Abstracts
Conference on Recent Developments in Algebraic Geometry,
Arithmetic and Dynamics
(21–25 August 2023)

## 1 Meng Chen

*Fudan University, China*
On projective varieties of general type with many global K-forms

Abstract

We prove the following results: (1) for any nonsingular projective 3-folds $X$ of general type with $\chi(\mathcal{O}_X) \neq 2, 3$, the canonical volume $\mathrm{Vol}(X)$ has the optimal lower bound $\frac{1}{420}$; (2) for nonsingular projective 3-folds (resp. 4-folds) $X$ of general type with $h^{2,0}(X) \geq 108 \cdot 42^3 + 4$ (resp. with sufficiently large $h^{2,0}(X)$), the 3-canonical map (resp. 5-canonical map) is stably birational; (3) for any nonsingular projective $n$-fold $X$ of general type with $q(X) > n \geq 4$, the canonical stability index $r_s(X)$ is upper bounded by the $(n-1)$-th canonical stability index $r_{n-1}$. This is a joint work with Zhi Jiang.

## 2 Jungkai Chen

*National Taiwan University, Taipei*
Toward classification of threefold divisorial contraction to curves

Abstract

In minimal model program, elementary birational maps consists of divisorial contractions, flips, and flops. In three dimensional minimal model program, it is expected that one can describe these elementary maps explicitly. The threefold divisorial contractions to points are classified mainly thanks to the works of Kawamata, Hayakawa and Kawakita. In this talk, we will present our approach toward the classification of threefold divisorial contraction to curves. This is a joint work in progress with Hsin-Ku Chen and Jheng-Jie Chen.

## 3 Ziyang Gao

*Leibniz Universität Hannover, Germany*
Degeneracy loci in families of abelian varieties and their applications

Abstract

Given an abelian scheme over char 0 and an irreducible subvariety X, one can define the t-th degeneracy locus of X for each integer t. This geometric concept of degeneracy loci has recently seen many applications in Diophantine Geometry, notably when t is 0 and 1, in the recent developments on the uniformity of the number of rational points on curves, on the solutions of the Uniform Mordell-Lang Conjecture and of the Relative Manin-Mumford Conjecture. In this talk, I will define the degeneracy loci in the universal abelian variety, and explain how they are used in the applications mentioned above.

## 4 Yoshinori Gongyo

*The University of Tokyo, Japan*
The Mukai type conjecture

Abstract

I introduce the total invariant for Fano manifolds and propose the new conjecture for the characterization of the product of projective space. And I will explain the relation with my joint work with J. Moraga for the generalized complexities for surfaces.

## 5 Fei Hu

*Nanjing University, China*
An upper bound for polynomial volume growth of automorphisms of zero entropy

Abstract

Let $X$ be a smooth complex projective variety of dimension $d$ and $f$ an automorphism of $X$. Suppose that the pullback $f^*|_{\mathsf{N}^1(X)_{\mathbf{R}}}$ of $f$ on the real Néron–Severi space $\mathsf{N}^1(X)_{\mathbf{R}}$ is unipotent and denote the index of the eigenvalue 1 by $k+1$. We prove an upper bound for the polynomial volume growth $\mathrm{plov}(f)$ of $f$ as follows:

$$\mathrm{plov}(f) \leq (k/2+1)d.$$

Combining with the inequality $k \leq 2(d-1)$ due to Dinh–Lin–Oguiso–Zhang, we obtain an optimal inequality that

$$\mathrm{plov}(f) \leq d^2,$$

which affirmatively answers questions of Cantat–Paris-Romaskevich and Lin–Oguiso–Zhang.
This is joint work with Chen Jiang.

## 6 Jun-Muk Hwang

*Institute for Basic Science, Korea*
Minimal rational curves whose VMRT at a geneal point is an adjoint variety

Abstract

VMRT is an invariant of the germ of a minimal rational curve on a uniruled projective manifold. When the VMRT at a general point is a homogenous projective variety Z of certain types, the germ of a minimal rational curve is uniquely determined by Z. This is the case when the homogenous variety Z is the VMRT of an irreducible Hermitian symmetric space (proved by Mok) or a homogeneous Legendrian submanifold (proved by Hwang-Li).

In a joint work with Qifeng Li, we show that when the VMRT at a general point is an adjoint variety of a simple Lie algebra of type different from $A_k$ (k different from 2) or $C_k$, germs of minimal rational curves have locally symmetric structures at general points. Nontrivial examples of such germs arise from the wonderful compactifications of simple algebraic groups, as discovered by Brion and Fu.

## 7 Chen Jiang

*Fudan University, China*
An effective upper bound for anti-canonical volumes of singular Fano 3-folds

Abstract

Fano varieties are natural geometric objects in algebraic geometry and differential geometry. Motivated by the classifications theory, we are interested in certain boundedness properties of Fano varieties. In this talk, I will explain recent progress on the study of anti-canonical volumes of Fano 3-folds. We show that for a Fano 3-fold with e-lc singularities, the anti-canonical volume is at most $3200/e^4$. This is based on a joint work with Yu Zou.

## 8 Shu Kawaguchi

*Doshisha University, Japan*
Eisenstein K3 surfaces and analytic torsion

Abstract

For Eisenstein K3 surfaces, we construct invariants using equivariant analytic torsions. The invariants give rise to an automorphic form on the coarse moduli spaces of Eisenstein K3 surfaces, which are complex ball quotients. As applications, we obtain reflective modular forms on some complex ball quotients, and we show that some of the moduli spaces of Eisenstein K3 surfaces are quasi-affine. This is joint work with Ken-Ichi Yoshikawa.

## 9 Yujiro Kawamata

*The University of Tokyo, Japan*
On deformations over non-commutative base

Abstract

It is natural to consider deformations of sheaves over non-commutative base, because they are controlled by non-commutative dg algebras. We will describe versal non-commutative deformations, and show some applications to birational geometry and derived categories.

## 10 Yujiro Kawamata

*The University of Tokyo, Japan*
On non-commutative deformations of complex manifolds

Abstract

We will describe infinitesimal deformations of complex manifolds to the direction of something having possibly non-commutative structure sheaves by using Hochschild cohomology. We will also describe global non-commutative deformations of some surfaces.

## 11 JongHae Keum

*Korea Institute for Advanced Study, Korea*
Mori dream surfaces

Abstract

The Cox ring of a variety is the total coordinate ring, i.e., the direct sum of all spaces of global sections of all divisors. When this ring is finitely generated, the variety is called Mori dream (MD). A necessary condition for being MD is the finite generatedness of Pic(X), i.e., the vanishing of the irregularity. Smooth rational surfaces with big anticanonical divisor are MD. Thus all del Pezzo surfaces of any degree are. A K3 surface or an Enriques surface is MD if its automorphism group is finite.

In this talk I will consider the case of surfaces of general type with $p_g$=0, and provide several examples that are MD. I will also provide non-minimal examples that are not MD. This is a joint work with Kyoung-Seog Lee.

## 12 Frank Kutzschebauch

*Universität Bern, Switzerland*
Factorization of holomorphic matrices

Abstract

Every complex symplectic matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factorized as a product of the following types of unipotent matrices (in interchanging order).

- (i): $\begin{pmatrix} I & B \\ 0 & I \end{pmatrix}$, upper triangular with symmetric $B = B^T$.
- (ii): $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$, lower triangular with symmetric $C = C^T$.

The optimal number $T(\mathbb{C})$ of such factors that any matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factored into a product of $T$ factors has recently been established to be 5 by Jin, P. Lin, Z. and Xiao, B.

If the matrices depend continuously or holomorphically on a parameter, equivalently their entries are continuous functions on a topological space or holomorphic functions on a Stein space $X$, it is by no means clear that such a factorization by continuous/holomorphic unipotent matrices exists. A necessary condition for the existence is the map $X \to \mathrm{Sp}_{2n}(\mathbb{C})$ to be null-homotopic. This problem of existence of a factorization is known as the symplectic Vaserstein problem or Gromov-Vaserstein problem. In this talk we report on the results of the speaker and his collaborators B. Ivarsson, E. Low and of his Ph.D. student J. Schott on the complete solution of this problem, establishing uniform bounds $T(d, n)$ for the number of factors depending on the dimension of the space $d$ and the size $n$ of the matrices. It seems difficult to establish the optimal bounds. However we obtain results for the numbers $T(1, n)$, $T(2, n)$ for all sizes of matrices in joint work with our Ph.D. students G. Huang and J. Schott. Finally we give an application to the problem of writing holomorphic symplectic matrices as product of exponentials.

## 13 Yongnam Lee

*Korea Advanced Institute of Science & Technology, Korea*
Positivity of the tangent bundle of smooth projective surfaces and Fano threefolds

Abstract

Recently, the positivity of the tangent bundle of del Pezzo surfaces and Fano threefold with Picard number 1 was studied by several authors. In this talk, I will present some works on the positivity of the tangent bundle of smooth projective surfaces and Fano threefolds. In the first part, I will explain the total dual VMRT and its application to the pseudo-effective cone of the tangent bundle of Fano threefolds. In the second part, the pseudo-effectiveness of the tangent bundle of fibered surfaces with non-negative Kodaira dimension will be discussed. The first part is based on joint work with Hosung Kim and Jeong-Seop Kim. The second part is based on joint work with Jia Jia and Guolei Zhong.

## 14 Hsueh-Yung Lin

*National Taiwan University, Taipei*
Dynamical filtrations, beyond zero entropy

Abstract

Dynamical filtrations are certain filtrations on the (1,1)-part of the second cohomology group $H^2(X)$ of a compact Kähler manifold X, originally discovered and studied by Dinh-Lin-Oguiso-Zhang in the context of zero entropy group actions. New upper bounds of various dynamical invariants were obtained along the way. In this talk, we extend the study of dynamical filtrations to arbitrary solvable group actions, without the zero entropy assumption.

## 15 Sheng Meng

*East China Normal University, China*
On projective threefolds with non-isomorphic surjective endomorphisms

Abstract

The equivariant minimal model program (EMMP) is known for polarized endomorphisms and non-isomorphic surjective endomorphisms of normal projective surfaces. However, one cannot run an EMMP for a smooth projective threefold X with a non-isomorphic surjective endomorphism. In this talk, I will introduce a newly developed technique of the f-Iitaka fibration. In this way, I will characterize X in terms of the first dynamical degree. Application on the Kawaguchi-Silverman conjecture will also be discussed which is a further approach after the work of Chen-Lin-Oguiso. This is a joint work with professor De-Qi Zhang.

## 16 Fabio Perroni

*University of Trieste, Italy*
On the motivic class of the moduli stack of admissible $G$-covers

Abstract

Given a finite group $G$, we study moduli spaces of $G$-covers $\pi\colon D \to C$ of smooth curves, and their compactifications given by admissible $G$-covers. In the case where $C$ is a pointed nodal curve of genus 0 we show that the class, in the Grothendieck group, of the stack of admissible $G$-covers is determined by the class of the open sub-stack corresponding to coverings of the projective line. This yields a formula that computes the Betti numbers of moduli spaces of admissible $G$-covers starting from the Betti numbers of the moduli spaces of $G$-covers of $\mathbb{P}^1$. This is a joint work with Massimo Bagnarol.

## 17 Ho Hai Phung

*Vietnam Academy of Science and Technology, Vietnam*
Finite torsors on projective schemes defined over a discrete valuation ring

Abstract

Given a Henselian and Japanese discrete valuation ring A and a flat and projective A-scheme X, we follow the approach of Biswas and dos Santos [J. Inst. Math. Jussieu 10 (2011), no. 2, 225–234] to introduce a full subcategory of coherent modules on X which is then shown to be Tannakian. We then prove that, under normality of the generic fibre, the associated affine and flat group is pro-finite in a strong sense (so that its ring of functions is a Mittag-Leffler A-module) and that it classifies finite torsors Q $\to$ X. This establishes an analogy to Nori's theory of the essentially finite fundamental group. In addition, we compare our theory with the ones recently developed by Mehta–Subramanian and Antei–Emsalem–Gasbarri. Using the comparison with the former, we show that any quasi-finite torsor Q $\to$ X has a reduction of the structure group to a finite one.

## 18 Xiaotao Sun

*Tianjin University, China*
Global F-regularity of moduli spaces of parabolic bundles

Abstract

Globally F-regular varieties have many nice properties, but it is not easy to prove global F-regularity of a variety in general. In this talk, I will show that the moduli space of rank $r$ semistable parabolic bundles on a generic curve is globally F-regular for generic choice of parabolic points when p>3r.

## 19 Behrouz Taji

*The University of New South Wales Sydney, Australia*
Higher dimensional Arakelov inequalities

Abstract

Given a smooth projective family of polarized varieties with a fixed numerical invariant (Hilbert polynomial) it is a natural to ask if there is a relation between the canonical divisor of the family (relative canonical class) and the canonical class of the base that only depends on the a-priori fixed numerical invariant. For families of high genus curves over one dimensional base schemes such numerical relations were first discovered by Arakelov. They turned out to be of fundamental importance in proving Shafarevich Finiteness Conjecture (and consequently Mordell's conjecture by Arakelov, Parshin and finally Faltings in full generality). Bedulev and Viehweg then extended Arakelov's inequalities to higher dimensional fibers (canonically polarized families of manifolds) over curves. Later on Viehweg asked if there are Arakelov-type inequalities over arbitrary base schemes. In a joint work with Kovács we give an affirmative answer to this question.

## 20 Sheng-Li Tan

*East China Normal University, China*
Foliated algebraic surfaces with low slope and Poincare problem

Abstract

For a holomorphic foliation on an algebraic surface, one can define its Chern numbers and slope. If the foliation is of general type, then these invariants are positive rational numbers. We will talk about the lower bounds of the volume and slope, and give a solution to Poincare problem about the algebraicity of foliations with low slope.

## 21 Junyi Xie

*Peking University, China*
Partial heights and the geometric Bombieri-Lang conjecture

Abstract

It is a joint work with Xinyi Yuan. We prove the geometric Bombieri-Lang conjecture for projective varieties which have finite morphisms to abelian varieties over function fields of characteristic 0. Our proof is complex analytic, which applies the classical Brody lemma to construct entire curves on complex varieties. Our key ingredients includes a new notion of partial height and its non-degeneracy in a suitable sense. The non-degeneracy is required in the application of the Brody lemma.

## 22 Xun Yu

*Tianjin University, China*
Automorphism groups of smooth hypersurfaces

Abstract

In this talk, I will discuss some recent results about classifying automorphism groups of smooth hypersurfaces in the projective space. This talk is based on joint works with Keiji Oguiso, Li Wei, Song Yang, and Zigang Zhu.