IMPRINTS
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Representation Theory, Combinatorics and Geometry

From 12 December 2022 to 7 January 2023, the Institute hosted a program on "Representation Theory, Combinatorics and Geometry". The organizers contributed this invited article to Imprints.

BY HUANCHEN BAO *(National University of Singapore)*, **JOSEPH CHUANG** *(City, University of London)*, **KARIN ERDMANN** *(University of Oxford)*, **KAY JIN LIM** *(Nanyang Technological University)*, **KAI MENG TAN** *(National University of Singapore) and* **WEIQIANG WANG** *(University of Virginia)*

The one month program on "Representation theory, Combinatorics and Geometry" was held in IMS from 12 December 2022 to 7 January 2023. It was organised by Huanchen Bao (National University of Singapore), Joseph Chuang (City, University of London), Karin Erdmann (University of Oxford), Kay Jin Lim (Nanyang Technological University), Kai Meng Tan (National University of Singapore) and Weiqiang Wang (University of Virginia). This program focused on representation theory of symmetric groups, as well as various interactions between representation theory, combinatorics, and geometry.

The program was originally planned in 2019 but was postponed due to the pandemic. Until mid 2022, it was not clear whether the program could be held as an in-person event. We were fortunate enough to hold this program entirely in person in the end. Despite the availability of online seminars, there is still no substitute for in-person interactions. Participants can engage in informal discussions, share insights and feedback, and moreover, build personal connections. The program provided an excellent opportunity for younger researchers, in particular for local graduate students and postdocs, to interact with leading researchers and be exposed to a much wider area of research.

The first week of the program was devoted to representation theory of symmetric groups. The representation theory of symmetric groups and related algebras is a vibrant and dynamic research area, with many unsolved problems and sometimes surprising connections to other areas, such as number theory and algebraic topology. Recent progress on fundamental questions about symmetric groups has been made through a blend of ideas approaching the representation theory from different perspectives:

- as a finite group to which one can apply character theory and modular representation theory
- as a special finite group closely connected with algebraic combinatorics
- as a prototypical diagrammatic algebra
- as one side of Schur-Weyl dualities with algebraic groups
- as a group acting naturally on algebraic varieties or on topological spaces

The remaining part of the program aimed to foster interactions between representation theory, combinatorics and geometry. The scope was designed to be diverse, aiming at introducing a wider range of topics. The workshop in the first week was followed by four six-hour-long mini-courses on the following topics:

1. Total positivity: combinatorics, geometry, logic and representation theory by Xuhua He (The Chinese University of Hong Kong)

2. Monoidal categorifications, quantum affine algebras and quiver Hecke algebras by Se-Jin Oh (Ewha Womans University)

3. An introduction to the geometric Satake equivalence by Xinwen Zhu (Caltech)

4. Coulomb branches of 3d N = 4 SUSY gauge theories and bow varieties by Hiraku Nakajima (KAVIL IPMU).

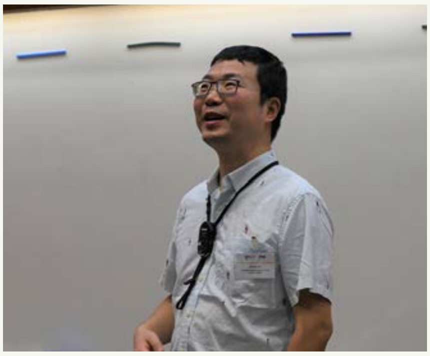
The first mini course on total positivity has been a great example revealing the connections between representation theory, combinatorics and geometry. The speaker (Xuhua He) assumed only minimal background knowledge from the audiences, barely above linear algebras, and discussed the connection between total positive matrices with combinatorics, geometry, logic and representation theory.

The mini courses were scheduled in a relaxed manner with only two lectures scheduled each day and with extended lunch breaks, providing participants with opportunities for discussion and closer interactions with the speakers. The Christmas and the New Year holidays provided a chance to rest and reflect between different mini-courses. Several speakers brought their families and spent extended time in Singapore and Southeast Asia. The organizers also planned hiking and sightseeing tours for the participants.

The final part of the program was another workshop from 3 to 7 January 2023, focusing on the interactions between representation theory, combinatorics and geometry. The highlights of the workshop were talks by two distinguished visitors: George Lusztig (MIT) gave a workshop talk on 3 January 2023 as well as a colloquium talk in the math department; Hiraku Nakajima (KAVIL IPMU) delivered a workshop talk on 5 January 2023 in addition to his lectures during the mini courses of the previous week.

The impact of the program continued even after it ended. Some participants extended their stay in Singapore and visited members of the mathematics department. In adddition, the seminars during the first two weeks after the program were delivered by some of the participants.

Arun Ram: Murphys, Casimirs, Transvections and Hecke algebras

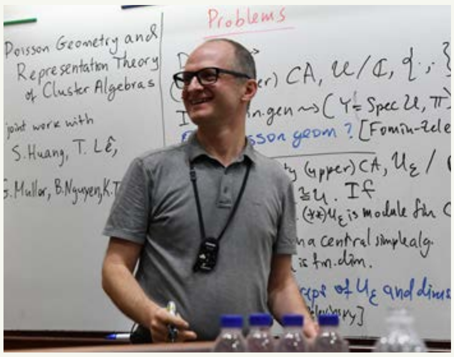


Xuhua He: Total positivity: combinatoics, geometry, logic and representation theory

Yaping Yang: Higher dimensional loop Grassmannians via fusion

Eric Vasserot: Critical convolution algebras and quantum loop groups

Weiqiang Wang: Relative braid group actions on quantum groups and modules



Milen Yakimov: Poisson geometry and representation theory of cluster algebras

SCIENTIFIC ADVISORY BOARD

New members

Susan Murphy: Professor Murphy is Mallinckrodt Professor of Statistics and of Computer Science, Radcliffe Alumnae Professor at the Radcliffe Institute, Harvard University. She obtained her PhD from University of North Carolina. Her awards include the Leo Breiman Senior Award (2022), the Van Wijngaarden Award (2021), the Royal Statistical Society Guy Medal in Silver (2019), the R.A. Fisher Award and Lectureship (2018), and the Precision Medicine World Conference Luminary Award (2018) and the MacArthur Award (2013). She is a member of the National Academy of Medicine (2014) and the National Academy of Sciences of the US National Academies (2016). She was president of the Institute of Mathematical Statistics (2018-2021) and president of the Bernoulli Society (2015-2021). Her research interests include experimental design and causal inference in sequential decision making, sequencing treatments in mobile health intervention, and inference for high dimensional models.

Ngaiming Mok: Professor Mok is Edmund and Peggy Tse Professor in Mathematics at The University of Hong Kong (HKU), and Chaired Professor in Mathematics and Director of the Institute of Mathematical Research at HKU. He obtained his MA from Yale University in 1978 and PhD from Stanford University in 1980, taught at Princeton University immediately afterwards, and was Professor at Columbia University and Université de Paris before returning to Hong Kong to take up the Chaired Professorship at HKU in 1994. His awards include the Future Science Prize in Mathematics and Computer Science (2022), the Tan Kah Kee Science Award in Mathematics and Physics (2022), the Chern Prize in Mathematics, ICCM (2022), the Distinguished Research Achievement Award by HKU (2011), the Bergman Prize (2009), the State Natural Science Award, China (2007), the Outstanding Researcher Award by HKU (2000), the Croucher Senior Research Fellowship, Hong Kong (1998), the Presidential Young Investigator Award, US (1985) and the Sloan Fellowship (1984). He is Member of the Selection Committee of the Shaw Prize in Mathematics (2022-23), Fellow of the American Mathematical Society (2019), Member of the Academy of Sciences of Hong Kong (2017), Member of the Chinese Academy of Sciences (2015) and Member of the Fields Medal Committee (2010). His research interests include complex differential geometry, several complex variables, algebraic geometry and functional transcendence theory.

APPRECIATION

The Institute would like to express its deep appreciation and gratitude to Mr Gim Pew Quek and Professor Emmanuel Ullmo, who served as members of IMS Scientific Advisory Board, and to Professor Mohan Kankanhalli and Professor Li-Shiuan Peh, who served as members of its Management Board, for their valuable advice and contributions to the Institute.

For more information about our institute, visit our webpage at
ims.nus.edu.sg

BOARD OF MANAGEMENT

New members

Kian-Lee Tan: Professor Tan is Tan Sri Runme Shaw Senior Professor and Dean of School of Computing at National University of Singapore (NUS), and Director of the Singapore Data Science Consortium (SDSC). Professor Tan obtained his PhD in Computer Science from NUS. His awards include the NUS Outstanding University Researchers Award (1998) and the NUS Graduate School Excellent Mentor Award (2011). He won the Singapore's President Science Award with Professor Beng Chin Ooi in 2011, and the IEEE Technical Achievement Award in 2013. He is a member of ACM and IEEE (and IEEE CS). He was a member of the VLDB Endowment Board (2012-2017) and PVLDB Advisory Committee (2014-2017). Professor Tan is an associate editor of the ACM Transactions on Database Systems (TODS). He has also served in the editorial board of the Very Large Data Base (VLDB) Journal, the WWW Journal, and the IEEE Transactions on Knowledge and Data Engineering. His current research interests include query processing and optimization in multiprocessor and distributed systems, database performance, data analytics, and database security.

Adrian Röllin: Professor Röllin is the Head of the Department of Statistics and Data Science at the National University of Singapore. He obtained his PhD at University of Zürich in 2006, and was the Deputy Director at IMS from 2018–2021. He holds honorary appointments with the Saw Swee Hock School of Public Health and Department of Mathematics. He is Fellow of the Institute of Mathematical Statistics (2019) and Elected Member of the International Statistical Institute (2019). Professor Röllin's research interests lie in distributional approximation, Stein's method, and mathematical and statistical modelling of infectious disease processes.

Wee Teck Gan: Professor Gan is Head-Designate of the Department of Mathematics at the National University of Singapore. He obtained his PhD at Harvard University. He is currently a Tan Chin Tuan Centennial Professor at NUS, and was previously affiliated with University of California, San Diego and Princeton University. He is Fellow of the Singapore National Academy of Science (2018) and was an invited speaker at ICM 2014. His awards include the Singapore President's Science Award (2017) and Sloan Research Fellowship (2003). His research interests are in Langland's program and automorphic representations.

Optimization in the Big Data Era

5–16 DEC 2022

ORGANIZING COMMITTEE

Stephen J. Wright | University of Wisconsin
Defeng Sun | The Hong Kong Polytechnic University
Kim Chuan Toh | National University of Singapore

The first workshop on Fast Optimization Algorithms in the Big Data Era had 23 invited talks from 5 to 9 December 2022. The second workshop on Structured Optimization Models in High-Dimensional Data Analysis had 22 invited talks from 12 to 16 December 2022.

Six distinguished lectures were delivered by Professor Stephen Wright ("Optimization in Theory and Practice" and "Primal-dual optimization methods for robust machine learning"), Professor Yurii Nesterov ("Set-Limited Functions and Polynomial-Time Interior-Point Methods" and New perspectives for higher-order methods in Convex Optimization"), and Professor Jong-shi Pang ("Nonconvex Stochastic Programs: Deterministic Constraints" and "Nonconvex Stochastic Programs: Chance Constraints").

There were close to 80 participants with more than 20 graduate students.

Kim-Chuan Toh (left) and Jong-Shi Pang

Yurii Nesterov (left) and Stephen Wright

Group photo

Representation Theory, Combinatorics and Geometry

12 DEC 2022–7 JAN 2023

CO-CHAIRS

Huanchen Bao | National University of Singapore
Joseph Chuang | City, University of London
Karin Erdmann | University of Oxford
Kay Jin Lim | Nanyang Technological University
Kai Meng Tan | National University of Singapore
Weiqiang Wang | University of Virginia

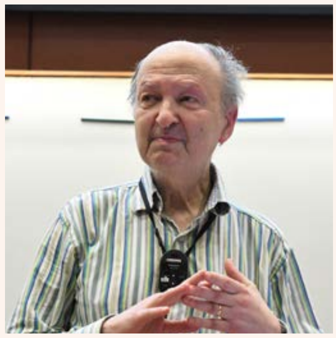
This one-month program focused on representation theory of symmetric groups and related algebras, as well as interactions between representation theory, combinatorics, and geometry.

The first workshop on representation theory of symmetric groups and related algebras (12 to 16 December 2022) had 21 talks. There were mini courses by Xuhua He (The Chinese University of Hong Kong, China), Se-Jin Oh (Ewha Womans University, Korea), Xinwen Zhu (Caltech, USA) and Hiraku Nakajima (KAVLI IPMU, Japan) from 19–21 and 27–29 December 2022. Each speaker gave six hours of lectures.

The second workshop on interactions between representation theory, combinatorics, and geometry (3 to 7 January 2023) had 17 talks. George Lusztig (Massachusetts Institute of Technology, USA) and Hiraku Nakajima (KAVLI IPMU, Japan) gave talks under the Distinguished Visitor Lecture Series.

There were more than 100 participants which included more than 20 graduate students.

Hiraku Nakajima



George Lusztig

Group photo

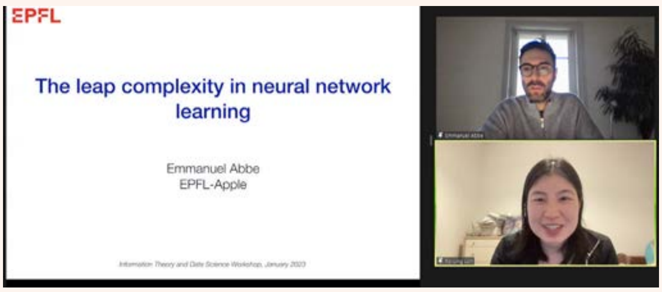
Information Theory and Data Science Workshop

16–27 JAN 2023

CO-CHAIRS

Po-Ling Loh | University of Cambridge)
Jonathan Scarlett | National University of Singapore)
Vincent Y. F. Tan | National University of Singapore)

The workshop had a total of 33 invited talks. The talks planned were a mixture of in-person interactions and zoom sessions. This workshop has helped researchers gain more insights on the role of information theory in data science. There were more than 120 participants, which included close to 50 graduate students



Zoom session by Emmanuel Abbe

Antonios Varvitsiotis

Arnab Bhattacharyya

Group photo

INTERVIEW

HIRAKU NAKAJIMA: EXCHANGING IDEAS BETWEEN GEOMETRY AND REPRESENTATION THEORY

Interview of Hiraku Nakajima by Chee Whye Chin

Hiraku Nakajima has made seminal contributions to geometry and representation theory through his study of certain moduli spaces appearing in gauge theory, now called Nakajima quiver varieties. Through a deep study of the properties of these quiver varieties, including their topology and the actions that they admit, he has found the natural framework for the geometric construction of certain representations of Kac-Moody Lie algebras and their quantum analogue. Subsequently, he has established further remarkable relations between the geometry of the Hilbert schemes of points on projective surfaces on the one hand, and the representation theory of the infinite-dimensional Heisenberg algebra on the other. In a joint work with Kōta Yoshioka, he proved Nekrasov's conjectured relation between supersymmetric Yang-Mills theory on $\mathbf{R}^4$ and the Seiberg-Witten prepotential. More recently, in joint works with Alexander Braverman and Michael Finkelberg, he has given a mathematically rigorous definition of the Coulomb branches of 3-dimensional $N=4$ supersymmetric gauge theories; these are hyper-Kähler manifolds that one can associate to a complex representation of a reductive group. His works have provided a firm mathematical setting for developing the rich ideas arising from theoretical physics that reveal deep relations between geometry and representation theory.

Nakajima received his PhD from the University of Tokyo in 1991 and has taught at Tohoku University and the University of Tokyo before moving to Kyoto University in 1997. He remained at Kyoto for over 20 years, during which he also served as the chairperson of the mathematics department for the year 2004—2005. Since 2018, he has been professor and principal investigator at the Kavli Institute for the Physics and Mathematics of the Universe (IPMU) and the University of Tokyo. He has published more than 80 research articles in both English and Japanese, among which includes an often-cited book on the Hilbert schemes of points on surfaces. He delivered a plenary talk at the International Congress of Mathematicians (ICM) in 2002, and he was awarded the Hardy Lectureship of the London Mathematical Society in 2010. His other awards include the 1997 Geometry Prize and the 2000 Spring Prize from the Mathematical Society of Japan, the 2003 Cole Prize in Algebra from the American Mathematical Society, 2006 JSPS Prize from the Japan Society for the Promotion of Science, the 2014 Japan Academy Prize and the 2016 Asahi Prize. He has also served on the editorial board of several top journals, and as a member of various committees related to the International Mathematical Union (IMU). He will take over as the President of the IMU from 2023.

Between 27 Dec 2022 and 8 Jan 2023, Nakajima visited the IMS as Distinguished Visitor for the programme on "*Representation Theory, Combinatorics and Geometry*". He gave a mini-course of three 2-hour lectures on 27-29 Dec 2022 titled "*Coulomb branches of 3d N=4 SUSY gauge theories and bow varieties*". This was followed by his Distinguished Visitor Lecture on 5 Jan 2023 titled "*Coulomb branches of orthosymplectic quiver gauge theories*". Chin CheeWhye took the opportunity of his presence at the programme to interview him on 29 Dec 2022 on behalf of the IMS newsletter Imprints. The following is an edited and vetted transcript of the interview, in which he talked about his mathematical background, his academic career, his mathematical work, as well as his IMU presidency.

IMPRINTS I Can you tell us how you got interested in mathematics when you were a student?

HIRAKU NAKAJIMA N I was not a motivated student. I know that many of my colleagues started to study mathematics seriously at an early stage, but I never tried to read, for example, the advanced books. I just followed all the lectures taught in school. I think the first time I realized that I might be interested in mathematics was when I prepared for the entrance exam to the university. Now, because the number of children has decreased, it has become much easier to enter the university. But at my age, the competition was still hard. In fact, after graduating from high school, many of my friends prepared for the entrance exam… they didn't succeed immediately after graduation, but they went to some kind of school — one more year between high school and university — just in order to prepare for the entrance exam. For my case, I didn't do that, and I got into the university immediately after graduation. I think I prepared when I was in high school; I solved many exercises and I enjoyed them, so at that time, I felt that maybe I was interested in mathematics. And maybe also in the sciences in the broad sense.

I Was it at the university that you made the decision to be a career mathematician?

N Yes. At the University of Tokyo, the first two years were undergraduate courses, and students didn't need to choose a subject, so we studied various things, general subjects. I did a little bit of experiments in physics and chemistry, and I just disliked them. I wanted a bit more theoretical stuff, so I studied a little bit of theoretical physics. But I thought I preferred mathematics, and so I decided to choose a mathematics course.

I So that this was around your third or fourth year as a student?

N Well, at the end of the second year, I chose mathematics as my field.

I Was there a teacher or professor who inspired you particularly?

N Not particularly. I mean, at my time, there were two campuses, Hongo and Komaba. We had four years for undergraduate, and they were separated into two stages. The first two years we studied in Komaba for the general subjects, and in the third and fourth years, we moved to Hongo, and we were specialized in mathematics. But somehow the professors were located in either campus. So, in the first year, I was taught by professors in the Komaba campus, and then after moving to Hongo, I was taught by different professors.

I And they didn't interact?

N They had some slight interactions, but… the two departments have combined in 1992 and now all the faculty members previously in Hongo have moved to Komaba. So all the faculty members in this new math department teach all the courses for general students.

I It's more like what is done in the US?

N Yeah, it's more like that. In some sense, I think it was a big decision for the department. I was an assistant professor when the decision was made, and the two departments were combined exactly when I moved to Tohoku University.

I So, you had four years of undergraduate studies, and…

N Four years of undergraduate, and I continued to graduate school in Hongo. It was supposed to be five years, but at my time, I finished only the master course, and I didn't go to the doctorate course. So immediately after graduation from the master course, I was offered an assistant professor position. But that was back then; now it is not possible anymore, because of the availability of the positions --- I think it has become more competitive than before.

I Tell us about your experience as a graduate student at Tōdai (University of Tokyo). How did you end up working with Ochiai?

N I liked most of the lectures taught at Tōdai. I didn't specialize at the beginning, so I studied various subjects. I think one of the attractive lectures was taught by Kazuya Kato, who is a number theorist. He's of course a talented mathematician, but he is an interesting character. He was, I think, the youngest faculty member at the time. Kato taught me Galois theory, but somehow afterwards, I found that maybe algebra might be too difficult for me. And if I excluded algebra, then geometry seemed to be the most natural. I liked Ochiai's lectures on geometry. In geometry, you have differential geometry and topology. I felt that topology was a little bit like algebra stuff and I didn't like much about that kind of thing, so I chose differential geometry as my subject.

I I see. You subsequently taught at Tohuku, Tōdai, Kyoto, and now you're back at Kavli. How do you find the different departments? How would you describe the differences in the mathematical environments at these different departments?

N When I was a student, I thought that basically the courses which were taught at the University of Tokyo covered all the subjects in mathematics, and that I didn't need to study other things. But in fact, after moving to other universities, I realized that each university has its strong subjects where many people study them. For example, there were a lot of people studying representation theory at the University of Tokyo, but representation theory is a huge subject, and there are various aspects. When I moved to Tohoku University, there was also a strong school in representation theory there, and I learned many new things after moving. I was already an associate professor then, but nevertheless, by attending seminars and talking with my colleagues, I learned many new things about representation theory.

I **Was the area of focus of your colleagues at Tohoku in a different area?**

N Yes, different from the strong areas at the University of Tokyo. And also, at both Kyoto University and the University of Tokyo, algebraic geometry is strong, but they study different aspects of algebraic geometry. In particular, at Kyoto University, people studied the moduli space of vector bundles, starting from Nagata, and Maruyama was the leading person in algebraic geometry at Kyoto University, and there were many younger people studying moduli spaces. I think they had a big influence on me.

I **You have worked at Kyoto for more than 20 years since 1997, during which you also served as the chairperson of the department for the year 2004—2005. You have also supervised many graduate students ….**

N I didn't supervise many students; only a small number. I supervised even less since I moved to the Kavli research institute.

I **I see. But you are also serving as an editor of several top journals. Between all these very different activities — your own research, your teaching and supervision work, your administrative duties, both as department chair and also as editor of journals, how do you manage your time?**

N Well, there was a kind of tradition at Kyoto University, that a new professor would be asked to be the chairperson, because then, he or she can learn the management of the department. And usually younger professors have more energy. And in fact, when I was the chairperson, I was helped by many of the senior colleagues. I think it was a really good experience for me, and I learned how to manage such kind of things. Of course, maybe I might be talented for such kind of things, but I never noticed this before. But I think when I was chairperson, I basically cannot concentrate on research, so I didn't publish any research paper at that time. But somehow, I think during this one year, I learned how to manage those things. Afterwards it became much easier.

I **But still, you have many other things on your plate. You have supervision work, you have editorial stuff, and you certainly need to devote time to your own research topics. Do you prioritize one over the other? Or do you schedule yourself every day to spend *X* amount of time on one, and then *Y* amount of time on the other?**

N I think I don't… At least when I start doing something for my research, I don't need to concentrate on my research project for a long period. At some point I do concentrate on a particular thing for research, but I think I can somehow postpone such things if I need to do something else.

I **So, your research questions are always working in the background?**

N Yeah, that's true. I think that if I study some problems and I feel that those are too difficult at that time, then I just try to keep them in my mind, and I just postpone pursuing those questions. Then after many years, those questions might really come back to me. I have had many such kind of experiences. I usually feel that maybe if I concentrate too much on a particular problem, it might not be successful, and maybe it might be better to put this aside and do something else. And if this something else is administrative stuff, I feel it's okay. But now that I've become a little old, I'm not sure I can still continue to do that, because I don't have much time to wait for the questions to come back to me!

I **You seem to let the questions take their own natural course… you don't try to push too hard on them.**

N Yes, I think that's my style.

I **Since 2018, you have moved to Kavli IPMU. What prompted the move? I think Kyoto now lists you as a professor emeritus…**

N Yes, officially. One reason is that this Kavli IPMU is an institute which consists of mathematicians and physicists, both theoretical and experimental physicists, and also astronomers. We come together to study the origin of the universe — that is the aim of this institute. I personally don't know if my mathematical research is helpful for understanding the origin of the universe, but at least my field is closely related to theoretical physics, and my recent work is about supersymmetric gauge theories. So that's one main reason.

Another reason is that my spouse, Yukari Ito, is also a mathematician. We lived in Kyoto and she worked at Nagoya University, there's a Shinkansen (high speed railway) between them, and she commuted daily between Kyoto and Nagoya for 20 years or more, so it was a little hard. She has managed to do that, but somehow when she got a little old, she seems to be tired of that.

I **I guess moving to Tokyo sort of saved her from the daily commuting?**

N Yeah. It's now much easier than before, but she seems to be very busy now with various activities for the society, so …

I **How do you find working at Kavli different compared to working at Kyoto? Do you have to teach courses at Kavli?**

N If I like to, I could teach a course at the University of Tokyo. At the institute, the director decides everything basically, so we don't have meetings... I mean, of course we hire many postdocs and so we have meetings for the choice of new postdocs, but otherwise we don't have much administrative work to do. Basically, the director and some small number committee members decide everything.

I **So, the meetings are more research or academic related rather than administrative?**

N Yes. That is a very different thing from Kyoto. The Kavli IPMU is a very new institute. It is only 15 years old, and somehow all the administrative systems are new…. First of all, English is the official language at this institute. As far as I know, this is the only institute in Japan that has English as the main language.

I **notice that there are many foreign faculty members at Kavli. I suppose most of the time when you meet these colleagues, the discussion will be in English?**

N Basically, all the postdocs… not all but most of postdocs are from abroad, and we communicate in English. So, in many aspects this institute is very new, very exceptional among the Japanese institutes.

I **Let's talk about your mathematical work. Your initial mathematical investigations were in differential geometry, specifically on the Yang-Mills equations and Yang-Mills connections. How did you get attracted into this area? I guess this was the time when you were still a master's student? How did you find this topic to be something that you want to do for your master's thesis?**

N Yeah. I chose Professor Ochiai as my supervisor, and he encouraged me to study PDEs on manifolds. He didn't talk about the Yang-Mills equations, but more broadly, he suggested me to study PDEs on manifolds. So I studied quite a lot of topics, for example, minimal surfaces, harmonic maps, and I also studied Kähler-Einstein metrics and various PDEs on manifolds. On the other hand, gauge theory had just become a hot topic around that time. Donaldson proved his astonishing theorem a few years before I became a master's student. Matsumoto gave a series of lectures on Donaldson's work. He's a topologist, and he was not strongly into PDEs, so in that sense, he didn't go too deep into Donaldson's theory, but I learned that this was a really interesting new topic in the interaction between topology and differential geometry. In particular, analysis was one of the very important tools in Donaldson's theory. So I decided to study the PDE aspects of gauge theory. At the end of Matsumoto's lectures, to get the credit for his course, he told us to write a report on anything we wanted. So I just used my knowledge on analysis and PDEs on manifolds and wrote a small thing. I thought that it was something new, but it was not deep enough because it was originally just a report for the course. But when I submitted this report, Matsumoto passed it to Ochiai, and Ochiai told me that this was interesting and I should write a paper about it. So that's how I chose the topic in math for the master's thesis. In that sense I didn't seriously consider what I should study, but somehow, by chance, I chose it…

I **It was decided for you!**

N Well, in a sense it's true. I didn't think the study in this report was deep enough, but somehow, yeah….

I **Can you outline the key ideas behind the use of the moduli spaces of Yang-Mills connections (instantons) to study the geometry and topology of 4-manifolds?**

N I mean, this was a really striking new idea of Donaldson. Before Donaldson, for example in the works of Yau, people did study many PDEs on manifolds, but somehow, the goal of such a study was usually to find the solutions of the partial differential equations. And once the solutions were found, then usually that would be the end. Of course, once you have nice solutions of PDEs, they have some geometric applications, but the applications were usually not the main thing; the main part of the study was to find the solutions. People classified this kind of study as geometry, but most of the parts were actually in analysis, hard analysis.

I **So the main focus was in the technical solution of the PDEs, and the geometrical applications were more like a by-product?**

N Yeah, it's something like that. But Donaldson's work was completely different. His goal was not just to find the solution — he considered the space of all solutions. The point is that the solution is not unique; there are many solutions, and he considered the moduli space — this is the space of all solutions of the partial differential equations. We usually think that solving the partial different equation is the main part, but he went much further, and he studied the geometric properties of the moduli space. He treated the moduli space as a new geometric object. That was a very new idea.

And I was quite attracted by that idea. I really wanted to … not only me, but many people started to study gauge theory. In some cases, people said that the analysis used in gauge theory was too hard, but I had already studied that because Professor Ochiai encouraged me to study PDEs, so somehow I already had enough background knowledge for the analysis for gauge theory.

I **I can imagine that this new idea of using the moduli space as a tool to study the underlying geometry of the original manifold, that was probably groundbreaking at that time. Today maybe people will take it for granted! Subsequently you started working on asymptotically locally Euclidean spaces, or ALE spaces. I think they were introduced by Kronheimer or maybe with you together in your joint paper?**

N Well, people studied them earlier. Kronheimer constructed all the examples, but certain examples were already studied earlier by Hitchin and Eguchi-Hanson, Gibbons-Hawking and various people. But in some sense Kronheimer constructed all the cases and classified them all.

I **I see. So what is the significance of ALE spaces in physics and mathematics? Why are people from both sides interested in them?**

N I think originally, people considered them as a kind of non-compact analogue of K3 surfaces. The K3 surfaces naturally appear in the classification of complex surfaces — K3 is named after Kummer, Kähler and Kodaira, and the mountain K2. Anyway, by the solution of the Calabi conjecture by Yau, a K3 surface has a Ricci flat metric, and this Ricci flat metric means that it is a solution of Einstein's equation in Euclidean signature. And this Ricci flat metric has various special properties. So K3 surfaces were the first non-trivial examples of Ricci flat manifolds. But because the solution of the Calabi conjecture is given by solving partial differential equations, the existence of the solution is guaranteed by functional analysis, so even now, people don't know how to write down the metric explicitly. It's not explicit, only abstract existence. On the other hand, if you replace the K3 surfaces by certain non-compact spaces, you can write down the solution explicitly. For example, for the Eguchi-Hanson metric, you have rotational symmetry, so the equations are reduced to ordinary differential equations, and you can basically solve them explicitly. So, instead of directly studying the difficult Ricci flat metric on K3 surfaces, people studied the solution of the Ricci flat equation on non-compact spaces — those are the ALE spaces.

And then, afterwards, the ALE spaces are classified by the finite subgroups of SU(2), and people have known that this classification coincidentally said the same with the classification of the simply-laced simple Lie algebras, the Lie algebras of type *A*, *D* or *E*. When I started to study this, that was not noticed, but people gradually understood that they are related to the representation theory of Lie algebras. In this classification, you can see the relation only at the combinatorial level, but afterwards, people gradually understood that there are deeper relations between the ALE spaces and the Lie algebras.

I **So from physics, the ALE spaces were motivated by being non-com compact instances of Ricci flat manifolds...**

N Mathematicians also... differential geometers also wanted to construct examples of Ricci flat geometries.

I **Right, and the connection to representation theory came from this sort of mysterious appearances of the *ADE* groups in the classification.**

N And at that time, there was the work of Brieskorn and Slodowy, not for the ALE spaces, but when the underlying complex surface is the resolution of the simple singularity in surfaces. People already knew that there are some relation of that with Lie theory, and that was the work of Brieskorn and Slodowy. So there was already some hint that there might be some relations, but people found many more relations afterwards.

I **You have introduced the quiver varieties named after you in your 1994 paper in the *Duke Math Journal*, and you developed the theory further in a follow-up paper in 1998. How were you led from your earlier work on the ALE spaces to quiver varieties? How did the notion of quiver arise in this game?**

N Maybe I should say a little bit more in detail. I was first interested in the ALE spaces during my study of the moduli space of Einstein metrics. For example, on a K3 surface, if you fix the complex structure and the Kähler class, then the solution is unique, but you can change the complex structure and also the Kähler class, which means that you have a family of solutions. So I considered the space of those solution as a moduli space, like Donaldson did, and I studied what happens at the end of the moduli space — this moduli space is not compact, so I studied what happens at the boundary of the moduli space. And then I was led naturally to the study the ALE spaces jointly with Bando and Kasue, because if the metric fails to converge to another metric, then somehow the manifold is broken into pieces; the compact piece remains, but there are some bubbling — non-compact spaces bubble out from the compact piece, and these bubbles are ALE spaces. So that's how I first encountered the ALE spaces.

Next, I knew Mukai's works on the moduli of vector bundle on K3 surfaces, and because ALE spaces are non-compact analogues of K3 surfaces, I just wondered if there might be a similar story. So I started to study gauge theory on ALE spaces.

Then, I met Kronheimer in 1989, and we discussed about this moduli space together, and we found that although the Yang-Mills equations on ALE space are partial differential equations, the moduli space of their solutions is equivalent to the moduli space of algebraic solutions of certain equations of matrices. Those matrices are determined in terms of quivers — a quiver consists of many arrows, and each arrow represents matrices. And you can have some algebraic equations which are the counterparts of the partial differential equations. This is something I cannot explain, but for miraculous reasons, partial differential equations are replaced by algebraic equations, which turned out to be quivers. And that I found with Peter Kronheimer.

This was already a nice result, but somehow, I initially thought that since the PDEs have been changed to algebraic equations, the study of their moduli space should become easier; that was my hope. But when I started to study those spaces for the algebra equations, it turned out to be a translation of the problem, and even though algebra equations might be easier, it turned out not to be true. So it was still difficult, and I lacked the tools to understand the spaces of those solutions.

And then at the ICM in 1990, I heard Lusztig's talk on geometric methods in representation theory, and he mentioned some of his results based on quivers. At that time, I didn't understand his work much, but I thought that maybe this was a new tool which might be useful for understanding those moduli spaces. Then afterwards, I found the relation with representation theory. Basically, the spaces were found with Kronheimer, so it's not only my contribution, but somehow at the time, I thought that they should be regarded as new objects which might be useful for representation theory. So I decided to name them quiver varieties.

I **I see. Recognizing the importance of certain concepts — that by itself is also very important because it focuses attention on what is decisive. So how are these quiver varieties related to the representation theory of Kac-Moody algebras?**

N Basically, I considered the homology groups of the quiver varieties. The quiver varieties are some new spaces, and we consider their homology groups. But the quiver varieties are not only a single space; they form a family, so we pick up a family of quiver varieties, and we consider all of their homology groups. And then on this direct sum of homology groups, you have an action of the Lie algebra. So this was my construction. I mean, Ringel and Lusztig already constructed something very similar but slightly different.

Of course, people already knew how to construct representations in a geometric way — for example, Borel-Weil theory. So, we consider a flag manifold and a line bundle on that, and we consider the space of holomorphic sections of this line bundle. Then, because the group acts on the flag manifold, the space of sections is a representation of the group, so you can realize finite dimensional representations of the group in this way. That is Borel-Weil theory.

But I was looking at the topological homology groups, so even if you have an action on the space, you may not have any interesting action on the homology groups, because the homology groups are invariant under homotopy, and the group is usually a continuous group, so the identity component of the group acts trivially on the homology groups. But the point is that I was taking the family of spaces and considering their homology groups simultaneously, and the action of the Lie algebra.... I mean, I choose a generator of the Lie algebra, and this generator sends the homology of one space to the homology of another space, and so it is possible to have something non-trivial. And this... I never imagined to have to use the Lie algebra defined in that way. In fact, when I was a student, I studied manifolds and I studied Lie groups and Lie algebras, but I never understood them as given by generators and relations. I mean, such a presentation of the Lie algebra can be a tool to do some computation, but I never thought that such a presentation could be fundamental. But nevertheless, in my definition, I was forced to use this presentation in very essential way, and there seems to be no other way to understand the Lie algebra as a whole without this presentation.

I **Would it be right to say that it is the generators of the Lie algebra acting on the parameters of your various quiver varieties?**

N Maybe I should explain that the quiver varieties have many connected components, and they are indexed by discrete parameters, and I use the so-called correspondence. So I consider the product of two spaces and have a sub-manifold in the product; that defines a kind of integral operator on the homology of those two spaces. That's how I defined the action. But anyway, as I said, the quiver varieties consist of various connected components, and the connected components are parametrized by discrete data. And the Lie algebra is a tangent space to the Lie group, so originally it came from something continuous. But if I use the presentation by generators and relations, such kind of things, the continuous parameters are lost. That's somehow puzzling for me. Afterwards, I was doing very similar things, but always, my works have such a flavor. And I still feel that there might be some other way to understand, but I never arrived at such a different realization.

I **This idea, or in some sense the shadow of it, also appeared in your work on the representation of infinite dimensional Heisenberg algebra and relating that to the Hilbert scheme of points. Can you outline how you were inspired to make this discovery?**

N Well, it turns out that the Hilbert schemes of points are not so far away from quiver varieties. In fact, if you choose a particular surface, for example an ALE space — it's not compact but it's an example — then the Hilbert schemes of points on an ALE space are examples of quiver varieties. And, because I used generators and relations… so, the Kac-Moody Lie algebra has finitely many generators and finitely many relations, but the Heisenberg algebra has infinitely many generators; that is the only difference. Fortunately, although the generators are infinite, the relations are not so complicated. So I managed to do a similar kind of computation for the Hilbert scheme. And also, at some point, I realized that these Hilbert schemes of points on surfaces have very nice properties even for arbitrary complex surfaces — that I noticed at some point.

In fact, many people have already studied the Hilbert schemes for many years. In gauge theory, there are many works that use algebraic geometry techniques to understand this moduli space of solutions of Yang-Mills equations. The Hilbert schemes are toy models of the moduli space of vector bundles, and in fact, in some cases, you can understand the moduli of vector bundle by using the Hilbert scheme of points. And people succeeded in understanding the Hilbert schemes. I wrote a book on the Hilbert schemes, but my own work only appeared in the final chapter; before that, I collected various earlier works on the Hilbert schemes. In fact, this book was based on my lecture series at the University of Tokyo, and I think during the preparation of this course, I learned quite a lot about Hilbert schemes. It was very useful afterwards.

I You sketched the basic ideas of this other way of constructing quiver varieties in the first half of your ICM plenary talk in 2002. It is like constructing quiver varieties without quivers…

N Yes. That is true. Quivers have become very popular now. But when I started to study quivers… I mean, one of the reasons quivers became popular was because of homological mirror symmetry. So, because of homological mirror symmetry, the derived category of coherent sheaves on algebraic manifolds became a very popular topic. And for some classes of algebraic manifolds, that derived category is equivalent to the derived category for a non-commutative algebra which is defined by a quiver. Then quivers become very popular. When I started to study quivers, it was not so… maybe it started to become popular around 2002, but I felt that maybe it was better to avoid quivers, and that was one of the reasons why…. I'm not sure this was a good decision or not.

I In the latter half of your ICM talk, you sketched this geometric construction of the representations of affine Lie algebras, which is also related to your JAMS paper…

N Not the affine Lie algebra; it is the representation of the so-called quantum toroidal algebra. The quantum toroidal algebra contains the affine Lie algebra, but it is much bigger. It is something like the loop algebra of the affine Lie algebra. The affine Lie algebra is already the loop algebra of the finite dimensional Lie algebra, so if you consider the loop algebra for the affine Lie algebra, then you have two loops for this. People then call it a toroidal algebra. And yes, in the latter half of the talk, I studied the representation theory of the quantum toroidal algebra using the quiver varieties.

I How are the ideas in Kazhdan-Lusztig theory related to the constructions that you have made?

N I think that at some point it became very clear that the quiver varieties are the analogues of the cotangent bundle of flag varieties. And Kazhdan-Lusztig realized representations of affine Hecke algebra using the equivalent K-theory of the cotangent bundle; more precisely, they used the so-called Steinberg variety. The Steinberg variety is a fiber product of two copies of the cotangent bundle of flag variety over the Lie algebra. You can define a similar fiber product for quiver varieties, so at some point it became clear that we should have a similar study of quiver varieties, and the corresponding algebra should be the quantum loop algebra of Kac-Moody algebra; and in particular, if the quiver is of affine type, then it should be the quantum toroidal algebra. I think through the discussion with various people, Ginzburg and Lusztig and many others, it became really clear that this should be the analogue. But at that point, I didn't have many tools to understand equivariant K-theory. Now there's a standard book by Chriss and Ginzburg about geometric techniques in representation theory, but it became available only in the late 90s.

So anyway, for the cotangent bundle of flag varieties, there is a big group action, and there are various tools to analyze those equivariant K-theory. But for quiver varieties, you have the group action, but this group action is not big enough, and it's usually far away from being a homogeneous space like the flag varieties. So I needed several years to understand how to compute equivariant K-theory. I think immediately after I did this work on the homology of the quiver varieties, I realized that there should be a similar story for equivariant K-theory, but I needed several years to know how to compute equivariant K-theory.

I Your more recent work concerns the mathematical definition of the Coulomb branches in supersymmetric gauge theory. What are the issues with the physicists' definition of the Coulomb branch? And what are the challenges faced in giving the mathematical definition?

N Yeah, physicists don't usually care what should be the definition. For some reason, they think the existence of the Coulomb branches is granted by some physical consideration. But as a mathematician, I don't understand what they think. And they usually don't start from the axioms and prove such kind of things — the mathematical way of thinking. So maybe they have use some hidden hypotheses or some implicit hypotheses to consider the Coulomb branches. In fact, I heard about the Coulomb branch in Witten's lecture in 1996, and it was a long time ago. He didn't explain what is the definition, but he mentioned that there is a concept of Coulomb branches, and he explained how to compute Coulomb branches using the so-called *D*-branes, but his use of *D*-branes was not mathematically rigorous at all. In fact, he identified various spaces as Coulomb branches, and those spaces were familiar to me; they were quite often moduli spaces of the solutions of those… not necessarily Yang-Mills instantons, but their cousins called monopoles — such moduli spaces appear as Coulomb branches. And also, quiver varieties themselves sometimes appeared as Coulomb branches. So, somehow I was provided with examples of Coulomb branches without understanding the definition! It was puzzling for many years. But at some point, the physicist Amihay Hanany explained to me that there is a formula for the characters of the coordinate ring of the Coulomb branch — that is called the monopole formula, and this is very general. He started with a choice of group and representation, and he had some recipe to write down the formula, and it was very explicit. I tried to realize the space which gives this formula, and I succeeded.

I think this is quite typical in the interaction between mathematics and physics. Physicists usually don't care what should be the definition, and they quite often use implicit hypotheses to consider something, without mentioning it; somehow that is obvious for them.

I think another instance which is related to my study is in the so-called Seiberg-Witten theory. They say that certain supersymmetric gauge theories in four dimensions are controlled by a family of elliptic curves; those are called Seiberg-Witten curves. This is very similar to mirror symmetry. Mirror symmetry gives mysterious relations between pairs of Calabi-Yau manifolds, and the famous application of the idea is that the Gromov-Witten invariants, which are coming from the symplectic geometry of a particular Calabi-Yau, can be computed from the complex algebraic geometry of another Calabi-Yau, which is the mirror. And the computation of Gromov-Witten invariants is usually regarded as a difficult problem — they come from the study of moduli spaces and are not easy to compute, usually. But on the other hand, this complex algebraic geometric side is about the study of periods, that is kind of…. I shouldn't say it is classical, but it is manageable, and many people have been studying that. And this Seiberg-Witten curve plays the role of the mirror of the gauge theory. You want to compute something in gauge theory, but instead of directory studying them, you can use the periods of elliptic curves, which is a really classical subject; so physicists use Seiberg-Witten curves to compute. In particular, Witten conjectured that the Donaldson invariants — which people think are difficult to compute — can be computed very easily by using this idea. But nevertheless, people didn't know… so these Seiberg-Witten curves appeared as a result of some study of the quantum field aspects of gauge theory, and people didn't understand the definition of the Seiberg-Witten curves. They are explicit elliptic curves and they are very useful for computations; those are classical computations on elliptic curves and theta functions, such kind of things. But the most mysterious part for mathematicians is how Seiberg-Witten curves arise from the gauge theory. The problem is not mathematically rigorous, but they have some answers. The Seiberg-Witten curve is an answer without the question! And for many years, people understood that they are very useful, but people didn't understand the fundamental reason how Seiberg-Witten curves arise. Several years afterwards, Nikita Nekrasov gave a conjecture on how Seiberg-Witten curves arise starting from gauge theory, and his conjecture was mathematically precise. And then it became clear how to understand Seiberg-Witten curves.

I This was in your joint work with Yoshioka?

N My work with Kōta Yoshioka was one of the solutions.

I Before we wrap up this interview, we would like you to talk about your IMU presidency. We congratulate you on your election as the president of the IMU. For the larger mathematical public, can you describe how the IMU operates? And what are the roles and responsibility of the IMU president?

N The IMU does various things, but one of the major things is the International Congress of Mathematicians (ICM). I mean, of course the local organizer of the ICM plays an important role, but the IMU in some sense supervises the ICM. The local organizer deals with the practical things, but for example, the choice of the speakers and the choice of the format of the sessions are decided by the committees formed by the IMU. In that sense, the IMU plays this important role. It is something similar to the Olympic games, where there are the local committee and the International Olympic Committee. So, the local organizer deals with all the practical things, but the IMU supervises. And in particular, the IMU chooses the program committees and the program committees choose the speakers. And the choice of speakers is very important. So, in that sense, the IMU plays a very important role.

I The term of the presidency is four years?

N Four years, yes, until the next ICM. The next ICM will be in 2026 and it will be in Philadelphia. And at the end of that year will be the end of my presidency.

And also, another important thing is the prize selection committee. I mean, now there are lots of other prizes in mathematics, for example, the Abel Prize and the Wolf prize. But traditionally, the Fields Medal is one of the most important prizes in mathematics, sometimes called the Nobel Prize in mathematics. Of course, I know that this is only for mathematicians younger than 40 years old. But anyway, the choice of the recipients of the Fields Medal is made by a special committee, and the committee members are chosen by the IMU. And in particular, the chairperson of this committee is myself, by a rule; the IMU president also chairs the prize selection committee. And because the Fields Medal is for young people, somehow it is more difficult than the choice for other prizes, because I mean, for other prizes, you can just choose the established people; but for young people, we really need to choose the right person whose work should be influential in the future, so that might be more difficult. But anyway, we have had the tradition of the Fields Medal for many years, and I think the previous committees have chosen the recipients very carefully, and I think they were basically accepted by the community. So, I would like to follow that.

I What do you foresee as the main challenges that you will face in the next four years? Especially given the current geo-political situation around the world...

N Yeah. So, in fact, I was asked to be the IMU president last year (2021), not this year, and at that time, people told me that the covid pandemic will end soon, and maybe the next four years will be bright. But then at the beginning of this year (2022), Russia invaded Ukraine, and the ICM was moved to be a virtual one. This was not expected at all. As the president-elect, I attended the committee meeting where this transition from ordinary to virtual was decided. At that time, it was clear that we could not have the usual ICM in St. Petersburg, so all of the committee members immediately agreed that we should move to virtual, but it was a really heavy decision. I attended as a guest, so I was not involved in the decision, but I really felt that it was a very heavy decision. In fact, I have collaborators in Russia and I have visited Russia many times. In that sense, it was really shocking to me.

And yeah, international relationship... I feel that mathematics should be independent of international conflicts. But nevertheless, the ICM is a really big event, and unless the government provides some support, we cannot have the ICM. In that sense, we cannot be really free from the international relationships. So, I really hope that this war will end soon, but I'm not so optimistic. This is really a big issue for me to consider.

I Well, you certainly have your work cut out for you for the next four years! And we wish you all the best in your presidency. Thank you very much for your time and the very enlightening discussion that we have had for the past hour. Thank you very much!

N Thank you.

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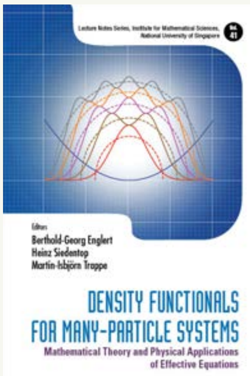

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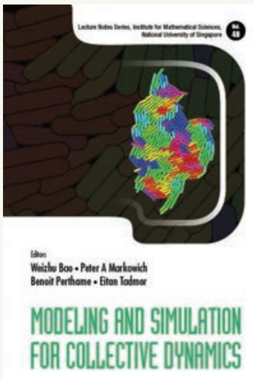

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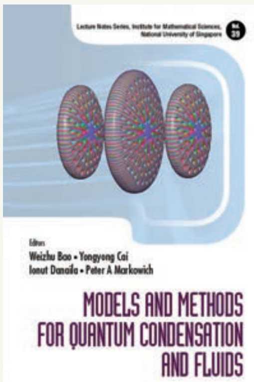

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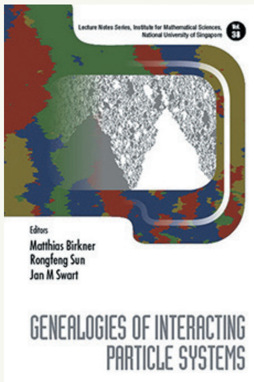

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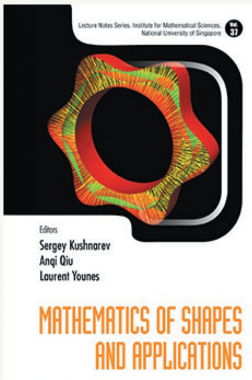

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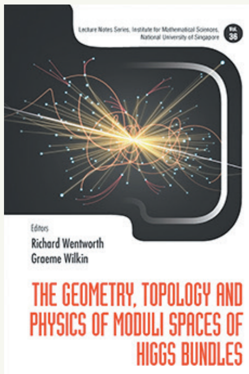

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