Speakers

Abstracts

Workshop on Set Theory

(3–7 Jul 2023)

1 David Asperó

University of East Anglia, UK
On the limits of properness

Abstract

The focus of the talk will be on limitations for obtaining high versions of classical forcing axioms.

Joint work with Tom Benhamou began with the observation that the Galvin property is equivalent to being not Tukey maximal; hence, Tukey types refine various Galvin properties. We initiate the development of the Tukey theory of ultrafilters on measurable cardinals, allowing the flow of results from the countable to the uncountable and vice versa. The situation for ultrafilters on measurable cardinals turns out to be quite different from that on ω, sometimes greatly simplifying the situation on ω and sometimes posing new obstacles. The structure of the Tukey classes also turns out to be sensitive to different large cardinal hypotheses. We will present results from our preprint arXiv:2304.07214 and ongoing work.

2 Gunther Fuchs

City University of New York, USA
On the strength of leap constellations

Abstract

The study of blurry ordinal definability is concerned with the hierarchy of the inner models of the form κ-HOD, for any cardinal κ. The idea is to replace the usual notion of ordinal definability of a set x, i.e., "x is the unique object with a property involving ordinal parameters," with κ-blurry ordinal definability, meaning "x is one of fewer than κ objects with a property involving ordinal parameters." In a model of ZFC, this hierarchy stretches from HOD to V as the union of all κ-HODs. When a set appears in κ-HOD that was not present in the earlier stages of the hierarchy, κ is called a leap. The leaps form a closed class of cardinals, and a limit cardinal is a leap iff it is a limit of leaps. I will talk about some constellations of leaps in terms of their large cardinal strength. It turns out that situations where the successor of a singular cardinal is a leap are of particular interest. While "the least leap is the successor of a singular cardinal" by itself has no consistency strength beyond ZFC, no matter the desired cofinality of the singular limit cardinal, adding the requirement that it be a strong limit boosts the large cardinal strength. The main results I want to discuss are that "there is a singular strong limit cardinal κ below which the leaps are bounded and κ^+ is a leap", "the least leap is the successor of a singular strong limit cardinal," and "$\aleph_\omega$ is a strong limit cardinal and $\aleph_{\omega+1}$ is the least leap" are all equiconsistent with the existence of a measurable cardinal. There are many open questions about the consistency strengths of other leap constellations.

3 Moti Gitik

Tel Aviv University, Israel

On negation of the Singular Cardinals Hypothesis with GCH below

Abstract

The purpose of this talk is to provide an attempt to understand the difficulty of getting a model where GCH breaks first time at a singular κ and there is an inner model in which κ is a regular cardinal but still with 2^κ big.

There is a tension between the negation of the Singular Cardinals Hypothesis the power function below it. A celebrated result of J. Silver [5] states that a singular cardinal of uncountable cofinality cannot be the first that violates GCH.

M. Magidor [3], using extremely sophisticated arguments, showed that this need not be the case with a singular of cofinality ω. Namely, starting

with a supercompact cardinal with a huge above, he constructed a model in which $2^{\aleph_\omega} = \aleph_{\omega+2}$ and $2^{\aleph_n} = \aleph_{n+1}$, for every $n < \omega$.

In early 80-th, Hugh Woodin came up with a beautiful construction of a model of $2^{\aleph_\omega} = \aleph_{\omega+2}$ and $2^{\aleph_n} = \aleph_{n+1}$, for every $n < \omega$. The initial assumptions of his construction were optimal.

He asked the following natural question:

Assuming that there is no inner model with a strong cardinal, is it possible to have a model M in which $2^{\aleph_\omega} > \aleph_{\omega+2}$ and $2^{\aleph_n} = \aleph_{n+1}$, for every $n < \omega$, and there is an inner model N such that $\kappa = \aleph_\omega$ is a measurable and $2^\kappa \geq (\aleph_{\omega+3})^M$?

A reasonable approach to this question was to use Extender based forcing over κ together with a suitable preparation which say adds many Cohen subsets to ν's below κ, and then, passing into a submodel in which κ is still regular, we combine this Cohen's from the preparation together, using Prikry sequences, in order to obtain $2^\kappa-$Cohens over the submodel.

It turned out to be realizable to some degree. Namely, as it was shown in [1], even the Prikry forcing (with carefully picked $\kappa-$complete ultrafilter) can add κ^+-many mutually generic Cohen subsets to κ over a submodel. However, by [1], neither the original ([2]) nor C. Merimovich ([4]) versions of Extender based Prikry forcings cannot produce the above type of inner models. Namely, if $_E$ denotes the Extender based forcing of [2] and $G \subseteq_E$ is generic, then:

For every $A \in V[G] \setminus V, A \subseteq \kappa$, κ changes its cofinality to ω in $V[A]$.

If $\mathbb{P}_E$ denotes the Extender based forcing of [4] and $G \subseteq \mathbb{P}_E$ is generic, then:

For every $lA_\alpha \mid \alpha < \kappa^{++} listofdifferentsubsetsof\kappa$ in $V[G]$, there is $I \subseteq \kappa^{++}, I \in V, |I| = \kappa$ such that κ changes its cofinality to ω in $V[lA_\alpha \mid \alpha \in \mathring{I}]$.

We would like to use The Mitchell Covering Lemma with Pcf-arguments in order through some more light on the reasons of the difficulty to have an inner model in which κ is regular, but still 2^κ is big. In particular, this will provide some progress on the question of Woodin.

References

[1] T. Benhamou, M. Gitik, On Cohen and Prikry forcing notions, preprint.

[2] M. Gitik and M. Magidor, The Singular Cardinal Hypothesis Revisited.

In: Judah, H., Just, W., Woodin, H. (eds) Set Theory of the Continuum. Mathematical Sciences Research Institute Publications, vol 26. Springer, 1992.

[3] M. Magidor, On the singular cardinal problem, II, Ann. of Math. 106 (1977), 517–647.

[4] C. Merimovich, Extender based Radin forcing, Transactions of the American Mathematical Society, volume 355, pages 1729–1772, January 2003.

[5] J. Silver, On the singular cardinals problem, Proc. of the International Congress of Math.; Vancouver, vol. 1, 1974, pp. 265–268.

4 Daisuke Ikegami

Shibaura Institute of Technology, Japan
Preserving AD via forcings

Abstract

The research in this talk was motivated by the following question:

Could there be an elementary embedding $j\colon V \to V[G]$ such that G is set-generic over V, $(V[G], \in, j)$ is a model of ZF, V is a model of AD, and the critical point of j is ω_1^V?

The positive answer to the above question would give us a poset which preserves AD while adding a new real. However, we still do not know if there is such a poset. To see whether there could be such a poset, we have been working on the question what kind of posets preserve AD.

In this talk, we present several results on posets preserving AD. Among them are the following:

1. Assume $\mathsf{ZF} + \mathsf{AD}^+ +$ "$V = \mathrm{L}\big(\wp(\mathbb{R})\big)$". Suppose that a poset $\mathbb{P}$ increases Θ, i.e., $\Theta^V < \Theta^{V[G]}$ for any $\mathbb{P}$-generic filter G over V. Then the poset $\mathbb{P}$ does not preserve AD.

2. Assume $\mathsf{ZF} + \mathsf{AD}$. Then any non-trivial poset which is a surjective image of $\mathbb{R}$ does not preserve AD.

3. Assume $\mathsf{ZF} + \mathsf{AD}^+ +$ "$V = \mathrm{L}\big(\wp(\mathbb{R})\big)$". Suppose that Θ is regular. Then there is a poset $\mathbb{P}$ on Θ which preserves AD and adds a new subset of Θ.

The item 2 above answers the question of Chan and Jackson. The item 3 in case of $V = \mathrm{L}(\mathbb{R})$ answers the question of Cunningham.

This is joint work with Nam Trang.

A preprint on this work is available: https://arxiv.org/abs/2304.00449.

5 Andreas Lietz

University of Münster, Germany
$\mathbb{Q}_{\max} - (*)$.is a forcing axiom

Abstract

The axiom $\mathbb{Q}_{max} - (*)$ states that-AD holds in $L(\mathbb{R})$ and there is a filter $g \subseteq \mathbb{Q}_{\max}$ generic over $L(\mathbb{R})$ so that $\mathcal{P}(\omega_1) \subseteq L(\mathbb{R})[g]$. Woodin has shown that this implies that NS_{ω_1} has the strongest possible saturation property, namely it is ω_1-dense. We introduce a forcing axiom and, building on work of Asperó-Schindler, show that it has $\mathbb{Q}_{max} - (*)$ as a consequence. Finally we force this axiom from a supercompact limit of supercompact cardinals. Ultimately this shows that assuming large cardinals, there is a stationary set preserving forcing which forces that NS_{ω_1} is ω_1-dense. This answers a question of Woodin positively.

6 Tadatoshi Miyamoto

Nanzan University, Japan
A simplified morass by finite mixtures of two types

Abstract

We present a poset that forces a simplified $(\omega_2, 1)$-morass of Velleman [3] assuming appropriate amount of the Generalized Continuum Hypothesis (GCH). It is a combination of ideas by Aspero-Mota [1] and Neeman [2]. The conditions are finite mixtures of countable and uncountable non-transitive

elementary substructures of a relevant relational structure. They are demanded to satisfy a list of internal structural properties and partially ordered s.t. no explicit use of fast functions are involved. The poset is proper, kind of a step higher proper, and has the right chain condition. In particular, it preserves the cofinalities and so the cardinalities. However, the status of the Continuum Hypothesis (CH) is not clear, though it would be violated.

References

[1] D. Aspero, M. Mota, *Forcing consequences of PFA together with the continuum large*, Trans. Amer. Math. Soc. 367 (2015), no. 9, 6103-6129.

[2] I. Neeman, *Forcing with Sequences of Models of Two Types*, Notre Dame J. Formal Logic, 55(2), 265-298, 2014.

[3] D. Velleman, *Simplified morasses*, J. Symbolic Logic 49 (1984), no. 1, 257-271.

7 Yinhe Peng

Chinese Academy of Sciences, China

A partition ordinal definable from a surjection

Abstract

This talk concerns ZF, without AC. There are interesting consequences if there is a surjection from AB to P(A)P(B). For example, we prove that there is a partition of A into ω pieces that is ordinal definable from the surjection. This sets some limitation of the existance of such surjection. Some other consequences and limitations will also be introduced. The consistency of such surjection will appear in another joint paper with Shen and Wu.

8 Hiroshi Sakai

Kobe University, Japan

Higher reflection principles and cardinal arithmetic

Abstract

So far, set theorists have extensively studied various reflection principles at "ω_2-level", such as Weak Reflection Principle (WRP), Rado Conjecture (RC), Game Reflection Principle (GRP), Fodor-type Reflection Principle (FRP) etc. These reflection principles hold if a supercompact cardinal is collapsed to ω_2 in a nice way. They are known to have many interesting consequences, especially on cardinal arithmetic. For example, WRP and RC implies the Singular Cardinal Hypothesis and that the continuum is less than or equal to ω_2. In this talk, we discuss higher analogs of these reflection principles. Among other things, we discuss their consequences on cardinal arithmetic.

9 Farmer Schlutzenberg

University of Münster, Germany

Full normalization and the initial segment condition for mice with long extenders

Abstract

Mice M containing ordinals κ such that $M \models$"κ is κ^+-supercompact" were introduced and analysed by Woodin in the early 2010s. Neeman and Steel also introduced an alternate but equivalent hierarchy. While the theory is largely parallel to that for short extender mice, there are some notable differences. For example, the initial segment condition for extenders in the extender sequence fails. We will describe an alternate hierarchy of mice at this level for which the initial segment condition holds in a manner very analogous to that in the short extender realm. Full normalization of stacks of normal trees holds (under natural strategy condensation hypotheses), and we will describe the key new feature that distinguishes full normalization at this level from that for short extender mice.

10 Xianghui Shi

Beijing Normal University, China

The structure of generalized degrees in $L[\mathcal{E}]$

Abstract

In this talk, we report some recent progress in the development of higher degree theory. The study of generalized degree structures at uncountable

cardinals can be traced back to Sy Friedman's 1978 paper on $\aleph_{\omega_1}$-degrees, and it was revisited by Shi and Yang about 10 years ago. Their works show that in L and $L[U]$-type inner models, where U is a certain sequence or matrix of measures, there is a deep connection between the complexity of the structure of Zermelo degrees at singular cardinals of countable cofinality and the strength of relevant large cardinals where the degree structures reside. But the analysis relies heavily on the classical covering lemmas. With Mitchell-Schimmerling's recent work on the covering at limit cardinals in Mitchell-Steel's K, we showed that similar phenomena occur in K as well, assuming there is no transitive inner model of ZFC for one Woodin cardinal. This is a joint work with Ralf Schindler.

11 Spencer Unger

University of Toronto, Canada
The tree property

Abstract

Combining elements from a long line of research on the tree property, we prove that it is consistent that every regular cardinal between $\aleph_2$ and $\aleph_{\omega^2+3}$ has the tree property while $\aleph_{\omega^2}$ is strong limit. In this talk, I'll give some background and give a sampling of some of the many ideas that go into the proof along with their connections to other research. This is joint work with James Cummings, Yair Hayut, Menachem Magidor, Itay Neeman and Dima Sinapova.

12 Liuzhen Wu

University of Chinese Academy of Sciences, China
A surjection from square onto power

Abstract

Cantor proves that for any set A, there is no surjection from A onto its power set P(A). In this talk, we describe a construction of a ZF model. In this model, there is a set A and a surjection from its square set A^2 onto its power set P(A). This answer a question posed by Truss. This is joint work with Guozhen Shen and Yinhe Peng.

13 Jindrich Zapletal

University of Florida, USA
Algebra and Axiom of Choice

Abstract

In a recent book “Geometric Set Theory” with Paul Larson, we developed a new machinery for proving consistency results in the choiceless set theory ZF+DC. The machinery is especially suitable for consistency results connected with analysis or algebra. I will explain the main ideas behind this new method and present several consistency results. For example, it is consistent with ZF+DC to have a transcendence basis for R over Q and to have no nonprincipal ultrafilters on natural numbers at the same time.