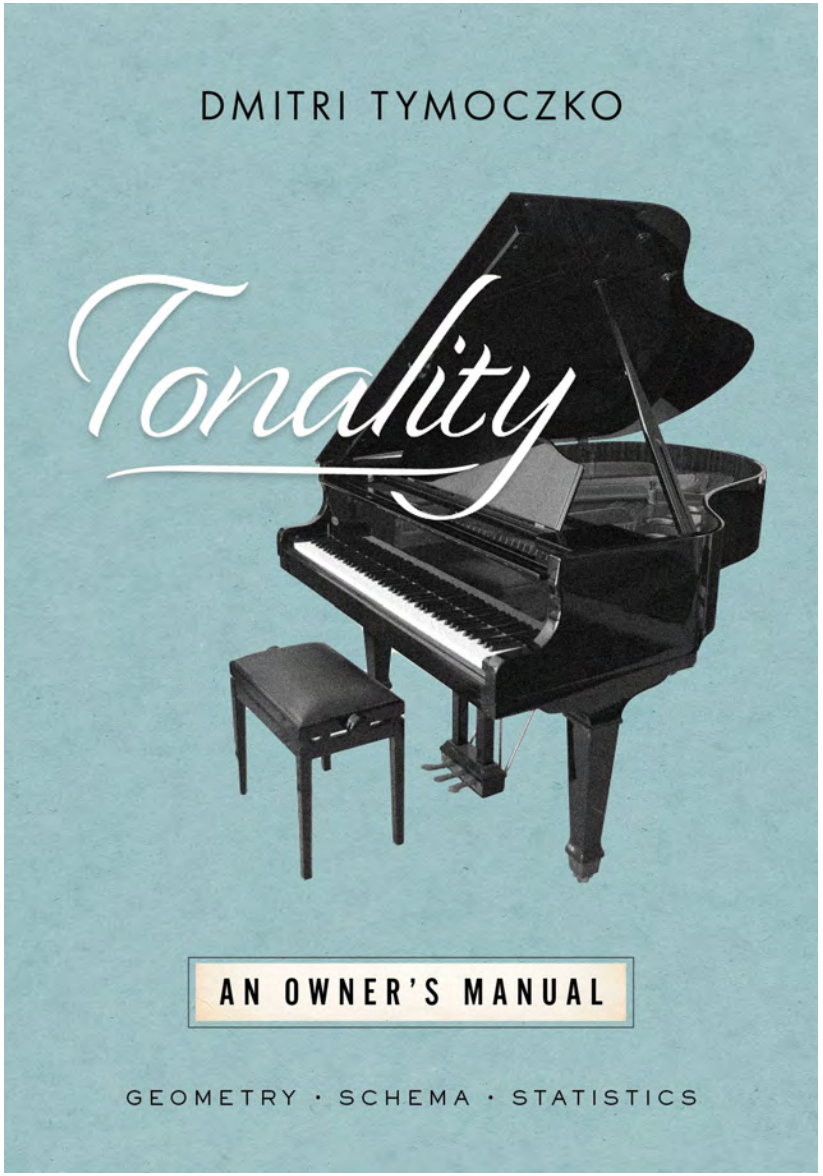
- I am working in the tradition called "American Set Theory."
  - I believe transposition is a fundamentally important *symmetry* of musical structure.
  - Inversion is also important though less so.
- My version of set theory is different from the one you know:
  - Hierarchical
  - Collection relative
  - Approximate
- Describes traditional and vernacular practice (folk, rock, jazz, etc.) as well as modern music.

DEPARTMENT of MUSIC AT PRINCETON

http://dmitri.tymoczko.com

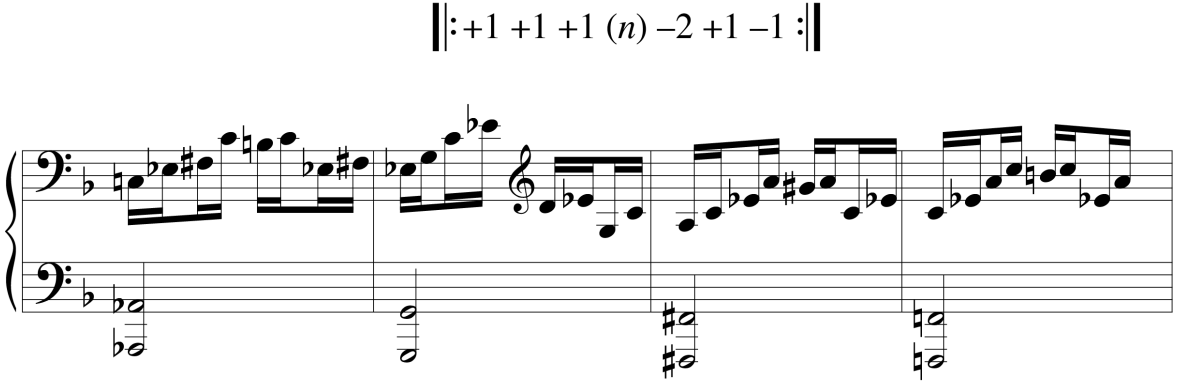


middle C is fixed at 60

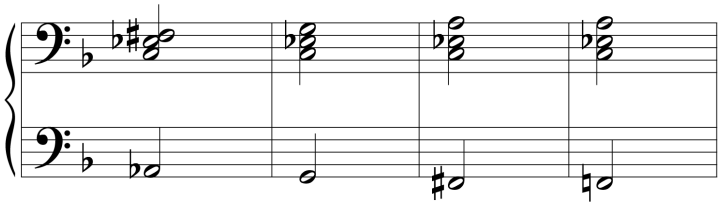


A CHORD IS AN ABSTRACT OBJECT

- Beethoven moves the upper voice systematically along a three-note chord that is itself moving.
  - Chords neither localized in pitch or time.
  - They are essentially scales.



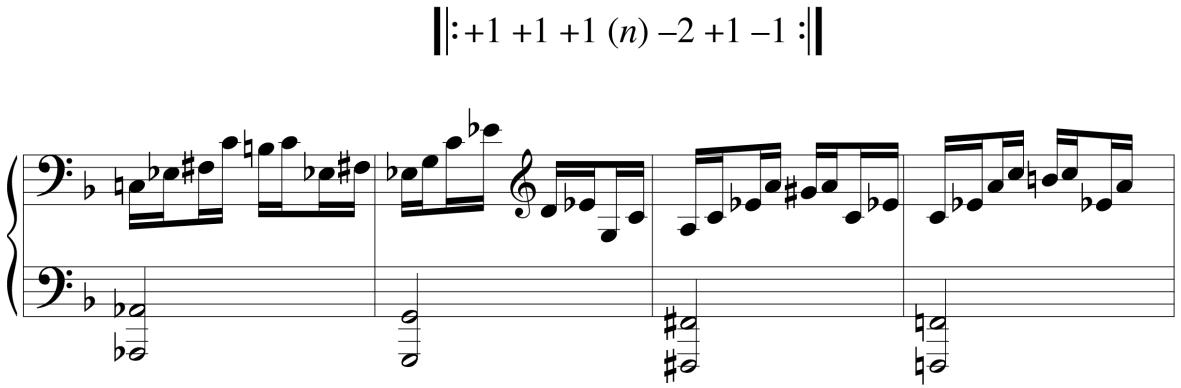
Beethoven, Op. 54, II, mm. 38–41.



⇐ *standard notation is misleading*

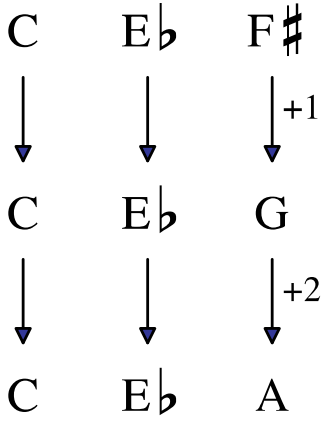
A CHORD IS AN ABSTRACT OBJECT

- Beethoven moves the upper voice systematically along a three-note chord that is itself moving.
  - Chords neither localized in pitch or time.
  - They are essentially scales.



better:

octave-free notation:

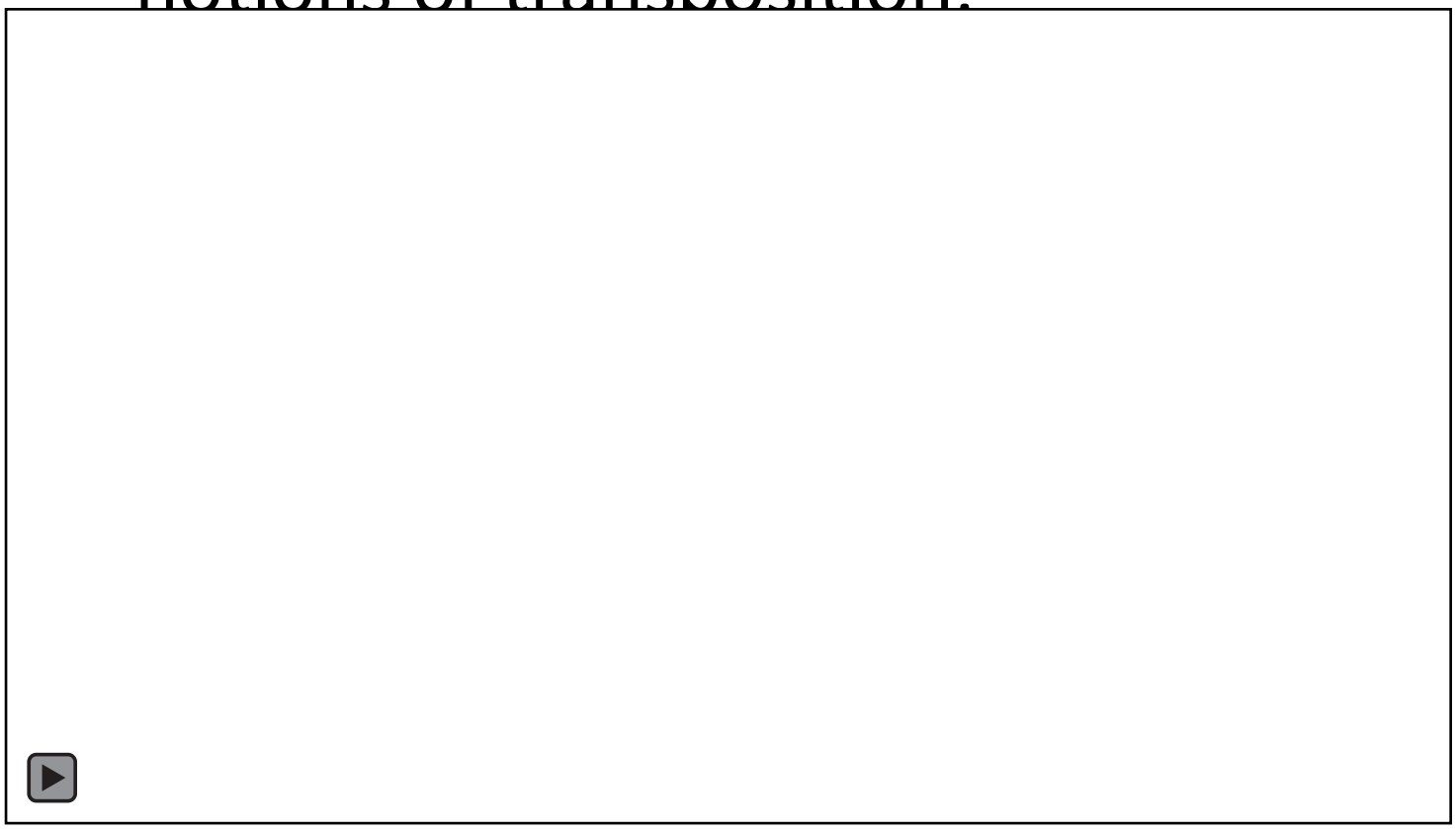


EXTRINSIC AND INTRINSIC

- Every musical object, including chords, sets, set classes, motives, etc., defines two scalar contexts:
  1. The *extrinsic* or enclosing scale that contains that object as a proper subset.
     - E.g. quarter-tone, chromatic, diatonic, pentatonic, etc. Even continuous unquantized space.
  2. The *intrinsic* scale consisting of that object's own notes.

EXTRINSIC AND INTRINSIC

- These two contexts define two separate notions of transposition.



BIG T little t

- T_x = transposition by *x* scale steps.
  - **Bold T** for chromatic transposition
  - Regular T for any other scale (e.g. diatonic)
- t_y = transposition by *y* chordal steps.
- $T_x t_y$ = simultaneous transposition along chord and scale.

BIG T little t

These turn out to have a simple and beautiful geometry which we will discuss later.