# Speakers

# Abstracts

Workshop on Computability Theory

(12–16 Jun 2023)

## 1 George Barmpalias

*Chinese Academy of Sciences, China*
Compression of enumerations and gain

Abstract

We study the compressibility of enumerations, and its role in the relative Kolmogorov complexity of computably enumerable sets, with respect to density. With respect to a strong and a weak form of compression, we examine the gain: the amount of auxiliary information embedded in the compressed enumeration. Strong compression and weak gainless compression is constructed for any computably enumerable set, and a positional game is studied toward understanding strong gainless compression.

Full paper at: `http://arxiv.org/abs/2304.03030`

## 2 Nikolay Bazhenov

*Sobolev Institute of Mathematics, Russia*
Learning families of algebraic structures from text

Abstract

A learner is a function $M$ which, given a finite amount of data about a countable algebraic structure $\mathcal{S}$, outputs a conjecture about the isomorphism type of the input structure. A family of structures $\mathcal{K}$ is learnable if there exists a learner which for any $\mathcal{S}$ from $\mathcal{K}$, given larger and larger amounts of

$\mathcal{S}$-data, eventually correctly identifies the isomorphism type of $\mathcal{S}$. If $\mathcal{S}$-data contains only positive information, then we talk about *learnability from text* (or $\mathbf{TxtEx}_{\cong}$-learnability) for the family $\mathcal{K}$.

We obtain a syntactic characterization of $\mathbf{TxtEx}_{\cong}$-learnability, following the ideas from [1]. The key ingredient of the proof is a new version of Lopez-Escobar Theorem for the Scott topology.

The talk is based on joint works with E. Fokina, D. Rossegger, A. Soskova, and S. Vatev.

## References


[1] N. Bazhenov, E. Fokina, and L. San Mauro. Learning families of algebraic structures from informant. Information and Computation, 275, article 104590, 2020.


# 3 Yong Liu


*Nanjing Xiaozhuang University, China*


Splitting property in 3-r.e. degrees


Abstract

Recursively enumerable (r.e.) degrees and its generalization n-r.e. degrees have been extensively studied in the history. Among all the interesting properties, the splitting property is the very basic one. An n-r.e. degree is splittable if it is a join of two other n-r.e. degrees. It is known that a nonrecursive r.e. degree is splittable (Sacks) and a proper 2-r.e. degree is splittable (Yamaleev). In this talk, we will discuss the splitting property in 3-r.e. degrees. This is a joint work with Ng Keng Meng.


# 4 Kenshi Miyabe


*Meiji University, Japan*


Solovay reducibility and signed-digit representation

Abstract

Solovay reducibility in the theory of algorithmic randomness compares two reals in terms of approximability, which is naturally related to computability and randomness. We give some characterizations of Solovay reducibility for weakly computable reals in terms of analysis.

We say that a real $x \in \mathbb{R}$ is weakly computable if there exists a computable sequence $(x_n)_{n\in\omega}$ of rationals such that $x = \lim_{n\to\infty} x_n$ and $\sum_n |x_n - x_{n+1}| < \infty$. Zheng and Rettinger (2004) extended Solovay reducibility to weakly computable reals as follows: Let $\alpha$ and $\beta$ be weakly computable reaals. We say that $\alpha$ is Solovay reducible to $\beta$ if, there exist computable sequences $(a_n)_n$ and $(b_n)_n$ of rationals converging to $\alpha, \beta$ respectively and $c \in \omega$ such that

$$|\alpha - a_n| \leq c(|\beta - b_n| + 2^{-n})$$

for all $n$.

We give a characterization of Solovay reducibility for weakly computable reals in terms of the signed-digit representation. Roughly speaking, $\alpha \leq_S \beta$ if and only if $\alpha$ is computable from $\beta$ with use bound $h(n) = n + O(1)$ when both of $\alpha$ and $\beta$ are represented by the signed-digit representation. This should be compared with the fact that computable Lipschitz reducibility is similarly defined with respec to the binary representation and that Solovay reducibility and computable Lipschitz reducibility are incomparable. Solovay reducibility is a continuous concept and has good compatibility with analysis. Also note that it is well-known that the binary expansion is not suitable in the study of computability in analysis while the signed-digit representation is.

Another characterization of Solovay reducibility for weakly computable reals we give is in terms of Lipshcitz functions. Solovay reducibility for left-c.e. reals has a natural characterization in terms of Lipschitz functions. In our previous work, we considered quasi Solovay reducibility for left-c.e. reals in terms of Hölder continuous functions. Roughly speaking, $\alpha \leq_S \beta$ if and only if there exists a Lipschitz function sandwiched by lower and upper semicomputable functions converging to $\alpha$ as the input goes to $\beta$.

We also studied related reducibilities and gave separation among them.

This is joint work with Masahiro Kumabe (The Open University of Japan) and Toshio Suzuki (Tokyo Metropolitan University).

# 5 Arno Pauly

*Swansea University, UK*

Observations and questions on the structure of the Weihrauch degrees

Abstract

Weihrauch reducibility is a computability-theoretic reductibility between multivalued functions on represented spaces. They can be used to compare the computational content of mathematical theorems. The Weihrauch degrees form a distributive lattice, into which the Medvedev degrees embed. But more is to be said about the structure of the Weihrauch lattice as such. Beyond the lattice operations, the Weihrauch degrees carry additional algebraic structure, which then leads to further questions as to what the theory of the Weihrauch degrees in various signatures is. I will summarize the existing research in the area and mention some work in progress and open questions.

# 6 Paul Shafer

*University of Leeds, UK*

Ordinal analysis of partial combinatory algebras

Abstract

We use the ordinals to define a hierarchy of extensionality relations ($\sim_\alpha$ : $\alpha \in \mathrm{On}$) on a given partial combinatory algebra (PCA). We then calculate the exact complexities of these relations for Kleene's first model, which is the PCA arising from ordinary Turing computability. The extensionality relations $\sim_\alpha$ exhaust the hyperarithmetical hierarchy as $\alpha$ advances through the recursive ordinals, and then they stabilize at the relation $\sim_{\omega_1^{\mathrm{CK}}}$, which is $\Pi^1_1$-complete. This work is joint with Sebastiaan A. Terwijn.

# 7 Richard A. Shore

*Cornell University, USA*

Basis Theorems, Zorn's Lemma, $\Sigma^1_1$ and $\Sigma^1_2$ Submodels, $\Pi^1_1$-$\mathrm{CA}_0$ and $\Pi^1_2$-$\mathrm{CA}_0$ and Applications in Combinatorics

Abstract

We discuss a number of old and some new reverse mathematical connections between basis theorems (Gandy and Shoenfield), restricted versions of Zorn's Lemma, strong $\Sigma^1_1$ and $\Sigma^1_2$ $DC_0$, $\Sigma^1_1$ and $\Sigma^1_2$ correct submodels and $\Pi^1_1$ and $\Pi^1_2$ $CA_0$. We then use these methods to prove some new and some old results in reverse mathematics. The results suggest several interesting open problems. The most attractive is whether a particular combinatorial result newly proven in $\Pi^1_2$-$CA_0$ by these methods is actually equivalent to $\Pi^1_2$-$CA_0$.

# 8 Mariya Soskova

*University of Wisconsin–Madison, USA*
Stratifying classes of enumeration degrees

Abstract

Turing reducibility gives a fine-grained approach to studying mathematical objects and relationships between them. For example, Moschovakis observed that every continuous function is computable relative to some fixed Turing oracle, thus understanding relatively computable functions gives a different perspective on the study of continuous functions on the reals. This point of view belongs to effective mathematics. Turing reducibility between sets of natural numbers allows us to gauge the algorithmic content of a mathematical object, such as real number or a continuous function. However, it is not well suited to handle partial information: suppose that instead of total access to the membership in the oracle, we are only given access to the positive information. Friedberg and Rogers capture this extended model of relative computability: X is enumeration reducible to Y if there is an algorithm to enumerate X given any enumeration of Y. The induced partial order of the enumeration degrees extends the Turing degrees and carries a variety of interesting aspects. The interplay between definability, structure, and effective mathematics gives rise to a zoo of classes that informs our understating of the partial order. We take a closer look at some examples of this phenomenon.