Workshop in Honour of Takashi Hara
Abstracts

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Morris Ang,
Columbia University, USA

Cutting Liouville quantum gravity by SLE with mismatched central charge

Beginning with the seminal work of Sheffield, there have been many deep and useful theorems relating Schramm-Loewner evolution (SLE) and Liouville quantum gravity (LQG) when their parameters are matched, meaning $\kappa \in \{ \gamma^2, 16/\gamma^2\}$. Roughly speaking, the SLE curve cuts the LQG surface into two or more independent LQG surfaces. We extend these theorems to the setting of mismatched parameters: an LQG disk is cut by an SLE curve into two or more LQG surfaces which are conditionally independent given the values along the SLE curve of a certain collection of auxiliary fields. These fields are sampled independently of the LQG and SLE, and have the property that the central charges of the LQG, SLE and auxiliary fields sum to 26. This central charge condition is natural from the perspective of bosonic string theory. Similar statements hold when the SLE curve is replaced by, e.g., an LQG metric ball or a Brownian motion path. These statements are continuum analogs of certain Markov properties of random planar maps decorated by two or more statistical physics models.

Based on joint work with Ewain Gwynne.

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Tim Banova
The University of Melbourne, Australia

Convergence of the voter model to historical Brownian motion in d > 2 dimensions

We apply recently-established sufficient conditions for f.d.d convergence to the measure-valued diffusion, canonical historical Brownian motion, to prove convergence of historical processes of the voter model on the d-dimensional integer lattice, when $d > 2$. Our approach is inspired by critical phenomena, specifically in our use of perturbations and diagrammatic estimates for coalescing random walk, in conjunction with standard probabilistic tools (ie. the (strong) Markov property and coupling).

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Federico Camia
NYU Abu Dhabi, UAE

Conformal covariance of connection probabilities and fields in 2D critical percolation

Critical site percolation on the triangular lattice was one of the first models where the emergence of conformal invariance in the scaling limit was demonstrated rigorously, thanks to Smirnov's celebrated proof of Cardy's formula for the scaling limit of crossing probabilities (between boundary arcs of a bounded domain). Much progress followed swiftly, but the conformal covariance of connection probabilities (between points in the interior of a domain), expected by physicists since the 1980s and explicitly conjectured by Aizenman in the 1990s, remained open. In this talk, I will discuss a recent proof of this conjecture and related results. The proof relies on the conformal invariance of the percolation full scaling limit constructed by Newman and myself in the early 2000s.

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Lung-Chi Chen,
National Chengchi University, Taipei

A local limit theorem for the long-range self-avoiding walk

Self-avoiding walk (SAW) is a walk that each path does not visit the same point more than once. It is known that taking an average on a ball, the spread-out finite-range SAW on $\mathbb{Z}^d$ with $d>4$ satisfies a certain type of a local limit theorem [van der Hofstad and Slade (2002)]. In this talk, we consider the spread-out long-range SAW on $\mathbb{Z}^d$, whose each step distribution $D(x)$ decays as $|x|^{-d-\alpha}$ for some $\alpha>0$. It is known that for $\alpha\neq 2$, the critical two-point function $G_{p_c}(x)$ decays as $|x|^{\alpha\wedge 2-d}$ above the upper-critical dimension $2(\alpha\wedge 2)$ [Chen and Sakai (2015)], and for $\alpha = 2$, $G_{p_c} (x)$ decays as$ |x|^{2-d}/ \log |x|$ whenever $d \geq 4$ (including equality) [Chen and Sakai (2019)]. In this talk, I present that the n-step connectivity function of SAW $c_n(x)$ is approximated by the corresponding n-step random walk distribution $s_n(x)$ in the asymptotic sense for $\alpha\in(0,2)$ and dimensions $d>2\alpha$.

This is a joint work with Yoshinori Kamijima (National Center for Theoretical Sciences) and Yuki Chino (National Yang Ming Chiao Tung University).

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Roberto Fernandez,
Utrecht University, Netherlands, and NYU Shanghai, China

High-temperature cluster expansion for quantum spin lattice systems

We develop a novel cluster expansion for finite-spin lattice systems subject to multi-body quantum ---and, in particular, classical--- interactions. Our approach is based on the use of the M\"obius transform instead of the usual ``$\pm$ trick", introduced by Mayer. Our treatment is inspired by work by Park, but leads to explicit expansions in a β-dependent effective fugacity, instead of a Kirwood-Salzburg system of equations. The resulting series provide an explicit perturbative evaluation of free energy and correlation functions at small β. To determine its convergence region we adopt a relatively recent cluster summation scheme that replaces the traditional use of Kikwood-Salzburg-like integral equations by more precise sums in terms of particular tree-diagrams.

Jointly with Nguyen Tong Xuan (NYU Shanghai).

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Ryoki Fukushima,
Tsukuba University, Japan

Number of paths in oriented percolation as zero temperature limit of directed polymer

We consider the number of open paths in a supercritical oriented site percolation. In the finite volume, it is the zero temperature limit of the directed polymer in the Bernoulli environment. Unlike in the positive temperature regime, even the existence of the "free energy" (=growth rate) is a non-trivial problem due to the problem of extinction. It was proved in 2017 by Garet, Gouere and Marchand but fundamental properties of the growth rate, such the continuity with respect to the percolation parameter, have been unknown. We prove that the free energy of the directed polymer in the Bernoulli environment converges to the growth rate for the number of open paths in super-critical oriented percolation as the temperature tends to zero. Our proof is based on the rate of convergence results which hold uniformly in the temperature. We also prove that the convergence rate is locally uniform in the percolation parameter inside the supercritical phase, which implies that the growth rate depends continuously on the percolation parameter.

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Cristian Giardina,
Università degli Studi di Modena e Reggio Emilia, Italy

Integrable interacting particle systems: from micro to macro

We consider integrable boundary-driven interacting particle systems, i.e. the Markov processes associated with the open integrable XXX chain with non-compact spins. We show that it is possible to explicitly compute the non-equilibrium steady state as a mixture of product measures. We identify the law of the mixture in terms of the Dirichlet process.

Next, by using the explicit knowledge of the non-equilibrium steady state, we establish formulas predicted by the Macroscopic Fluctuation Theory for several quantities of interest: the pressure (by Varadhan's lemma), the density large deviation function (by contraction principle), the additivity principle (by using the Markov property of the mixing law). To our knowledge, these results constitute the first rigorous derivation of these macroscopic properties for models of energy transport with unbounded state space, starting from the microscopic structure of the non-equilibrium steady state.

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Markus Heydenreich,
Universität Augsburg, Germany

Percolation phase transition for the marked random connection model

We investigate a spatial random graph model whose vertices are given as a marked Poisson process on R^d. Edges are inserted between any pair of points independently with probability depending on the Euclidean distance of the two endpoints and their marks. Upon variation of the Poisson density, a percolation phase transition occurs under mild conditions: for low density there are finite connected components only, while for large density there is an infinite component almost surely.

Our interest is on the transition between the low- and high-density phase, where the system is critical. We establish that if dimension is high enough and the mark distribution satisfies certain conditions, then an infrared bound for the critical connection function is valid. As a consequence, we obtain that various critical exponents exist and take on their mean-field values.

The method to achieve this result is the lace expansion, which in the context of bond percolation has been pioneered by Takashi Hara together with Gordon Slade in a series of papers in the 1990s. For our new results, we give this classical work a new twist by applying it to Poisson processes and combining the analysis with spectral estimates in Hilbert spaces.

We finally present an asymptotic expansion of the critical density as a function of the dimension.

Based on joint work with Matthew Dickson.

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Fumio Hiroshima,
Kyushu University, Japan

Localization of a renormalized Hamiltonian in QFT by a path measure

This is a joint work with Oliver Matte.

We study the spectral analysis of a renormalized Hamiltonian in QFT by a path measure. The so-called Nelson Hamiltonian describes a linear interaction between a scalar quantum field and a non-relativistic quantum particle. Suppose that a single boson is massless. The ultraviolet cutoff is imposed on the Hamiltonian to define it as a self-adjoint operator. By a renormalization procedure one can remove the ultraviolet cutoff and define the renormalized Hamiltonian H. In this talk we show that

(1) H has a ground state and it is unique,

(2) localizations of the ground state are shown.

The key ingredient in the investigation is an application of a Gibbs measure.

The references are

(1) FH, Adv. in Math. 259 (2014), 784-840.

(2) FH and O.Matte, RMP. (2021) 84 pages, online.

(3) FH and O.Matte, Point-wise spatial decay of eigenvectors in the Nelson model, preprint in 2022.

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Mark Holmes,
The University of Melbourne, Australia

Lattice trees and friends in high dimensions

We'll discuss recent results concerning the limiting behaviour (in high dimensions) of critical lattice trees and other models conditioned to survive for a long time.

In particular we'll discuss the so-called historical processes (which are measure-valued processes that encode genealogy) in the context of critical lattice trees and the voter model.

This is based on joint work with Ed Perkins and others, and work of Tim Banova.

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Tom Hutchcroft,
Caltech, USA

The near-critical two-point function and the torus plateau in high-dimensional percolation

We consider percolation on Z^d at and near the critical point in high dimensions, where the two-point function has been proved to have power law decay at the critical point and exponential decay below the critical point. We obtain an upper bound on the two-point function that interpolates between these two regimes and which is essentially optimal. A similar result is obtained for the slightly subcritical one-arm probability. As an application, we use the near-critical decay of the two-point function to prove that throughout the critical window for percolation on a high-dimensional torus, the torus two-point function has a plateau: it decays at small distances with the same power as on Z^d but at larger distances is essentially constant and of order V^(2/3) where V is the volume of the torus. The proof uses both the classical methods of high-dimensional percolation and newer tools based on randomized algorithms and the OSSS inequality.

Joint work with Gordon Slade and Emmanuel Michta.

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Noe Kawamoto,
Hokkaido University, Japan

Rate of convergence of the critical point for the memory-τ self-avoiding walk in dimension $d>4$

We consider the spread-out models of the self-avoiding walk and its finite-memory version, called the memory-τ walk. For both models, each step is uniformly distributed over the d-dimensional box $\{x\in\Zd:0<\|x\|_{\infty} \le L\}$. The critical point p_c^{τ} for the memory-τ walk is increasing in τ and converges to the critical point for the self-avoiding walk as τ goes to ∞. The speaker proved that the rate of convergence of p_c^{τ} in terms of τ is order of $\tau^{-(d-2)/2}$. Moreover, the speaker identified the exact expression of the coefficient of the dominant term of it. This improves the previous results obtained by Madras and Slade [Birkh\"auser, The Self-Avoiding Walk, Lemma~6.8.6, 1993].

This talk is based on the speaker's own work (http://arxiv.org/abs/2306.13936).

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Takashi Kumagai,
Waseda University, Japan

Heat kernel fluctuations and quantitative homogenization for the one-dimensional Bouchaud trap model

The Bouchaud trap model (BTM) is a random walk in a random medium given by a landscape of traps which retain the walk for some amount of time. In this talk, we consider one-dimensional versions of the model, and discuss scaling limits, heat kernel fluctuations and quantitative homogenization. We study how these properties change when the parameter of the distributions of the traps change.

This is a joint work with S. Andres and D. Croydon.

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Yucheng Liu,
The University of British Columbia, Canada

Gaussian deconvolution and the lace expansion

We give conditions on a real-valued function F on $\mathbb{Z}^d$, for $d>2$, which ensure that the solution G to the convolution equation $(F*G)(x) = \delta_{0,x}$ has Gaussian decay $|x|^{-(d-2)}$ for large $|x|$. Precursors of our results were obtained by Hara in the 2000s, using intricate Fourier analysis. We give a new, very simple proof using H\"older's inequality and basic Fourier theory in L^p space. Our motivation comes from critical phenomena in equilibrium statistical mechanics, where the convolution equation is provided by the lace expansion and the deconvolution G is a critical two-point function. Our results significantly simplify existing proofs of critical $|x|^{-(d-2)}$ decay in high dimensions for self-avoiding walk, Ising and φ^4 models, percolation, and lattice trees and lattice animals.

This is based on a joint work with Gordon Slade (arXiv:2310.07635).

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Pierre Nolin,
City University of Hong Kong, China

Self-organized criticality and avalanches in 2D forest fires

Bernoulli percolation can be used to analyse planar forest fire (or epidemics) processes. In such processes, all vertices of a lattice are initially vacant, and then become occupied at rate 1. If an occupied vertex is hit by lightning, which occurs at a (typically very small) rate, all the vertices connected to it burn immediately, i.e. they become vacant.

We want to analyse the behaviour of such processes near and beyond criticality, that is, when large components of occupied sites appear. They display a form of self-organized criticality, where the phase transition of Bernoulli percolation plays an important role. In particular, a peculiar and striking phenomenon arises, that we call "near-critical avalanches".

This talk is based on joint works with Rob van den Berg (CWI and VU, Amsterdam) and with Wai-Kit Lam (National Taiwan University, Taipei).

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Hirofumi Osada,
Chubu University, Japan

Stochastic analysis for strongly correlated, infinite particle systems

A strongly correlated infinite particle system in Euclidean space is typically an infinite number of particles interacting through the Coulomb potential. Conventionally, a Coulomb point process in two-dimensional space is known as the Ginibre point process only when the inverse temperature β is 2. The Coulomb point process for each inverse temperature $\beta >0$ has been recently constructed. A more strongly correlated model is the set of the zero points of the planar Gaussian analytic function (GAF). These point processes have different geometric properties from Gibbs measure and Poisson point processes, reflecting their strong correlation. Namely, the Ginibre point process has the small variance property of the number of particles in disks, the dichotomy of the reduced Palm measures, and the number rigidity. The planar GAF has the mean and number rigidity.

In this talk, we will clarify how the rigidity reflects the global dynamical behavior of each tagged particle and overview a general theory of stochastic analysis of infinite particle systems. This theory is necessary for that purpose and can be applied to strongly correlated, infinite particle systems.

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Tomoyuki Shirai,
Kyushu University, Japan

Zeros of random power series with stationary Gaussian coefficients

It is known that the zeros of random power series with i.i.d. complex Gaussian coefficients form the determinantal point process associated with the Bergman kernel.

As a generalization of this model, we are concerned with zeros of random power series with coefficients being stationary, centered, complex Gaussian process. We discuss the expected number of zeros in a disk and compare it with the i.i.d. coefficients case. When the spectral density of the Gaussian process of coefficients is a trigonometric polynomial, we discuss the precise asymptotics of the expected number of zeros inside the disk of radius r centered at the origin as r tends to the radius of convergence.

Also, we discuss the relationships between the intensity and spectral density.

This is based on a joint work with Kohei Noda (Kyushu).

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Daisuke Shiraishi,
Kyoto University, Japan

Loop-erased random walk in three dimensions

Loop-erased random walk (LERW) is a random simple path obtained through erasing all loops chronologically from a simple random walk path. In this talk, I will discuss some progress toward the convergence of 3D LERW as a stochastic process along arbitrary lattice spacings. This is joint work with Xinyi Li (Peking University).

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Gordon Slade,
The University of British Columbia, Canada

Boundary conditions and universal finite-size scaling in high dimensions

Above the upper critical dimension, boundary conditions play a dramatic role in finite-size scaling for spin systems and related models, as has been widely discussed in the physics literature. We present recent work (joint with Emmanuel Michta and Jiwoon Park arxiv:2306.00896) which provides a thorough and precise account of the effect of free vs periodic boundary conditions on the universal finite-size scaling of the weakly coupled hierarchical $|\varphi|^4$ spin system in dimensions 4 and higher, and offers precise conjectures for other spin systems and self-avoiding walk in high dimensions.

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Xin Sun,
Peking University, China

Percolation Exponent, Conformal Radius for SLE, and Liouville Structure Constant

In recent years, a technique has been developed to compute the conformal radii of random domains defined by SLE curves, which is based on the coupling between SLE and Liouville quantum gravity (LQG). Compared to prior methods that compute SLE related quantities via its coupling with LQG, the crucial new input is the exact solvability of structure constants in Liouville conformal field theory. It appears that various percolation exponents can be expressed in terms of conformal radii that can be computed this way. This includes known exponents such as the one-arm and polychromatic two-arm exponents, as well as the backbone exponents, which is unknown previously. In this talk we will review this method using the derivation of the backbone exponent as an example, based on a joint work with Nolin, Qian, and Zhuang.

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Kenkichi Tsunoda,
Kyushu University, Japan

Incompressible limit for a weakly asymmetric simple exclusion process with collision

We consider in this talk the so-called incompressible limit for a weakly asymmetric simple exclusion process (WASEP) with collision. One of fundamental questions in mathematical physics is the derivation of the master equation of fluid such as the Burgers equation or the Navier-Stokes equation. This model has been introduced by Esposito, Marra and Yau (1996) and they have derived the Navier-Stokes equation as an incompressible limit in dimensions strictly larger than two. The derivation of these equations in low dimension is achieved only from a lattice gas which admits mesoscopically long jumps. In the previous work, Jara, Landim and Tsunoda (2021), we considered the WASEP without collision. We discuss in this talk the incompressible limit for the WASEP with collision. This talk is based on joint work with Patrick van Meurs (Kanazawa University) and Lu Xu (Gran Sasso Science Institute).

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Remco van der Hofstad,
Eindhoven University of Technology, Netherlands

x-space asymptotics in high-dimensional percolation, lattice trees and lattice animals

The x-space asymptotics of two-point functions gives a powerful control of the critical behavior of statistical mechanics systems. In this talk, I will review results on their asymptotics for high-dimensional percolation, lattice trees and lattice animals. The best methods are by Takashi Hara, who was the first to use the lace expansion for such problems, both for spread-out (with Slade and myself) as well as nearest-neighbour systems. For the latter, with Robert Fitzner, we improved the results to at least 11 dimensions for percolation, and 16 and 17 dimensions for lattice trees and animals, respectively. In that work, Takashi's methods, developed first with Gordon Slade for five-dimensional self-avoiding walks, played a central role.

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