Speakers

1 **Damir Dzhafarov**

2 **Ziyuan Gao**

3 **Bakh Khoussainov**

4 **Takayuki Kihara**

5 **Patrick Lutz**

6 **Alexander Melnikov**

7 **Joseph S. Miller**

8 **Andre Nies**

9 **Keita Yokoyama**

Abstracts

Workshop on Computability Theory, Set Theory and their interactions

(19–30 Jun 2023)

1 Damir Dzhafarov

University of Connecticut, USA

The tree theorem for singletons in the Weihrauch degrees

Abstract

The tree theorem (TT), introduced by Chubb, Hirst, and McNicholl, has been studied extensively in reverse mathematics. Much of the interest lies in its relationship with Ramsey's theorem (RT), with which it shares many features, but from which it also differs in a number of ways. This talk will focus on the tree theorem for singletons, TT^1. This version asserts that for every finite coloring of the full binary tree, there is an infinite monochromatic set of strings isomorphic to the full binary tree as a partial order under the prefix relation. By a famous result of Hirst, Ramsey's theorem for singletons, RT^1, is equivalent over the theory RCA_0 to a first-order principle, bounding for Σ^0_2 formulas. On the other hand, Corduan, Groszek, and Mileti showed that TT^1 is strictly stronger than Σ^0_2 bounding, and hence than RT^1. We investigate TT^1 as a formal problem using Weihrauch reducibility. In this framework, we show that even TT^1 for 2-colorings is not equivalent to any first-order problem. We also show that first-order part of TT^1 is precisely RT^1. The results are joint work with Solomon and Valenti.

2 Ziyuan Gao

National University of Singapore, Singapore

Quasi-Isometric Reductions Between Infinite Strings

Abstract

This talk is on the recursion-theoretic aspects of large-scale geometriesof infinite strings, a subject initiated by Khoussainov and Takisaka (2017).We investigate several notions of quasi-isometric reductions between recursive infinite strings and prove various results on the equivalence classes ofsuch reductions. The main contribution is the construction of two infinite recursive strings such that one is quasi-isometrically reducible to the other butneither string is recursively quasiisometrically reducible to the other. The talk is based on a joint paper with Karen Frilya Celine, Sanjay Jain, Lou Chun Kit Ryan and Frank Stephan.

3 Bakh Khoussainov

University of Electronic Science and Technology of China, China
Quasi-axiomatizability of algorithmically presented structures

Abstract

We aim to describe the isomorphism types of algebraic structures in the language of first order logic. We define the notion of quasi-axiomatizablity that describes, in a precise sense, the isomorphism types of structures in first order logic. We focus on two classes of structures. The first is the class of structures for which positive atomic diagrams are computably enumerable. These structures are called positive structures. The second is the class of structures for which negative atomic diagrams are computably enumerable. These structures are called negative structures. Using expansions of languages, we investigate quasi-axiomatability of positive and negative structures by sets of $\exists$, $\forall$, $\exists\forall$, and $\forall\exists$-sentences in expansions of languages.

4 Takayuki Kihara

Nagoya University, Japan
Topos-theoretic aspect of the degrees of unsolvability

Abstract

In this talk, we examine the topos-theoretic aspect of the degrees of (computable) unsolvability. One of the main interpretations of constructive mathematics is Kleene's realizability interpretation, which, as is well known, can

be relativized by an oracle. In this sense, an oracle can be a factor that causes a change in a model of constructive mathematics. Let us review this observation from another point of view: there is a topos, called the effective topos, based on Kleene's realizability interpretation. And relativizing the realizability interpretation to an oracle yields a subtopos of the effective topos. Thus, the structure of oracles, i.e., the structure of the degrees of unsolvability, is expected to be closely related to the structure of the subtoposes of the effective topos. Indeed, we give a complete correspondence, in a strict sense, between the structure of the degrees of unsolvability and the structure of subtoposes of the effective topos (or its relatives). Through this bridge, we apply the theory of degrees of unsolvability to a detailed analysis of the structure of the realizability subtoposes of the effective topos. For instance, we show the existence of a "quasi-maximal realizability subtopos" (an analogue of a quasi-minimal degree), which is used to give a model of constructive mathematics separating "the formal Church thesis" and "the extended formal Church thesis". Degree-theoretic techniques such as a priority argument are used to construct some of subtoposes for separating the strength of some constructive principles in constructive mathematics.

5 Patrick Lutz

UCLA, USA

Martin's conjecture for order preserving functions above the hyperjump

Abstract

Slaman and Steel proved that Martin's conjecture—a proposed classification of definable functions on the Turing degrees—holds for all order preserving functions between the identity and the hyperjump. We will discuss recent efforts to extend this result to functions above the hyperjump. A key role is played by functions from the Turing degrees to the ordinals and we will mention some open questions about such functions. This is joint work with Benjamin Siskind.

6 Alexander Melnikov

Victoria University of Wellington, New Zealand

A recursion-theoretic approach to classification in topology

Abstract

A classical result of Markov states that, given two finite simplicial complexes, it is undecidable whether they represent homeomorphic compact spaces. In contrast, it is well-known that it is decidable whether two 2-dimensional compact surfaces, represented as simplicial complexes, are homeomorphic. There are many theorems of this sort in the literature. Nonetheless, results of this kind clearly make sense only for spaces that can be triangulated. Can we extend this approach beyond simplicial complexes?

I will discuss one possible answer to this question that uses index sets and tools of computable analysis. In the relatively tame case where all spaces are compact Polish, a wide array of tools is already required. However, set theory and higher recursion theory are not among these tools. The non-compact case is essentially a completely open topic; it contains many (seemingly challenging) questions. This is where **higher recursion theory** and **set theory** may (should!) become extremely helpful.

(Based on several papers jointly written with: Ng, Harrison-Trainor, Lupini, Downey, Koh, and Nies.)

7 Joseph S. Miller

University of Wisconsin–Madison, USA

The Hausdorff dimension of continuous images

Abstract

What effect do continuous functions have on Hausdorff dimension? An uncountable analytic set $E \subseteq \mathbb{R}^2$ must have a perfect subset, so it can be mapped continuously onto $[0,1]^2$—a set of dimension 2—regardless of the Hausdorff dimension of E. On the other hand, assuming CH, Patrick Lutz and I constructed a set $E \subseteq \mathbb{R}^2$ of Hausdorff dimension 1 such that any continuous image of E in $\mathbb{R}^2$ has dimension at most 1. Moreover, we can ensure that if $f\colon \mathbb{R}^2 \to \mathbb{R}$ is continuous, then $f[E]$ has Hausdorff dimension 0. The first fact generalizes to any Hausdorff dimension $s \in [0,2]$, but the second does not. Don Stull and I constructed a continuous function $j\colon \mathbb{R}^2 \to \mathbb{R}$ such that if $E \subseteq \mathbb{R}^2$ has Hausdorff dimension $s > 1$, then $j[E]$ has dimension at least $s/2$. In other words, j (at worst) preserves the relative dimension of sets of Hausdorff dimension greater that 1. The work with Partick Lutz implies that, in general, this is best possible.

These results are proved in the effective setting and then "classicalized" using the point-to-set principle of Jack Lutz and Niel Lutz.

8 Andre Nies

University of Auckland, New Zealand
Martin-Loef randomness, 2-randomness, and reverse mathematics

Abstract

ML-randomness of reals can be characterized by differentiability of computable functions of bounded variation. 2-randomness of infinite bit sequences can be characterized by the condition that the plain Kolmogorov complexity of initial segments is maximal infinitely often. We discuss how these results can be formulated and proved over RCA_0.

The talk is based on two JSL papers:

1. The reverse mathematics of theorems of Jordan and Lebesgue, 2021, with Triplett and Yokoyama.
2. Randomness notions and reverse mathematics, 2020, with Shafer.

9 Keita Yokoyama

Tohoku University, Japan
Determinacy and reflection principles in second-order arithmetic

Abstract

It is known that several variations of the axiom of determinacy play important roles in the study of reverse mathematics, and the relation between the hierarchy of determinacy and comprehension are revealed by Tanaka, Nemoto, Montalbán, Shore, and others. In [1], Kołodziejczyk and Michalewski relates the determinacy of arbitrary boolean combinations of Σ^0_2 sets and syntactic reflection in second-order arithmetic. Here, we sharpen their result and see the general relations between the determinacy and the reflection principle. This is joint work with Leonardo Pacheco.

[1] Leszek Aleksander Kołodziejczyk and Henryk Michalewski, How unprovable is Rabin's decidability theorem?, 2016 31st Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), 2016, pp. 1–10.