IMPRINTS

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NEWSLETTER OF THE INSTITUTE FOR MATHEMATICAL SCIENCES, NATIONAL UNIVERSITY OF SINGAPORE

Leaving a Legacy: Professor Chi Tat Chong's Tenure at IMS

BY ALEXANDRE THIERY *(National University of Singapore)*

Professor Chi Tat Chong recently concluded his distinguished tenure as the Director of the Institute for Mathematical Sciences (IMS), a role he held for over a decade. This marks the conclusion of a transformative era that commenced in early 2013, leaving an enduring legacy defined by his influential position as Director and substantial contributions during the foundational years of the Institute.

Chi Tat's commitment to IMS traces back to the 1980s when, in collaboration with a cohort of mathematicians, he crafted the initial proposal for a mathematical institute. This proposal, inspired by analogous institutions at globally renowned academic centers, led to the inauguration of IMS in 2000, with robust support from the Ministry of Education (MOE) and the University. During his tenure as the Provost of NUS, Chi Tat appointed Professor Louis Chen as the founding director while concurrently guiding the Institute's activities as the Chair of the Management Board.

A distinguished logician, Chi Tat played a pivotal role in spearheading Logic Summer Schools and programs at IMS over the past two decades. Collaborations with esteemed colleagues, such as Theodore Slaman and Hugh Woodin from UC Berkeley, significantly contributed to the advancement of logic in Asia and the nurturing of successive generations of students in the field. Notably, these efforts garnered attention from the John Templeton Foundation, which granted one million US dollars in 2009 to create the Asian Initiative for Infinity (AII) at IMS, complemented by matching funds from the Ministry of Education.

As Director of IMS, Chi Tat introduced innovative initiatives that enriched the Institute's scholarly activities. These included the Long-Term Visitor scheme, benefiting nearly 60 young researchers, and the IMS Distinguished Visitor scheme, which hosted over 40 high-profile academics globally. Additionally, in collaboration with the Department of Mathematics at NUS,

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IMS established the Oppenheim Lecture series. This series features distinguished mathematicians delivering lectures, alongside a public lecture series that has become integral to IMS, serving a diverse community in Singapore and garnering public appreciation. The Institute received an endowment of S$250,000, leading to the creation of the Ng Kong Beng fund, primarily allocated for public lectures.

Strategic partnerships were forged with esteemed institutions such as the Korean National Institute for Mathematical Sciences and the Vietnam Institute for Advanced Study in Mathematics, fostering scientific exchange in the region. Recognizing the significance of applied skills in education and industry engagement, Chi Tat initiated a collaboration with the Institute for Pure and Applied Mathematics (IPAM) at UCLA to offer the joint summer program Research in Industrial Projects for Students in Singapore, inaugurated in 2019.

Under Chi Tat's stewardship, IMS experienced substantial expansion, hosting approximately 150 workshops, some extending up to five months, thereby amplifying the institute's global reputation. The annual participation of around 1,000 scholars attests to the heightened activity levels. Furthermore, 19 volumes of the Lecture Notes Series were published during his tenure.

Under Chi Tat's leadership, IMS played a important role in fostering a culture of excellence within NUS and Singapore, significantly contributing to the flourishing of mathematical sciences and applications. His dedication garnered recognition and support at all levels of NUS, culminating recently in the establishment of an endowment fund generously supported across the university, ensuring IMS's continued success.

While relinquishing his role as Director, Chi Tat will persist in offering guidance as the Chairman of the Management Board, ensuring a promising future for IMS. His unwavering dedication has propelled the Institute to a prominent position among the world's leading institutes, leaving an indelible mark on its trajectory and setting a steadfast course for its future.

As of July 2023, Professor Zuowei Shen of the Department of Mathematics at NUS has assumed the role of IMS Director.

First row seated from left to right: Theodore Slaman, Chi Tat Chong, President Eng Chye Tan, Iain Johnstone
Second row from left to right: Kim Chuan Toh, Aaron Thean, Bin Liu, Choy Heng Lai, Louis Chen, Yeneng Sun

Emerging New Topics in Functional Data Analysis

From 10 to 21 July 2023, the Institute hosted a program on "Emerging New Topics in Functional Data Analysis". The organizers contributed this invited article to Imprints.

BY JIALIANG LI *(National University of Singapore)*

Advancements in modern technology have generated large data sets with high volumes of repeated measurements indexed by time points or spatial locations. Measurements on distinct entities, often in the forms of curves or images, are considered as realizations of random functions. Functional data analysis (FDA) is the collection of methodologies to analyze such data. As the field continues to expand, we face new types of functional data with more complicated structures, higher dimensions, and larger volumes. These new data structures also bring new methodological challenges in estimation, inference, prediction, and computation. In this 2-week workshop, we aimed to identify emerging new methodological frontlines of FDA and brought together some of the top researchers in this area to share information and ideas, nurture new collaborations to tackle emerging important research problems, and to make a greater impact on science, medicine, and business. The organizing committee included Jialiang Li from National University of Singapore, Yehua Li from University of California, Riverside, David Nott from National University of Singapore, Jian Qing Shi from Southern University of Science and Technology and Newcastle University, Wanjie Wang from National University of Singapore, and Jin-Ting Zhang from National University of Singapore.

The workshop involved a total of 30 one-hour research talks from invited speakers spread over two weeks, from 10–14 July 2023 and 17–20 July 2023. A Public Lecture on Functional Data Analysis in Practice was given by

Qiwei Yao from London School of Economics, UK on 13 July 2023.

We brought together theoretical researchers who work on mathematical foundations and statistical theories of functional data analysis (FDA) as well as applied statisticians who apply FDA in scientific domains and have first-hand experience with real-world data. Participants actively communicated with each other and discussed new issues and problems in FDA during the workshop period. For example, Anderson Zhang from the University of Pennsylvania, Anru Zhang from Duke University, Sonja Greven from Humboldt-Universität zu Berlin, and Rong Chen from Rutgers University all discussed new aspects in network problems where FDA approaches are needed and could potentially make improvements in modeling and prediction of the network data or tensor data analysis. Several participants mentioned the integration of the latest statistical learning methods. For example, Annie Qu from the University of California Irvine addressed the multi-resolution data; Hans Mueller from the University of California Davis talked about distributional time series; Jian Qing Shi from the University of Newcastle carried out a deep study on batch data on Riemann manifold; Lan Wang from the University of Miami discussed the off-policy evaluation with dynamic data. Their works are all directly related to machine learning and artificial intelligence and can cast additional insights from a functional data point of view.

This workshop also promoted pure theoretical development. In particular, Hang Zhou from the University of California Davis considered the diverging number of eigen-components in his talk; Alois Kneip from the University of Bonn combined the concurrent and cumulative functional coefficients models; Aurore Delaigle from the University of Melbourne was working on incomplete and fragment functional data; Bing Li from Penn State University investigated the properties of functional data on the reproducing kernel Hilbert space. All participants exchanged their views on these complex problems and carried out meaningful discussions.

This was a face-to-face meeting. There was a total of 56 participants from Singapore, the USA, the UK, Germany, China, Korea, and Australia.

Annie Qu: Individualized Dynamic Model for Multi-resolutional Data

Jian Qing Shi: Wrapped Gaussian Process Functional Regression Model for Batch Data on Riemannian Manifold

Wanjie Wang: Temporal Ordering and Manifold Recovery on Noisy Data

Tailen Hsing: A functional-data perspective in spatial data analysis

Yehua Li: Bayesian Spatially Varying Coefficient Models with Functional Predictors

Jin-Ting Zhang: A fast and accurate kernel-based independence test with applications to functional and high-dimensional data

For more information about our institute, visit our webpage at **ims.nus.edu.sg**

Multiscale Analysis and Methods for Quantum and Kinetic Problems

30 Jan–10 Mar 2023

CO-CHAIRS:

Weizhu Bao | National University of Singapore
Peter A. Markowich | University of Vienna and King Abdullah University of Science and Technology
Benoît Perthame | Sorbonne Université
Eitan Tadmor | University of Maryland

This program aimed to promote interdisciplinary research among applied and pure mathematicians, theoretical physicists, computational materials scientists and other applied scientists. It has provided a forum to highlight progress on multiscale modeling, mathematical analysis and numerical simulation for quantum and kinetic problems with emerging applications in quantum physics and chemistry, degenerate quantum gas and quantum fluids, graphene and 2D materials, network formation, collective motion in biology and social science, active matter dynamics, epidemiology, etc.

During the program, there were three workshops and two tutorial sessions and one session for junior researchers with high quality scientific talks. Christian Lubich (Universität Tübingen, Germany), Benoit Perthame (Sorbonne-Université, France) and Eitan Tadmor (University of Maryland, USA) gave talks under the IMS Distinguished Visitor Lecture Series.

The program started with tutorial lectures by Eitan Tadmor (University of Maryland, USA) and Yao Yao (NUS) on 2 and 3 February 2023. The workshop from 6 to 10 February 2023 on Multiscale analysis and methods for PDEs: fluids and active matter dynamics had close to 30 talks.

The tutorial sessions continued from 13 to 17 February 2023. The speakers were Benoit Perthame (Sorbonne-Université, France), Alexander Ostermann (Universität Innsbruck, Austria), Chunmei Su (Tsinghua University, China), Blair Blakie (University of Otago, New Zealand)

Benoît Perthame

Christian Lubich

Eitan Tadmor

Chiara Villa

Peter A. Markowich

Weizhu Bao

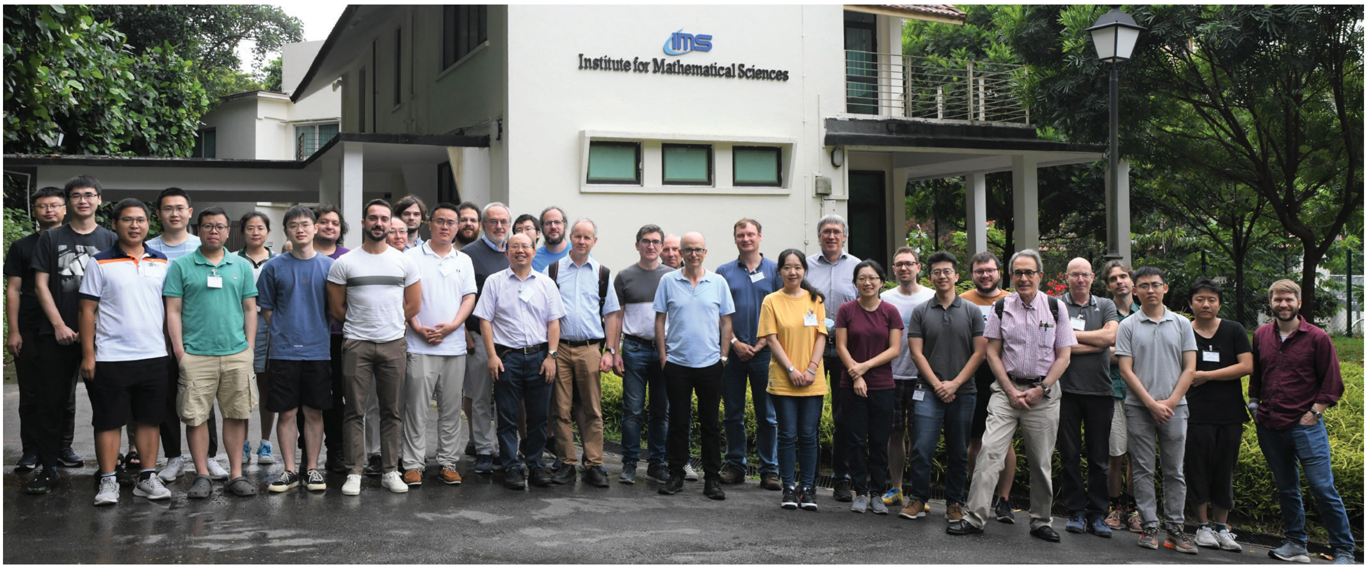
Zuowei Shen

and Yongyong Cai (Beijing Normal University, China). There were four talks by junior researchers on 15 February afternoon.

The second workshop on Multiscale methods and analysis for dispersive PDEs and applications to quantum materials had close to 30 talks from 20 to 24 February 2023. There were six talks for a workshop on Scientific computing and applications on 1 March 2023.

This program fostered local collaborations for researchers within NUS (Center for Quantum Technologies and Center for 2D Advanced Materials (C2DAM) and Graphene Research Center, and A*STAR (Institute of High Performance Computing and Institute of Materials Research and Engineering).

There were more than 100 participants, including more than 20 graduate students.



Games, Learning, and Networks

03 Apr–21 Apr 2023

CO-CHAIRS:
Georgios Piliouras | Singapore University of Technology and Design
Marco Scarsini | Libera Università Internazionale degli Studi Sociali Guido Carli

The first workshop (3–6 April 2023), which had 26 talks, brought together scholars interested in various strategic aspects of networks.

The tutorial on Quantum Games by Antonios Varvitsiotis (Singapore University of Technology & Design, Singapore) on 13 April 2023, and another two-part tutorial on Blockchain Mechanism Design by Barnabé Monnot and Davide Crapis (Ethereum Foundation) on 14 April 2023 provided several interesting avenues for future research. There were also five invited talks that afternoon, followed by a RIG open problem session after the talks.

The second workshop on Learning in games (17–21 April 2023), which was planned with 22 talks, brought together researchers from a wide array of fields – mathematics, theoretical computer science, optimization, learning theory, control theory, and economics. There was an open problem session and two work sessions planned on Monday and Tuesday afternoon, where participants brainstormed research questions.

The different perspectives converged on the study of the different aspects of multi-agent learning and equilibrium computation in games and offered a lot of complementary techniques and ideas. There were close to 100 participants, which included 27 graduate students.

Antonios Varvitsiotis:
Tutorial on Quantum Games
Data-Scarce Identification of Learning Dynamics via Sum-of Squares Optimization

Barnabé Monnot:
Tutorial on Blockchain Mechanism Design Part I
RIG Open Problems and closing thoughts

Davide Crapis:
Tutorial on Blockchain Mechanism Design Part II
RIG Open Problems and closing thoughts

Delane Foo:
Explain MEV Like I'm 5

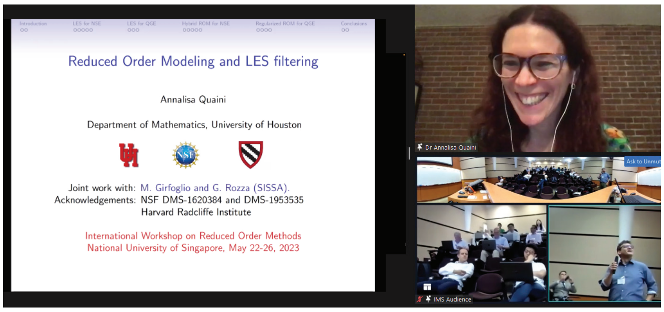
Francesca Parise:
Network games with large populations: nonuniqueness and higher-order interactions

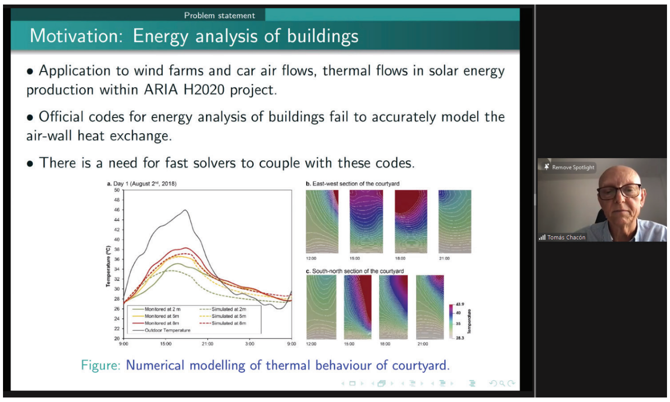
International Workshop on Reduced Order Methods

22 May–26 May 2023

CO-CHAIRS:
Annalisa Quaini | University of Houston
Gianmarco Mengaldo | National University of Singapore
Gianluigi Rozza | Scuola Internazionale Superiore di Studi Avanzati

There were more than 25 talks over five days, which focused on topics such as reduced order methods (ROMs) with applications in computational fluid dynamics, computational mechanics, urban planning, climate science, industrial engineering problems, as well as earthquake geo-sciences. This workshop also highlighted a new perspective on the integration of emerging disciplines: machine learning, data science, HPC, model reduction, and uncertainty quantification. There were close to 50 participants, including more than 15 graduate students.





Research in Industrial Projects for Students (RIPS) 2023 – Singapore

22 May–21 Jul 2023

Sixteen undergraduate students (eight local, eight overseas) were selected for the program this year. Students worked on research projects with industry sponsors Google, MOH Office for Healthcare Transformation (MOHT), Procter & Gamble (P&G) Singapore Innovation Center (SgIC) and Cubist Systematic Strategies.

From omega to Omega

12 Jun–07 Jul 2023

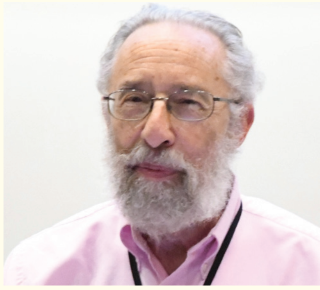
CO-CHAIRS:
James W Cummings | Carnegie Mellon University
Noam Greenberg | Victoria University of Wellington
Ralf Schindler | University of Münster
Yue Yang | National University of Singapore
Liang Yu | Nanjing University

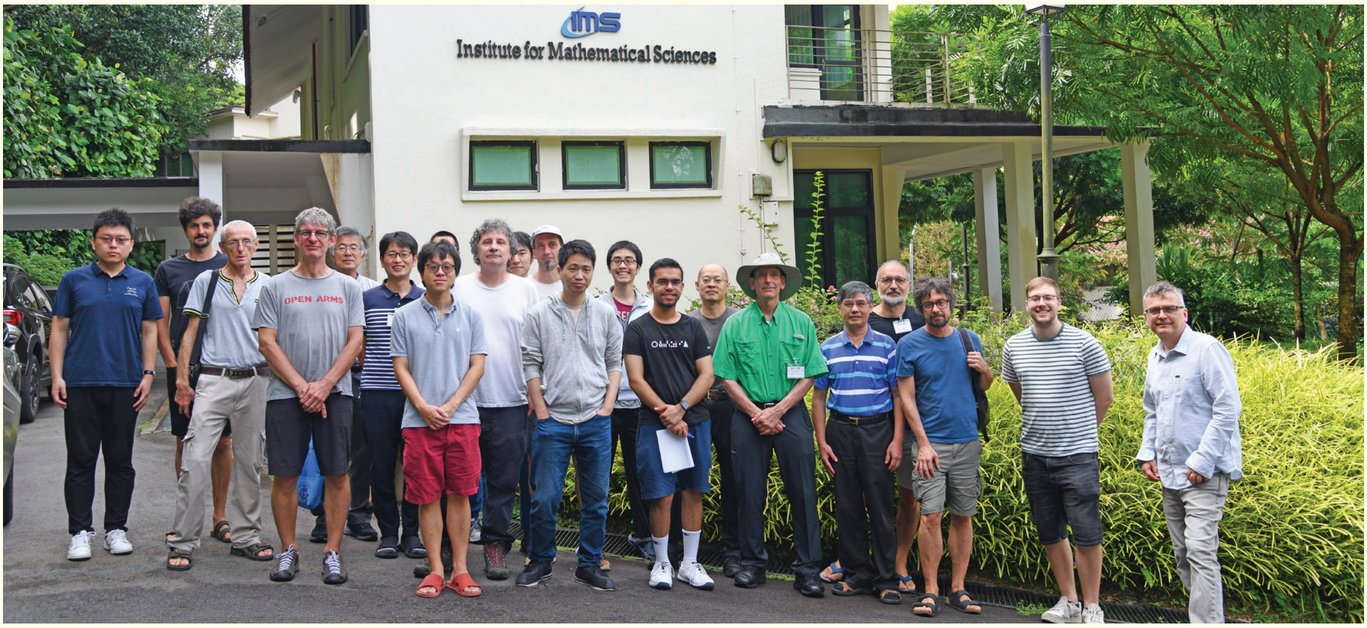
There were a total of eight talks in the first workshop on Computability Theory from 12 to 16 June 2023. The following week facilitated interactions on the connections between computability theory and set theory, and had nine talks. The Workshop on Set Theory (3–7 July 2023) had 13 speakers.

Richard Shore discussed old and new reverse mathematical connections between basis theorems, restricted versions of Zorn's Lemma, strong - and - DC, - and - correct submodels and - and -CA. Paul Shafer spoke on "Ordinal analysis of partial combinatory algebras", which connects the higher recursion theory, reverse mathematics in recursion theory to ordinal analysis in proof theory. Patrick Lutz reported his recent progress on Martin's Conjecture. André Nies gave a talk on the interaction between algorithmic randomness and analysis. Moti Gitik gave two talks about his recent progress on a problem in cardinal arithmetic proposed by Hugh Woodin.

There were close to 70 participants, which included more than ten graduate students.


Moti Gitik


Richard Shore



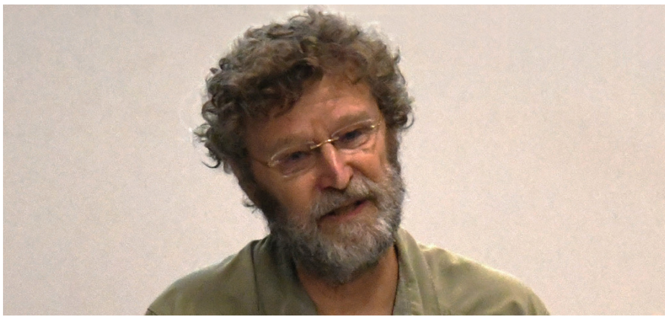
IMS Graduate Summer School in Logic

26 Jun–14 Jul 2023

Tutorials were given by W Hugh Woodin (Harvard University, USA), Theodore A Slaman (University of California at Berkeley, USA) and Joris van der Hoeven (Centre National de la Recherche Scientifique CNRS, France). Professor Woodin gave 10 hours of tutorials in the first week. Professor Slaman and Professor van der Hoeven each gave 12.5 hours of tutorials in the second and third week of the summer school. There were also 12 talks by the graduate students. Over 100 participants attended, including more than 60 graduate students.


Hugh Woodin:
Exotic models, Ultimate L, and the Ultrapower Axiom

Joris van der Hoeven:
Model theory of asymptotic differential algebra



Theodore Slaman:
Recursion Theoretic Degree Structures and Borel Equivalence Relations

Emerging New Topics in Functional Data Analysis

10 Jul–21 Jul 2023

CO-CHAIRS:
Jialiang Li | National University of Singapore
Yehua Li | University of California, Riverside
David Nott | National University of Singapore
Jian Qing Shi | Southern University of Science and Technology and Newcastle University
Wanjie Wang | National University of Singapore
Jin-Ting Zhang | National University of Singapore

This workshop brought together researchers who work on mathematical foundations and statistical theories of functional data analysis (FDA) as well as applied statisticians who apply FDA in scientific domains with real world data. This event aimed to broaden the impact of FDA by applying the cutting edge methodology to solve real scientific problems, motivating new functional data theory and methods from new scientific questions and new data structures, and identifying future research directions. New topics and problems that can be learned from the recent developments in these related fields include factor models in high dimensional multivariate time series data and tensor regression for image data and microbiome data.

The first workshop from 10 to 14 July 2023 focused on recent developments in dependent functional data and factor models, and had 16 talks. Professor Qiwei Yao (London School of Economics and Political Science, UK) gave a public lecture titled "Functional Data Analysis in Practice" on 13 July 2023. The second workshop on recent developments in high dimensional functional data analysis and applications focused on new methodology and applications had a total of 14 talks from 17 to 20 July 2023.

There were close to 60 participants, including 12 graduate students.

Ng Kong Beng Public Lecture Series

Secrets of Mental Math: An astonishing performance of Magical Mathematics

23 March 2023

Professor Arthur Benjamin of Harvey Mudd College, USA delivered a public lecture entitled “Secrets of Mental Math: An astonishing performance of Magical Mathematics” at NUS University Town on 23 March 2023.

Professor Arthur Benjamin, a renowned mathematician and magician, delivered an entertaining and energetic public lecture titled "Secrets of Mental Math" on 23 March 2023. The event was a joint effort by the Department of Mathematics and RC4 College at NUS. In his performance, characterized by a mix of speed and spectacle, Professor Benjamin demonstrated techniques for performing rapid mental math calculations. He showed participants how to add and multiply numbers in their head faster than a calculator. Professor Benjamin went on to unveil methods for memorizing 100 digits of pi. Moreover, those participating in the public lecture learned how to determine the day of the week for any given date in history, adding to the impressive mental skills exhibited during the talk.

Professor Benjamin's lecture was not just an academic event, but also a celebration of the human mind's power and the magic of mathematics. The unique blend of mathematics and magic drew an impressive attendance, with 252 people tuning in to experience the event.

Victor Tan and Arthur Benjamin

Functional Data Analysis in Practice

13 July 2023

Professor Qiwei Yao from the London School of Economics and Political Science delivered a public lecture entitled "Functional Data Analysis in Practice". The event took place on the 13th of July 2023 and was part of the workshop on Emerging New Topics in Functional Data Analysis (10–21 July 2023).

In his lecture, Professor Yao explored Functional Data Analysis (FDA), a statistical branch that deals with data in forms like curves, surfaces, and less regular structures like trees and graphs. He explained how FDA offers significant improvements over traditional data analysis methods, demonstrating this through two practical examples. The first example focused on forecasting daily electricity load curves, showcasing FDA's effectiveness in managing dynamic datasets. The second example involved estimating extreme quantiles for random functions from small samples, illustrating the method's versatility and applicability in various fields, extending beyond conventional statistical applications.

Known for his extensive research in statistical inference for complex time series, Professor Yao effectively highlighted the practicality and wide-ranging potential of FDA, citing applications from forecasting electricity demand to evaluating pricing models.

The public lecture was attended by 32 people.

Qiwei Yao and Chen-bo Zhu

INTERVIEW

JONG-SHI PANG

NAVIGATING THE COMPLEX TERRAIN OF OPTIMIZATION

Interview of Jong-Shi Pang by Sanjay Chaudhuri

Professor Jong-Shi Pang is well-known for his seminal contributions to the field of multi-agent optimization and equilibrium theory. He is famous for his work on finite-dimensional variational inequalities, complementarity problems, non-convex non-differentiable optimization, and their applications in the field of decision sciences, energy modeling, friction contact problems, structural mechanics, electricity market design, and option pricing.

Professor Pang was born in Saigon (now Ho Chi Minh city) and finished his school education in Vietnam. The family moved to Taiwan in the late sixties, where he finished his B.S. in Mathematics from National Taiwan University. Professor Pang got his Ph. D. in Operations Research from Stanford University in 1976. He also holds an M.S. degree in Statistics from Stanford.

In 2013, Professor Pang joined the Viterbi School of Engineering at the University of Southern California as the Epstein Family Chair and Professor of Industrial and Systems Engineering. He was named a Distinguished Professor in April 2023. He has held academic positions at many prestigious universities including Johns Hopkins University, the University of Texas at Dallas, and Carnegie-Mellon University. He was the Caterpillar Professor and Head of the Department of Industrial and Enterprise Systems Engineering at the University of Illinois at Urbana-Champaign between 2007 and 2013, and the Margaret A. Darrin Distinguished Professor in Applied Mathematics in the Department of Mathematical Sciences and a Professor of Decision Sciences and Engineering Systems at Rensselaer Polytechnic Institute from 2003 to 2007. Professor Pang has also been the Program Director in the Division of Mathematical Sciences at the National Science Foundation. He serves as the Editor-in-Chief of the prestigious SIAM Journal on Optimization and was the Editor-in-Chief of Mathematical Programming, Series B from 1999 to 2006.

Professor Pang is an elected member of the National Academy of Engineering. He was inducted as a fellow of the Institute for Operations Research and the Management Sciences (INFORMS) in October 2019. He received the John von Neumann Theory Prize in 2019 and George B. Dantzig Prize in 2003. Professor Pang is a member of the inaugural 2009 class of Fellows of the Society for Industrial and Applied Mathematics. He was also a co-winner of the 1994 Frederick W. Lanchester Prize awarded by INFORMS. So far Professor Pang has co-authored four monographs, edited several special volumes, and published more than one hundred and sixty articles on mathematical modeling and analysis of a wide range of complex engineering and economics systems. His work is highly cited. He has been an ISI Highly Cited Researcher in the Mathematics Category.

From 17th to 19th December 2022, Professor Pang visited the IMS as a Distinguished Visitor for the program titled "Optimization in the Big Data Era". He delivered two lectures as part of the IMS Distinguished Visitor Lecture Series on 14th December 2022. The lectures were entitled "Nonconvex Stochastic Programs: Deterministic Constraints" and "Nonconvex Stochastic Programs: Chance Constraints".

On 13th December 2022, in the lead-up to his presentations, Sanjay Chaudhuri conducted an interview with Professor Pang on behalf of the IMS newsletter Imprints. The transcript of this interview has been edited and vetted for publication. In the interview, Professor Pang provides insight into his mathematical background, his academic career, and his research.

IMPRINTS I We have Professor Jong-Shi Pang from the Viterbi School of the University of Southern California. He is the Epstein Family Chair and Distinguished Professor in the Department of Industrial Engineering and System Engineering. First of all, I would like to thank you for agreeing to give this interview.

I You grew up in Vietnam in the fifties and sixties in the middle of the Vietnam War. What was growing up in Vietnam like?

JONG-SHI PANG P Well, I was lucky. My family, particularly my parents, protected us well so that we were not so much involved in what was going on. I was born in the mid-fifties, and things were not really that bad. Life was quite normal. Things started to become bad, really bad, towards the end of the sixties. I can tell you many interesting stories about my life from that time. My brother and I went to school in Vietnam. In a nutshell, since we were protected well by the family, things were really not that bad.

I What motivated you to take up mathematics as a subject at the university?

P It's hard to pinpoint that. I attribute that to sort of a natural process. I finished high school at the age of 14. I skipped a lot of classes. In high school, I took all the courses. When I applied to college, I chose mathematics. I got my BS in Taiwan. As you probably know, mathematics in college is very different from that in high school. Especially in the math department, the proofs were very much emphasized. I can tell you that in the beginning, I had no idea; I was really not used to abstract concepts. They were totally new to me. My first calculus test was miserable. I failed it. Then after the first year, sort of halfway into the undergraduate programme in mathematics, we learned about integration. That's when I started to excel and then things just developed naturally from there. So, I don't think I can give you any sort of specific reason for pursuing Mathematics. It more or less developed naturally.

I My next question is why did you choose Taiwan? Vietnamese mathematicians we know, even today, mostly go to France.

P You are absolutely right. But now we need to sort of understand a little bit of history. Before the fifties, and even in the early fifties, Vietnam was very much under the occupation of the French. A lot of people, even my cousins, actually went to France. That is true.

At that time, somehow my family decided not to send us to France because my parents probably felt that France was too far away. However, my brother and I had to leave Vietnam because, and I hate to say this, they really started drafting. They started drafting fourteen and fifteen years olds. The draft was serious, involving actual combat. There was basically no other opportunity for young people. We urgently needed to find a new home.

Taiwan accepted Chinese-speaking people from overseas. In those days, if you could speak Chinese, you could easily get a passport in Taiwan. This was a key reason why we ended up in Taiwan. It was very natural, and very easy to move there in those days.

I So how was Taiwan in the seventies?

P Well, I was a student. Very naïve, very young, not even twenty years old. Again, my brother and I were very well protected. My parents actually moved to Taiwan with us. We were shielded from a lot of social issues. It was fine. I mean, it was very political at that time. Taiwan was under the control of Kuomintang. There were a lot of propaganda materials around. There was even a course on military service. But it was not bad. I mean, it was not bad at all. It was not like they were really trying to brainwash you or something. Not like that. So, no, life was good.

I Was there any separate curriculum in applied mathematics or operations research?

P I think that the two areas you mentioned are quite different. Nowadays, things have changed. At that time when I was in my junior year, the operation research field had just started. That was in the early seventies. The operations research (OR) subjects were not there. Even applied mathematics was not considered applied, it was not a subject by itself.

I It was mostly special functions, partial differential equations etc.

P Right. And unlike statistics. Even in the seventies statistics was already a discipline by itself. But Applied Math and Operations Research, at that time, were not separate disciplines. They were all embedded in mathematics. In terms of differences, even today, applied math is very much PDE, numerical analysis, whereas OR, especially in the beginning, was concerned a lot with planning, and decision-making. OR is an area where mathematics is heavily used, but the emphasis is different. In some sense, the subject is a lot more applied. It deals with real problems. For example, problems like financial management, inventory control, scheduling, optimal resource allocations, etc. are not traditionally dealt with in Applied Mathematics.

I What was the reason behind moving to the US for graduate studies?

P Again, I think it was a natural path. I finished college, and I was interested in further studies, and at that time, there were some very good universities in the US. I think the decision just came. As soon as I decided that I needed further study, the US became a natural choice.

I Many statisticians like Jeff Wu went to the US for their graduate studies around that time from Taiwan. Were you among the first group of students to go to the US from Taiwan? Was there any specific catalyst behind such a move?

P I know Jeff very well. He was only two years senior to me. We came to the United States at the same time, actually, on the same flight. He went to Berkeley and I went to Stanford. Statistics was quite popular as a discipline. Jeff was definitely not in the first group of statistics students to go to the US for Ph.D. from Taiwan. Now, if you ask me for names, I would need to think a little and do a little bit of research. On the other hand, if you're talking about the first group studying Operations Research, I was certainly among the earliest. When I was admitted to Stanford, I was admitted to the math department. But then I felt that it was a little bit too pure for me and I wanted to find another area. At that time, the OR department at Stanford was really very good. I mean, the top of the world. So, I immediately tried to switch there. Also, OR was very close to mathematics because they use so many mathematical tools. In a way, it was a natural path for me.

I can mention T. C. Hu, who was senior to me and made a similar move. But in terms of people in the Chinese community who actually ended up in the OR disciplines, I was among the earliest.

I Were there any specific catalysts behind?

P In a way, it was natural. I was interested in mathematics, and pure mathematics was a bit over my head. Although I did spend one year in the math department at Stanford. But then I felt that OR was more attractive to me. I somehow felt that it was more "practical". At that time, I didn't know about, for example, electrical engineering and the fact that they would use so much mathematics. The rest is history. I have a master's degree in statistics as well.

I What was Stanford like in the seventies? It must be very different from what it is now.

P It was a dream. It is very different now. We (me and my peers) used to call Stanford a country club. I am sure you understand that as a graduate student you don't have a lot of things to worry about, unless you're married. I was not married. There were quite a few married graduate students. But as a single student, I had everything I needed. There were good sports facilities, beautiful. The campus was quite spacious, unlike today. Today it is quite crowded. The only problem was that, as a poor graduate student, traveling was a problem. I mean, in that area, you needed your car to go anywhere. That was the only thing. But other than that, it was really good. I mean, the food was good on campus and outside. Basically, it was just a wonderful place. Stanford was a wonderful place.

I Who motivated you the most during your graduate studies at Stanford, and then later as a young faculty?

P I always say that I have three mentors who were really very supportive of me. I was very lucky in that respect. The first person, of course, is my advisor Dick Cottle. I will always remember him. Academically, he did so much for me. Then when I graduated, I was very lucky to have two extremely supportive people. I also remember them because I think that they were just wonderful human beings. One is Olvi Mangasarian at the computer science department in Wisconsin, Madison. A wonderful, wonderful human being. I think that's very important. I mean, nowadays, there are just not enough wonderful human beings. The third person is Steve Robinson. Although he was not the director, I'm sure that he was the person behind the scenes who actually hired me as a postdoc at the mathematics research center at the University of Wisconsin. So, there are three people I would credit to be the most influential and the most helpful, most kind. That's one thing that I want to emphasize. They were most wonderful, most supportive, and most kind. They were the people who have really impacted my life. I always remember them now.

In Academia, I can name a fourth person, and this person I will always remember. His area is not the same as mine. He was a professor at Carnegie Mellon, Egon Balas. I don't know whether you know him or not, but he himself was really a wonderful person. He has written this book, I forgot what the name was. He started off as a communist, I think in Romania. He was jailed. Later, he made a 180-degree turn from a pro-communist to someone who's against them. Eventually, he came to the States. He was extremely helpful to immigrants. I will credit him for orchestrating my first employment at Carnegie Mellon. A wonderful human person. Professionally and academically, these are the four people I give the most gratitude to.

Moving away from academics, I won't delve into details about my parents, but I'd like to share my non-academic experiences in the US, particularly at Stanford, which you inquired about earlier. Stanford offers a host program where families serve as hosts for students arriving on campus. I had a wonderful host family. They were exceptionally kind and truly wonderful. You won't find too many families like this today. So, in a way, I was very lucky. I had very supportive mentors and then, non-academically, I got this tremendous family who took me up. They didn't know me from Taiwan. They took me up and let me stay with their family a few weeks before school started and treated me as a wonderful guest. I mean, this is just a memory that I'll never, never forget. And this is something that unfortunately we don't have a lot of nowadays.

I Why did you choose to work on the complementarity problem?

P I think the complementarity problem is something that sort of connects classical mathematics and contemporary mathematics. In classical mathematics, you very much deal with equations. For example, linear equations, Ax=b, differential equations, and partial differential equations. Complementarity problems sort of go beyond that because you need to decide which system of linear equations, that's the linear part, is really the system whose solution you want. I believe the key reason I developed an interest in complementarity problems, particularly the linear ones, is their connection to matrix theory. I found that I had a natural aptitude for and enjoyed working with matrices. Complementarity problems involve studying various matrix classes, which was particularly appealing to me. This field serves as a bridge between traditional and modern mathematics, a realization I came to appreciate later. Initially, it was the matrix theory aspect that drew me to this area. My success and enjoyment in working with matrices played a significant role in this interest.

I In a nutshell, what is the complementary problem?

P Let me try to explain it from the perspective of equation solving. That is sort of a more modern view. Consider a system with, say, an exponential number of equations. The challenge lies in determining which specific system will yield the correct solution. On the classical side, you have equations, linear equations. Everyone knows about linear equations. But on the non-classical side, the modern perspective is that you have this combinatorial feature because you need to decide which system of linear equations will have solutions that are signed non-negative. It bridges classical mathematics in the form of equations with modern applied mathematics with this junction of combinatorial features. That's one perspective. Now, the other way to sort of explain the complementarity is as follows. Although one can look at it from the perspective of equations, some of the equations in the system actually involve non-differentiable functions. Another modern viewpoint on complementarity is that it involves analyzing a system of non-differentiable equations. While every mathematician is familiar with equations, the unique aspect here is that these equations are characterized by non-differentiable functions. This is fundamental because, though linear algebra isn't as dominant as it once was, it acts like a bridge. Traditionally, mathematics focused on smooth, differentiable aspects, but now there's a shift towards non-differentiable elements. The linear complementarity problem is at the forefront of this transition. It moves us from classical mathematics to a modern framework where combinatorial and non-differentiable elements are integral. That is in a nutshell, without really going into any details, a big picture of the fundamental role of this problem in the world of mathematics.

I Since there is a computational aspect to the problem, I wonder how good was the computational facility at Stanford in the seventies? Many computational statisticians, I have heard, were connected to SLAC. Were there enough computational resources available then?

P The statistics department at Stanford has always been good. But I don’t think many of them were particularly connected to SLAC. What was closest to me in terms of computations at that time was the numerical analysis group, which actually did a lot of numerical computations. They had a very strong numerical analysis group within the computer science department. In particular, there was Gene Gollub. He was sort of like the king. This is not exactly connected to these computational facilities, but they definitely have a very strong computer science department.

In fact, I can tell you something from a historical perspective. Nowadays, everyone is talking about artificial intelligence. When I was a student in the seventies, there was this guy Feigenbaum — I'm not sure if you're familiar with him. Although I'm not in computer science, he was a pivotal figure in artificial intelligence at that time. Then there was a period where AI seemed dormant. However, I think Feigenbaum and his group continued to advance in the field of AI. At that time, when considering computational facilities, email was just starting to emerge.

I Yes, exactly. But my question was more on the implementation. You could do the mathematical deductions but how did you actually implement them?

P Oh, yes, the computational power was enough, definitely enough. I'm not a too computational person. But we definitely had enough computational power. I was affiliated with this numerical analysis group, which was, of course, very computational. This allowed me to be closely associated with the students there. In my time, this group was housed in a place known as the Sarah House. I recall going there and using their terminal to write programs and such. To be precise, the computational facilities back then were quite different. We didn't have all these programming languages available; it was mainly Fortran at that time.

I Was the Fortran compiler available in those days?

P Of course. Fortran, I think, was from Waterloo. But in the seventies and eighties, you can imagine Fortran was the dominating language. Matlab had just started. The computational environment at that time was very different. These days, computation sort of dominates in many disciplines. It was not like that at that time.

I Has the improvement in computational tools brought about any significant changes in how the complementarity problem is approached?

P It is just like any area. Computational advances have brought as many changes to my area as they have in all areas. One can take tremendous advantage of computational tools. The impact is tremendous, not just on this particular problem, but on the optimization field in general. When you trace back how computers have impacted this problem and optimization in general, you can see that without the advances in computation, optimization would have never reached today's status. I mean, you can propose a lot, and you can write down a lot of algorithms, but without these advances in the hardware part and software parts of computational systems, there's just no way that you can solve hard problems. The impact was tremendous.

I Your article in SIAM Review on the applications of the complementarity problem lists a lot of interesting applications in engineering and economics. In recent times you have been interested in Nash and similar equilibria and portfolio optimization problems. Do you have a favorite application?

P Not exactly. In a way that I'm still very much sort of a "theoretician". But not quite that either. I'm interested in building a solid foundation that is inspired by applications. About Nash Equilibrium, there are lots of applications in electricity markets and in many different fields. We talked a lot about complementarity and it's interesting to note the practical connections of this concept, such as its relation to friction.

I I noticed you have written a lot on friction.

P Here's an interesting phenomenon to consider: when you drop a ball on a table, it doesn't penetrate the surface but bounces back. Why? This is explainable through complementarity relations. To understand complementarity, consider this bouncing ball. Before the ball hits the table, there's a positive distance between them. However, as it contacts the table, the distance becomes zero. At this moment, a counterforce, as per Newton's third law, comes into play and pushes back. In other words, that reactive force becomes positive. So, before the ball reaches the table, there's no reaction, there's no reactive force because there's no need. But the moment it reaches, there is a force that we don't see sort of pushes back, and that force becomes positive. So, you have this complementary relationship between the distance and the magnitude of the force. That is one interesting concept and one way to think about complementarity. In this case, when the distance between the ball and the table is positive the counter force is zero, and when the distance is zero, the counter force is positive.

I It is similar to the complementary slackness condition.

P Exactly. But this is a sort of physical thing that you don't think comes from the complementarity problem.

I In your view, what is the most important and interesting development in optimization theory in recent times?

P Recently, you might say, though I don't fully agree, that the popularity of optimization has surged due to its application in numerous machine learning problems. However, I think we need to exercise caution. I'm hesitant to make this claim because young people might then overlook the significant impact of optimization in various other areas. Essentially, regarding advances, particularly in computational tools, the interest in machine learning and statistics has led to tremendous advances in the computational refinement and computational methodology of convex programming. However, I must emphasize this point, as I don't want to mislead young people who might read the article: convex programming, despite its uses, has a major drawback that is often overlooked. The paradigm itself is very restrictive, to the extent that it becomes a detriment to the advances of the field. What we have here is a powerful toolset that many people use, especially in machine learning and artificial intelligence. However, many fail to recognize that there are many other applications in general optimal decision-making that employs this methodology. Then one must realize that this concept of convexity is really a simplification. I mean, the world is not convex. What is happening now is that, on one hand, this is a major advancement, but on the other hand, it's impeding further progress. You have this very powerful set of tools because of the advances. Everyone tries to use it without really thinking outside the box. Even in situations that are not quite convex, people try to adjust them so that they can apply this methodology. To the extent that it sounds like one doesn't really need to learn anything else besides convexity. I believe that mindset, despite the recent advances, is very dangerous. I think it's important for people to realize that convexity is a very powerful set of tools, but one cannot fit everything into it. That is probably not the right approach. That is going to hurt the real advances. If you start to do things just using this set of tools at the expense of simplifying or mis-formulating the problems, when things are on a smooth path, nothing will happen. Everyone is happy. But when you lose, you're going to lose a lot. I mean, just imagine, it's like a tall building on soft sand. The building may survive, what, 20 years. But it is on soft sand. So ultimately, the whole building will collapse. The tremendous advances are also bringing this danger that unfortunately a lot of young people do not recognize.

I What do you think is going to be the most exciting future development?

P I think, from a mathematical perspective, the challenge lies in handling non-convex and non-differentiable functions. You can't simply overlook the complexities. For instance, if a function isn't differentiable, you can't treat it as if it were, hoping the non-differentiable points won't arise. That's a risky assumption. The key advancement needed is the recognition of this issue. Once acknowledged, we need to start thinking about solutions. When faced with equations involving non-differentiable functions, what do you do? A lot of times people simply abuse them.

Another aspect I'd like to discuss is what I refer to as non-problems, which are essentially non-traditional topics. It's important to think beyond conventional subjects. My interest lies in the areas of randomness or uncertainty, which might align with your interests. When you combine the two things, you have topics that are non-traditional, like you have non-differentiable functions, and then you need to deal with all those non-traditional parts within the paradigm or context of uncertainty and randomness. That is, I think, really the area that people should focus on.

This is relevant for statistics as well. You cannot take derivatives of functions that are not differentiable. Then, additionally, you have randomness in your observations. When you need to make optimal decisions, when you are faced with this kind of non-traditional objectives, I think it is important to study various methods which can handle nontraditional features in the presence of randomness.

I Optimization and big data. Any thoughts?

P Yes, it is a good marriage. It is also a very fruitful marriage. So far, the marriage seems to be doing well and I like that. Anyway, things are very rosy now. Optimization contributes a lot to big data and big data also inspires a lot of advances and optimization. There is no question about that. But I think the key point is, and I want to come back to what I said, one should recognize the limitations of this marriage. And then one cannot be stuck in the present. One must think outside of the box. The two areas will continue to inspire each other. There is no question that optimization will continue to make significant contributions to solving problems in the big-data area. On the other hand, they're also very challenging data science problems that inspire people to think about how to improve the methodologies in optimization. But in this whole process, at least for optimizers, they need to think outside the box. It should not be just that we have this powerful set of tools, then why not make it better? But then you're still within this, this box. You make the box even fancier, but you're still within the box.

I Optimization techniques have a large role to play in statistics. However, most statisticians are not familiar with the subject. On the other hand, operation researchers don’t seem to be that interested in core statistical applications. As someone who is also interested in modern statistical estimation problems, do you have any suggestions about how these two groups can interact more?

P To some extent, you are certainly right. One can say that, 30 years ago, statisticians did not talk a lot to optimization people. However, consider my case as a possible exception. I have a bachelor's degree in math and a master's in statistics, so I've long recognized the value optimization brings to statistics, and vice versa.

I The opposite thing is true because operation researchers talk to statisticians much more often than statisticians talk to operation researchers.

P I agree that what you've said is correct, especially regarding training. To put it plainly, more statistics students should take optimization courses. There should be cross-disciplinary learning, where students from one field take sufficient courses in the other, and vice versa. Now, traditionally that is not quite the case. A lot of statisticians really are not very familiar with some of the basic concepts in optimization. Conversely, while engineers are required to take courses in probability and statistics, they don't have a similar requirement for optimization. That's where the problem is. That is not right. All the undergrads, in all engineering disciplines, even in economics and social sciences, take probability and statistics courses, but very few of them take courses in optimization. I can tell you that there are no such requirements except in my department of Industrial and Systems Engineering. That's already where the problem is because people know at least a little bit about probability and statistics, but you can get a PhD degree in engineering or economics without having taken an optimization course. Statistics and optimization should be on equal footing, but currently, there's a disparity: more people are familiar with statistics than with optimization. Optimizers at least have some basic understanding of statistics, but the reverse isn't as common. I think that to bridge this and make this kind of exchange more fruitful, it is essential that this situation must balance itself.

I'm not saying optimization should be mandatory for everyone but consider this: people often misuse terms like 'local minimizer' or 'local minimum' without fully understanding how to verify these points. This misuse of language shows a lack of understanding. To address your question, it's crucial for people to have a solid grasp of the basics, to realize the need for deeper knowledge. Every engineer and social scientist takes a statistics course, but in many departments, there's no requirement for optimization. Consider your department as an example; it probably doesn't have any specific requirements in this regard.

I We have a couple of optimization modules in our program. I am not sure whether they are required.

P Take the economists. They definitely take an econometrics class, but they don't really take an optimization class. I think it’s a major problem. If we want the two areas to really interact, even advance together, I think that this barrier needs to go, and a fusion must smoothly emerge. Otherwise, you are always going to get people who are only one-sided.

I Professor Pang, thank you very much for your time.

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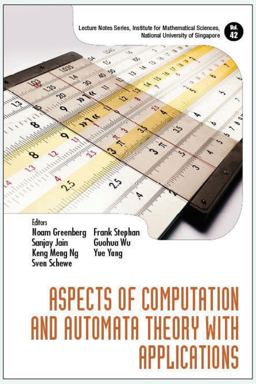

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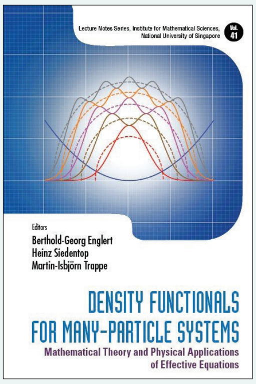

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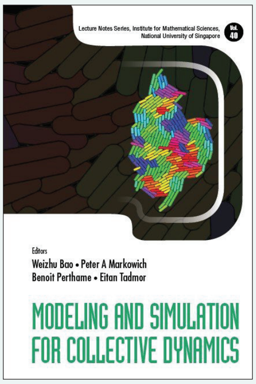

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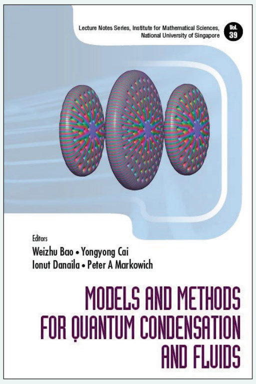

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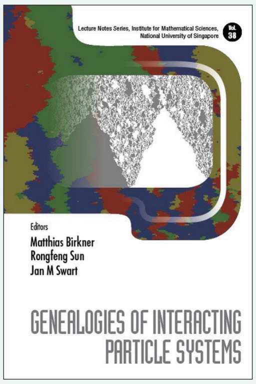

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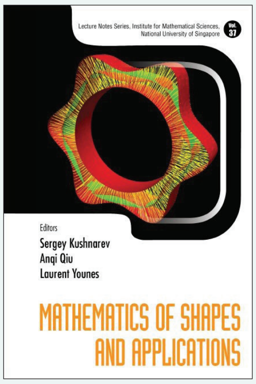

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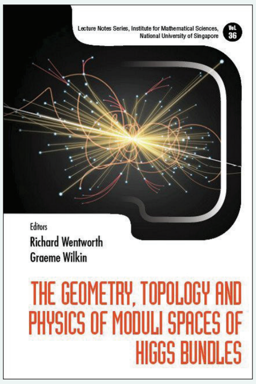

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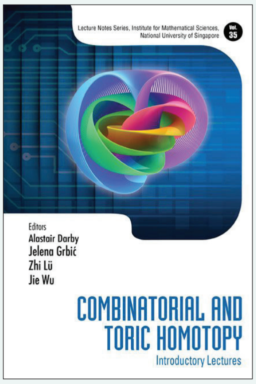

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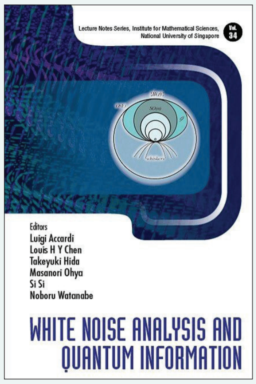

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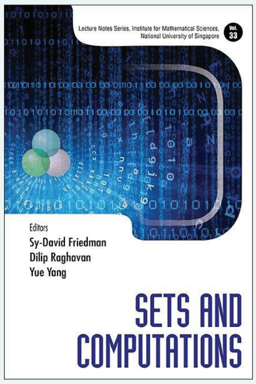

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