

# IMS DISTINGUISHED VISITOR LECTURE SERIES

22 February 2023  
9.00–9.45am, GMT +8 (Singapore Time)  
IMS Auditorium

## Time integration of tree tensor networks Christian Lubich

I first report on recent numerical experiments with time-dependent tree tensor network algorithms for the approximation of quantum spin systems. I will then describe the basics in the design of time integration methods that are robust to the usual presence of small singular values, that have good structure-preserving properties (norm, energy conservation or dissipation), and that allow for rank (= bond dimension) adaptivity and also have some parallelism. This discussion of basic concepts will be done for the smallest possible type of tensor network differential equations, namely low-rank matrix differential equations. Once this simplest case is understood, there is a systematic path to the extension of the integrators and their favourable properties to general tree tensor networks.

This talk is based on joint work with many colleagues and former and present students, among which I wish to single out Othmar Koch for the first mathematical work on dynamical low-rank approximation (DLRA) in 2007, Ivan Oseledets for jointly finding the first robust DLRA integrator (the projector-splitting integrator) in 2014, Gianluca Ceruti for jointly finding the Basis Update & Galerkin (BUG) integrators in 2021, and him and Hanna Walach and Dominik Sulz for the recent systematic extension from low-rank matrices to general tree tensor networks.

1 March 2023  
10.00–11.00am, GMT +8 (Singapore Time)  
IMS Auditorium

## Convergent evolving surface finite element algorithms for geometric evolution equations Christian Lubich

Geometric flows of closed surfaces are important in a variety of applications, ranging from the diffusion-driven motion of the surface of a crystal to models for biomembranes and tumor growth. Basic geometric flows are mean curvature flow (described by a spatially second-order evolution equation) and Willmore flow and the closely related surface diffusion flow (described by spatially fourth-order evolution equations).

Devising provably convergent surface finite element algorithms for such geometric flows of closed two-dimensional surfaces has long remained an open problem, going back to pioneering work by Dziuk in 1988. Recently, Balázs Kovács, Buyang Li and I arrived at a solution to this problem for various geometric flows including those mentioned above. The proposed algorithms discretize nonlinear parabolic evolution equations for geometric quantities along the flow, in our cases for the normal vector and mean curvature, and use these evolving geometric quantities in the velocity law interpolated to the finite element space. This numerical approach admits a stability and convergence analysis with optimal rates of convergence for finite elements of polynomial degree at least two.

The mini courses and talk are part of the program on  
*Multiscale Analysis and Methods for Quantum and Kinetic Problems*  
30 January–10 March 2023

Program webpage  
<https://ims.nus.edu.sg/events/qkp2023/>

Registration  
<https://tinyurl.com/multiscaleanalysisreg>



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Professor Christian Lubich is Professor at the Mathematisches Institut at the University of Tübingen, Germany. He has been at this position since 1994, after being Assistant Professor at ETH Zurich (1991–1992) and Associate Professor at University of Würzburg (1992–1994) before. He received his PhD from University of Innsbruck.

His research interests lie in the numerical analysis for a large variety of time-dependent problems formulated as ordinary or partial differential equations and/or integral equations, in recent years mostly in the areas of geometric flows, wave scattering, quantum dynamics, and charged-particle dynamics. He is coauthor of the book “Geometric Numerical Integration” with E. Hairer and G. Wanner and author of a book on numerical methods for quantum dynamics.

His awards include the Dahlquist Prize of the Society for Industrial and Applied Mathematics (2001). He was a plenary speaker at the 2018 International Congress of Mathematicians (ICM) in Rio de Janeiro.

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