Mini-Courses & Short Talks
Abstracts

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Tom Hutchcroft
Caltech, USA

Mini Course:
Topics in critical percolation

Bernoulli percolation is one of the simplest to define models of a system undergoing a phase transition. In this course I will give an introduction to the study of percolation at and near the critical point, i.e., the point at which it undergoes its phase transition, focusing on the distinction between critical behaviour below and above the so-called upper critical dimension. In particular, I intend to give a complete derivation from first principles of various critical exponents for long-range and hierarchical models of percolation.

Course References on critical exponents for (long-range) percolation:

Sharpness of the phase transition: https://arxiv.org/abs/1502.03050

FKG, BK, tree graph inequalities are all covered in Grimmett's book https://link.springer.com/book/10.1007/978-3-662-03981-6 (where you can also read the old proofs of sharpness). You can also read Aizenman and Newman's original paper https://link.springer.com/article/10.1007/BF01015729.

Grimmett also has an excellent discussion of non-rigorous scaling theory.

Triangle condition implies mean-field exponents: https://arxiv.org/abs/2106.06400

The universal tightness theorem and associated hyperscaling inequalities are proven in this paper: https://arxiv.org/abs/2008.11197

The two-point function on the hierarchical lattice: https://arxiv.org/abs/2103.17013

Upper bounds for the two-point function for LRP on Zd: https://arxiv.org/abs/2202.07634

"Renormalization group" analysis of hierarchical percolation: https://arxiv.org/abs/2211.05686

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Xin Sun
Peking University, China

Mini Course:
SLE, LQG and LCFT

We will give an introductory course on the interaction between three central topics in conformal probability: Schramm-Loewner evolution (SLE), Liouville quantum gravity (LQG), and Liouville conformal field theory (LCFT). The content of the four lectures are:

1. SLE and LQG as the scaling limits of lattice models and random planar maps.

2. Coupling of SLE and LQG: quantum zipper and mating of trees.

3. LCFT: integrability and conformal bootstrap.

4. Coupling of SLE and LCFT.

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Gefei Cai
Peking University, China

Proof of Delfino-Viti conjecture for 3-point connectivity of 2D critical percolation

In the 2D critical Bernoulli percolation, the n-point connectivity $P_n(x_1,...,x_n)$ is defined to be the probability that $x_1,...,x_n$ are in the same cluster. Delfino-Viti'10 conjectures that the ratio $R=P_3(x_1,x_2,x_3)/\sqrt{P_2(x_1,x_2)P_2(x_1,x_3)P_2(x_2,x_3)}$ will converge to a universal constant, independent of x_1,x_2,x_3, which can be expressed through the imaginary DOZZ formula. This formula gives $R\approx 1.022$ and matches the numerical simulation. In this talk we rigorously prove Delfino-Viti conjecture, from the coupling results of Liouville quantum gravity and conformal loop ensemble developed recently. Our approach also solves 3-point connectivities of 2D critical FK-q percolations and 3-point connectivity of spin clusters in 2D critical Ising model.

This talk is based on the joint work with Morris Ang (Columbia), Xin Sun (BICMR) and Baojun Wu (BICMR).

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Francesca Cottini
National University of Singapore, Singapore & University of Luxembourg, Luxembourg

Quasi-critical fluctuations for the 2d directed polymers

The model of directed polymer in random environment describes a perturbation of the simple random walk given by a random environment (disorder). The partition functions of this model have been widely investigated in recent years, also motivated by their link with the solution of the Stochastic Heat Equation. In this talk we focus on the 2d case and we show that Gaussian fluctuations hold in the so-called quasi-critical regime, which interpolates between the sub-critical regime (where a deep understanding has by now been obtained) and the critical regime (where many key questions are still open). Our results identify the most extended regime where Gaussian fluctuations can hold, before reaching the critical regime where they fail.

This is a joint work with Francesco Caravenna and Maurizia Rossi (University of Milano-Bicocca).

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Ayana Ezoe
Chuo University, Japan

Switching particle systems for foraging ants showing phase transitions in path selections

Switching interacting particle systems studied in probability theory are the stochastic processes of hopping particles on a lattice consisting of slow particles and fast particles, where the switching between these two types of particles happens randomly with a given transition rate. In the present talk, we show that such stochastic processes of many particles are useful in modeling group behaviors of ants. Recently the situation-depending switching between two distinct types of primarily relied cues for ants in selecting foraging paths has been experimentally studied by the research group of Nishimori. We propose a discrete-time interacting random-walk model on a square lattice, in which two kinds of hopping rules are included. We report the numerical simulation results which exhibit the global changes in selected homing paths from trailing paths of the 'pheromone road' to nearly-optimal paths depending on the switching parameters. By introducing two kinds of order parameters characterizing the switching-parameter dependence of homing-duration distributions, we discuss such global changes as phase transitions realized in path selections of ants. Critical phenomena associated with the continuous phase transitions are also studied.

The present talk is based on the joint work with S. Morimoto, Y. Tanaka, M. Katori (Chuo University), and H. Nishimori (Meiji University); arXiv:cond-mat.stat-mech/2311.01946

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Yifan Gao
City University of Hong Kong, China

Scaling limit of the occupation measure of random walk cut points

In this talk, we consider the occupation measure of the cut points of a simple random walk on a d-dimensional cubic lattice for $d = 2, 3$, and we show that the scaling limit of the occupation measure in weak topology is the natural fractal measure on the Brownian cut points defined via its Minkowski content.

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Kohei Hayashi
RIKEN, Japan

Kardar-Parisi-Zhang equation from Bernardin-Stoltz model

We study the so-called Bernardin-Stoltz (BS) model which admits two conserved quantities: volume and energy. We consider the BS model driven by a general nonlinear potential and then asymptotically let the inverse temperature of the system go to zero. As a consequence, by a Taylor expansion argument, we can extract a principal part driven by the harmonic potential, and we can control the strength of a reminder part which is governed by a cubic polynomial. Under this situation, we consider two fluctuation fields defined as linear combinations of the fluctuation fields of volume and energy, and as a main result, we show that the fluctuations of one field converge to a solution of the stochastic Burgers equation.

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Luca Makowiec
National University of Singapore, Singapore

Diameter of Random Spanning Trees in Random Environment

We will introduce Random Spanning Trees in Random Environment, a disordered system on spanning trees. Our primary goal is to determine the order of the diameter of a (typical) spanning tree, a crucial step towards the pursuit of a non-trivial scaling limit. For the complete graph, we will give upper and lower bounds for a phase transition where we either observe the diameter of the Uniform Spanning Tree or that of the Minimum Spanning Tree, which scale with different power laws. Lastly, we discuss a conjecture about the order of the diameter inside the critical window.

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Saori Morimoto
Chuo University, Japan

Eigenvalue and pseudospectrum processes generated by the shift matrix with perturbation

Motivated by the recent study of non-Hermitian matrix-valued stochastic processes, we consider the dynamical processes of $n \times n$ matrix generated by multiplication of a shift matrix. We add a constant δ to all matrix elements at time $m \leq n-1$ as perturbation. In the present talk, first we show a movie of the "eigenvalue processes" on a complex plane obtained by computer simulation. We prove the theorems which identify and control the eigenvalues in the observed process. We report the numerical evidence for that the observed process includes pseudospectra. We will also discuss the case that perturbation is random, which is given by adding an independent Gaussian random variable to each matrix element.

The present talk is based on the joint work with M. Katori (Chuo University) and T. Shirai (Kyushu University, IMI).

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Ryoichiro Noda
Kyoto University, Japan

Convergence of local times of stochastic processes associated with resistance forms

A resistance metric on a space characterizes, as the electrical energy, a corresponding bilinear form (a resistance form) and, combined with a measure on the space (and under certain technical conditions), determines uniquely a Dirichlet form and a stochastic process on the space. Croydon-Hambly-Kumagai (2017) showed that if a sequence of spaces equipped with resistance metrics and measures converge with respect to the Gromov-Hausdorff-vague topology and a uniform volume doubling (UVD) condition is satisfied, then the associated stochastic processes and local times also converge. However, the UVD condition is too strong for many sequences of random graphs. In the subsequent work of Croydon (2018), the UVD condition was relaxed and the convergence of the processes was established under a weaker non-explosion condition. However, the convergence of local times was left open. In this talk, we show that if the spaces additionally satisfy a certain metric-entropy condition, which is weaker than the UVD condition, then the local times of the processes also converge. The metric-entropy condition can be checked using volume estimates of balls in the spaces, and we also present some example applications.

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Takumu Ooi
Kyoto University, Japan

Scaling limit of Liouville simple random walk on Z^2

As represented by Liouville measure, Gaussian multiplicative chaos (GMC) is a random measure constructed from a Gaussian field. In this talk, we consider the convergence of processes time-changed by GMC under certain assumptions. In particular, we provide that the scaling limit of the Liouville simple random walk on Z^2 is Liouville Brownian motion.

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Ryosuke Sato
Chuo University, Japan

Stochastic dynamics on DPPs and GICAR algebras

Determinantal point processes (DPPs) are mathematical descriptions of interacting particle systems with repulsive force, and they appear in diverse fields of mathematics and mathematical physics. Moreover, it is known that they are intimately related to operator algebras, specially called GICAR algebras. In this talk, we extend the connection between DPPs and operator algebras to encompass dynamical aspects. Namely, we discuss how dynamics on operator algebras give rise to stochastic dynamics on DPPs.

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Ryosuke Shimizu
Waseda University, Japan

Construction of first-order Sobolev spaces on the planar Sierpinski carpet

In this talk, I will explain how to construct a (1, p)-Sobolev space on the Sierpinski carpet for every p > 1 via subsequential scaling limits of discrete p-energies on approximating graphs.

I will also explain a relation between our Sobolev spaces and the Ahlfors regular conformal dimension.

This is joint work with Mathav Murugan (The University of British Columbia).

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Yuya Tanaka
Chuo University, Japan

Numerical study of eigenvector-overlaps and the regularized FK determinants of the non-Hermitian matrix-valued stochastic processes

We consider a one-parameter family of non-Hermitian matrix-valued processes, which can be regarded as a dynamical extension of Girko's ensemble interpolating the Gaussian Unitary Ensemble and the Ginibre ensemble of random matrices. There eigenvalue process is coupled with the eigenvector overlap process. We consider the time dependent random field defined on the two-dimensional complex space using the notion of the regularized Fuglede-Kadison (FK) determinant of the matrix-valued stochastic process. We made Python programs to simulate the eigenvalue processes, the eigenvector overlap processes, and the logarithmic versions of the FK determinant random fields. We have numerically studied the time evolutionary processes of the FK fields and compare their singular behaviors with the point processes of eigenvalues weighted by the diagonal elements of the eigenvector-overlap matrix-valued processes. We will show the movies of our computer simulation results.

The present talk is based on the joint work with M. Katori (Chuo University).

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Aoteng Xia
Peking University, China

Some progress on Random Field Ising Model

We studied the random field Ising model(RFIM) on the box [-N,N]^d\cap Z^d with external field {\epsilon h_v} where h_v will be i.i.d. normal variables. Our main interest is the behaviour boundary influence m(T,N,\epsilon). In dimension d\geq 3, we proved that for any T<T_c and \epsilon fixed but small enough, m(T,N,\epsilon) has a positive lower bound as N goes to infinity. In dimension 2, it has been proved that m(T,N,\epsilon) decay to 0 with exponential rate for any temperature T and any \epsilon>0 fixed. And we focus on the critical temperature T_c and the case that \epsilon decays with N. The main result is that if \epsilon<<N^{-7/8} it has little influence, i.e., m(T_c,N,\epsilon) has the same order with m(T_c,N,0); if \epsilon>>N^{-7/8}, the ratio between m(T_c,N,\epsilon) and m(T_c,N,0) will be of order exp(-c\epsilon^{8/7}N).

This talk is based on two joint works with Jian Ding, Fenglin Huang, Yu Liu.

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Zijie Zhuang
University of Pennsylvania, USA

Tightness of exponential metrics for log-correlated Gaussian fields in arbitrary dimension

In this talk, I will present a result proving the tightness of a natural approximation scheme for an analog of the Liouville quantum gravity metric on R^d for arbitrary $d \geq 2$. More precisely, let $\{h_n\}_{n\geq 1}$ be a suitable sequence of Gaussian random functions which approximates a log-correlated Gaussian field on R^d. Consider the family of random metrics on R^d obtained by weighting the lengths of paths by $e^{\xi h_n}$, where $\xi > 0$ is a parameter. We prove that if ξ belongs to the subcritical phase (which is defined by the condition that the distance exponent $Q(\xi)$ is greater than $\sqrt{2d}$), then after appropriate re-scaling, these metrics are tight and that every subsequential limit is a metric on R^d which induces the Euclidean topology.

This is based on a joint work with J. Ding and E. Gwynne.

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