Abstracts
IMS Graduate Summer School in Logic
(26 June–14 July 2023)

1 W Hugh Woodin

Harvard University, USA
Exotic models, Ultimate L, and the Ultrapower Axiom

Abstract

The LSA models of AD+ are an important exotic class of models. A recent and surprising theorem Goldberg's Ultrapower Axiom holds in the HOD of an LSA model.

We will develop the necessary background material from Goldberg's analysis of the Ultrapower Axiom and from the theory of LSA models, necessary to sketch the proof. The basic method also proves the HOD Ultrafilter Conjecture holds in all LSA models, which is a basic conjecture from AD+ theory.

Also recently discovered is a class of exotic models of ZFC which are obtained by forcing over ZF models, and which exhibit features arguably not obtainable by forcing over ZFC models. This involves the approximation and cover properties of Hamkins, and the forcing axiom Martin's Maximum.

All of this is part of the emerging picture on Ultimate L.

2 Theodore Slaman

University of California at Berkeley, USA
Recursion Theoretic Degree Structures and Borel Equivalence Relations

Abstract

We will explore the connections between recursion-theoretic degree structures and countable Borel equivalence relations. We will discuss Martin's Conjecture, which asserts that all Turing-degree invariant functions are iterations of the Turing jump, and Kechris's Question, which suggests that there is an abundance of other functions. As time allows, we will explore how equivalence relations based on equidefinability fit within the general picture of countable Borel equivalence relations. We will emphasize (1) the recent progress by Lutz and Siskin, which confirms a part of Martin's Conjecture, and (2) open questions, both old and new.

3 Joris van der Hoeven

Centre National de la Recherche Scientifique (CNRS), France
Model theory of asymptotic differential algebra

Abstract

Consider an algebraic differential equation with real coefficients. A solution at infinity is said to be tame, if it does not involve any oscillation, either directly (like sin x) or indirectly (like x^2 + sin x).

The subject of asymptotic differential algebra studies such solutions from a formal standpoint. This

can be done in the language of differential algebra enriched with an ordering or a valuation. Imposing a few natural compatibility axioms between the derivation and the ordering leads to the theory of H-fields.

Three important examples of H-fields are Hardy fields, transseries, and the surreal numbers. Hardy fields are differential fields of germs of real functions at infinity. Transseries are formal expressions that can be obtained from real numbers and an infinitely large variable x, using exponentiation, logarithms, and infinite summation. Surreal numbers were introduced by Conway and Berarducci and Mantova recently constructed a derivation on the field of surreal numbers.

Transseries satisfy several strong closure properties: they are closed under solving linear first order equations and they satisfy a differential intermediate value property. These properties are captured through the notion of H-closed H-fields. The theory of H-closed H-fields is model complete and is the model companion of the theory of H-fields.

The surreal numbers with Berarducci-Mantova's derivation is another examples of an H-closed H-field. Recently, we showed that all so-called maximal Hardy fields are also H-closed.

In our course, we will give an introduction to transseries, Hardy fields, the theory of asymptotic differential algebra, and the model theory of H-fields. The aim is to provide a good entry point for the more voluminous books that Matthias Aschenbrenner, Lou van den Dries, and I wrote on this topic.

4 Nathaniel Bannister

Carnegie Mellon University, USA

Coherence, triviality, and derived limits

Abstract

Derived limits are designed to measure and correct the failures of exactness of taking inverse limits by turning short exact sequences into long exact sequences. Computing derived limits can turn out to have significant set theoretic content, particularly for systems indexed by real numbers or ordinals. We will outline some recent results in this area with a focus on the system A and note interesting applications to the theory of strong homology.

5 Oriola Gjetaj

Ghent University, Belgium

A Goodstein principle- independence result for ID_2

Abstract

The Goodstein principle is a natural number-theoretic theorem which is unprovable in Peano arithmetic. Since the original process definition there have been different canonical representation using Ackermann function or the Grzegorczyk hierarchy. These representations give a natural Goodstein process independent from different theories of reverse mathematics. In this talk we consider a normal form for which we get an independent Goodstein theorem from the ordinal of ID_2. This is a joint ongoing work with A. Weiermann on exploring normal form notations for the Goodstein principle.

6 Josiah Jacobsen-Grocott

University of Wisconsin-Madison, USA
The failure of Selman's Theorem for hyperenumeration reducibility

Abstract

Hyperenumeration reducibility was first introduced by Sanchis. A set A is hyperenumeration reducible to B if there is a c.e. set W such that $x \in A \iff \forall f \in \omega^\omega \exists v, n \in \omega[\langle f\restriction n, x, v\rangle \in W \wedge D_v \subseteq B]$. The relationship between hyperenumeration and hyperarithmetic reducibility shares many parallels with the relationship between enumeration and Turning reducibility. We ask if this relationship can be pushed to prove and analog of Selman's Theorem for hyperenumeration reducibility. By studying e-pointed trees in Baire space we are able to get a counter example. An e-pointed tree T is a tree with no dead ends and the property that every path in T enumerates T. If T is an e-pointed tree, then for all X if T is Π^1_1 in X then $\overline{T}$ is Π^1_1 in X. We build an e-pointed tree T such that $\overline{T}$ is not hyperenumeration reducible to T.

7 Katarzyna W. Kowalik

University of Warsaw, Poland
Reverse mathematics of Ramsey-theoretic statements over a weaker base theory

Abstract

Reverse Mathematics of Ramsey-theoretic statements over a weaker base theory Reverse Mathematics is a program in mathematical logic that aims to classify mathematical theorems according to their logical strength. This is done by translating a given mathematical theorem into the language of second-order arithmetic, and then comparing it with some well-understood arithmetical axioms over a weak base theory. The usual choice for the base theory has been RCA_0, but we chose to work with its weakening, RCA_0^*, which makes it possible to track nontrivial uses of $Sigma^0_1$ induction in mathematical proofs. I will present some results on the logical strength of Ramsey's theorem and some of its combinatorial consequences over RCA_0^*. I will also explain some technical and conceptual challenges that appear when one works with a weaker base theory.
This is joint work with Marta Fiori Carones, Leszek Kołodziejczyk and Keita Yokoyama.

8 Clark Lyons

UCLA, USA
Descriptive combinatorics and distributed computing

Abstract

Given a graph labeling problem, we can measure the complexity of solving it in many ways. In the LOCAL model of distributed computing we can measure this complexity by asking how many rounds of communication are required asymptotically to solve the problem if the vertices all communicate with their neighbors and run the same local algorithm. And we can also ask whether there always exist measurable labelings that solve the problem on graphs with a Borel structure. It turns out that these two measures of complexity have a deep relationship, whereby upper and lower bound results for the complexity of a problem can be translated from one side to the other. In this talk we will see how this connection motivates some new applications of determinacy to impossibility results for labeling problems, including the 1-star and 3-star cover problem and the hairy paths cover problem.

9 Antonio Nakid Cordero

University of Wisconsin-Madison, USA
Enumeration degrees and effective topology

Abstract

In this talk, we will discuss the relation between subclasses of the enumeration degrees and effective topological spaces. This gives a powerful bridge to solve computability-theoretic problems using topological tools and topological problems using computability-theoretic tools.

10 Xuanzhi Ren

Nankai University, China
The amalgamation property and Urysohn structures in continuous logic

Abstract

In this talk we briefly present two proofs of the existence of Urysohn's universal space, one of which is Katětov's construction and the other relies on Fraïsse theory. We also introduce the notion of Urysohn's continuous structure and give a characterization of when such a structure exists for a continuous signature L. The characterization is related to an analytical property of the continuity modulus of L, which we call properness. Finally, we introduce Hall's group and present a result which states for proper continuous signature L, the automorphism group of Urysohn's continuous structure contains Hall's group as a dense subgroup.

11 Linus Richter

Victoria University of Wellington, New Zealand
The Borel-Definable Group Cohomology of $\mathbb{R}^n$ Is (Hopefully) Simple

Abstract

I will motivate group extensions, which allow us to decompose any finite group into a set of finite simple groups, its building blocks. Using descriptive set theory, we can study the special class of *definable* group extensions: those induced by Borel functions. Kanovei and Reeken have made inroads into classifying these Borel extensions—I will outline their result for abelian Borel extensions of $(\mathbb{R}, +)$ by any countable abelian group, and present work in progress on the case $\mathbb{R}^n$. This is joint work with Dan Turetsky.

12 Jaruwat Rodbanjong

Chulalongkorn University, Thailand
Definable Modules In O-Minimal Structures

Abstract

Let $\mathfrak{M}$ be an o-minimal expansion of a densely linearly ordered set. It is well-known that a definable group (ring) always admits a unique definable group (ring) manifold topology. Moreover, we have characterizations of definable groups (rings) in some special cases. In this talk, we discuss about the study of characterizations of definable modules in $\mathfrak{M}$; for examples, we can show that (1) every module definable in $\mathfrak{M}$ admits a unique definable module manifold topology, and (2) every definable module in $\mathfrak{M}$ is finitely generated.

13 Mengzhou Sun

National University of Singapore, Singapore
Non-elementary cofinal extensions and correspondence between first-order and second-order arithmetic

Abstract

Compared with end extensions, much little is known about cofinal extensions for models of fragments of PA. It is not even known whether the elementarity of cofinal extensions can fail at some specific level. In this talk, I will present a partial answer for this question for a large class of models. Our proof is based on a correspondence theorem between the first-order theory of M and the second-order theory of its definable initial segment, which is also interesting in its own right.

14 Sebastiano Thei

Università degli Studi di Udine, Italy
Cardinal preserving embeddings and a cardinal correct inner model

Abstract

An elementary embedding between two inner models M and N is cardinal preserving if M and N correctly compute the class of cardinals. In light of Kunen inconsistency, Caicedo asked whether there is such a cardinal preserving embedding. Taking the first step towards this question, we'll consider the case where either M or N is V. For instance, assuming the existence of a cardinal preserving embedding from V into N, one can obtain an inner model of DC which is both cofinal and cardinal correct. This is joint work with Gabriel Goldberg.

15 Wojciech Woloszyn

University of Oxford, UK
Resurrection principles

Abstract

The resurrection principle (RP) asserts that, in each forcing extension, all true assertions with real parameters retain their forceability throughout subsequent forcing extensions. This property exhibits deep connections with set-theoretic geology, the modal logic of forcing, and inner model theory. I will discuss classical and more recent results on RP, as well as its natural refinements.