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Abstracts

Workshop on interactions between representation theory, combinatorics, and geometry

(3–7 Jan 2023)

1 Jon Brundan

University of Oregon, USA

Non-degeneracy of the odd categorification of sl_2

Abstract

The odd analog of the Kac-Moody 2-category for the Lie algebra sl_2 was introduced by Ellis and Lauda a decade ago. It is no less remarkable than the usual even Kac-Moody 2-category for sl_2. It categorifies the "double cover" of the quantum group of sl_2 which was introduced by Clark and Wang. The best way to prove this statement requires bases for the 2-morphism spaces so as to be able to check that their graded dimensions match the bilinear form on the quantum group ("non-degeneracy"). In the last couple of years, two proofs of this have finally emerged, one by Dupont, Ebert and Lauda and one by Kleshchev and I. I'll talk about the latter which is based on an explicit 2-functor to a category of so-called odd Grassmannian bimodules.

2 Tsao-Hsien Chen

University of Minnesota, USA

Real groups, symmetric varieties, and derived Satake equivalence

Abstract

In an ongoing project of D. Ben-Zvi, Y. Sakellaridis and A. Venkatesh, the authors propose a conjectural generalization of the derived geometric Satake

equivalence for complex reductive groups to spherical varieties. I will describe a program aimed at establishing their conjecture in the case of symmetric varieties (an important class of spherical varieties). A key ingredient is the relation between the derived Satake equivalence for symmetric varieties and the geometric Langlands for real groups.
This is joint work with David Nadler, Mark Macerato, John O'Brien

3 Shun-Jen Cheng

Academia Sinica, Taipei
Categories of Whittaker Modules over Lie superalgebras and Categorification of Fock Spaces

Abstract

We explain certain categories of Whittaker modules over the Lie superalgebras that can have properly stratified structures. For integral blocks, we show that in the case of the general linear and ortho-symplectic Lie superalgebras these Whittaker categories categorify certain q-symmetric Fock spaces over the infinite-rank quantum group of type A and certain infinite-rank quasi-split quantum symmetric pairs of type AIII, respectively. In this picture, the canonical and dual canonical bases in these q-symmetric Fock spaces correspond to tilting and simple objects in these Whittaker categories, respectively. This talk is based on joint works with C.-W. Chen and V. Mazorchuk.

4 Dan Ciubotaru

University of Oxford, UK
A nonabelian Fourier transform for tempered unipotent representations of reductive p-adic groups

Abstract

In joint work with A.-M. Aubert and B. Romano, we define an involution on the space of compact tempered unipotent representations of inner twists of a split simple p-adic group. This generalizes a construction by Waldspurger for elliptic tempered representations of split orthogonal groups and it should also

be related to Lusztig's unipotent almost characters for p-adic groups. We conjecture that the restriction to reductive quotients of maximal compact open subgroups intertwines this involution with a disconnected version of Lusztig's nonabelian Fourier transform for finite reductive groups. I will explain the definitions, the relation with Lusztig's proposed unipotent almost characters of semisimple p-adic groups, and give supporting evidence for the conjectures.

5 Ryo Fujita

Kyoto University, Japan

Isomorphisms among quantum Grothendieck rings and their applications

Abstract

Quantum Grothendieck ring is a one-parameter deformation of the Grothendieck ring of the monoidal category of finite-dimensional modules over the quantum loop algebras, endowed with a canonical basis. In the case of type ADE, thanks to the geometry of quiver varieties, the canonical basis is known to compute the q-characters of simple modules (via the analog of Kazhdan-Lusztig algorithm) and enjoy some positivity properties. In this talk, we discuss a collection of isomorphisms between the quantum Grothendieck ring of type BCFG and that of "unfolded" type ADE, which respect the canonical bases and admit a cluster theoretic interpretation. They can be applied to propagate the positivity properties to type BCFG and to verify the analog of Kazhdan-Lusztig conjecture for several new cases. This talk is based on a joint work (partly in progress) with David Hernandez, Se-jin Oh, and Hironori Oya.

6 Christof Geiss

Instituto de Matemáticas (UNAM), Mexico

Geometric construction of the positive part of the Kac-Moody Lie algebra of affine type $\tilde{C}_n$.

Abstract

This is a report on joint work with my student Alberto Castillo. If C is a symmetric generalized Cartan matrix, by work of Lusztig and Schofield

the positive part n(C) of the Kac-Moody Lie algebra g(C) can be realized in terms of certain constructible functions on the representation varieties of a quiver of type C. Geiss, Leclerc and Schröer showed, how to extend this result to al Cartan matrices of finite type by considering locally free modules over a 1-Iwanaga Gorenstein Algebra H(C, D,). We show here, that the same approach works for C of affine type $\tilde{C}_n$. Moreover, we can describe the corresponding constructible functions quite explicitly.

7 Syu Kato

Kyoto University, Japan
Higher level BGG reciprocity for current algebras

Abstract

We prove the ext-orthogonality between thick and thin Demazure modules of the (twisted) affinization of a simple Lie algebra. This yields a higher level analogue of the Bernstein-Gelfand-Gelfand (BGG) reciprocity for (twisted) current algebras for each positive integer, that recovers the BGG reciprocity for current algebras in the sense of Chari and her collaborators as its level one case. We also discuss several problems about Demazure modules and orthogonal polynomials from our perspective. This talk is (mainly) based on arXiv:2207.07447.

8 Stefan Kolb

Newcastle University, UK
Quantum symmetric pairs and q-Pollaczek polynomials

Abstract

The coideal subalgebras appearing in the theory of quantum symmetric pairs can be described in terms of generators and relations. However, finding an explicit form of the defining relations is a difficult task. In this talk, I will explain how the star-product interpretation of quantum symmetric pairs can be used to obtain the defining relations in terms of families of orthogonal polynomials. I will particularly focus on the subtle case where a black dot neighbours a white dot in the Satake diagram. In this case the defining

relations involve a special instance of q-Pollaczek polynomials considered by Charris and Ismail in 1987. The talk is based on joint work with Milen Yakimov.

9 George Lusztig

MIT, USA

Unipotent character sheaves and strata of a reductive group

Abstract

Let G be a connected reductive group over an algebraically closed field. There are two interesting finite sets associated to G. One, denoted by CS(G), is the set of unipotent character sheaves on G; these are certain simple perverse sheaves on G which are useful for computing characters of irreducible representations of finite reductive groups. The other, denoted by St(G), is the indexing set of a partition of G into strata, each of which is a union of conjugacy classes of fixed dimension. It turns out that there is a natural surjective map from CS(G) to St(G). It is defined using Springer correspondence in bad characteristic.

10 George Lusztig

MIT, USA
Semisimple groups and the theory of total positivity

Abstract

According to Chevalley, semisimple groups can be defined over any field. More recently they were defined over some structures called semifields (which include the set of strictly positive real numbers under the usual addition and multiplication). It turns out that the theory over semifields can be used to get a better understanding of the theory over fields.

11 Hiraku Nakajima

Kavli IPMU, Japan
Coulomb branches of orthosymplectic quiver gauge theories

Abstract

Orthosymplectic quivers were used by Kraft-Procesi and later by myself to realize intersections of nilpotent orbits and slices to other orbits for classical groups as symplectic reduction, in other words as Higgs branches. We consider their Coulomb branches and identify them with orthosymplectic bow varieties. This is a joint work in progress with Hanany and Finkelberg.

12 Linhui Shen

Michigan State University, USA
Cluster nature of quantum groups

Abstract

We present a rigid cluster model to realize the quantum group $U_q(g)$ for g of type ADE. That is, we prove that there is a natural Hopf algebra isomorphism from the quantum group to a quotient algebra of the Weyl group invariants of a Fock-Goncharov quantum cluster algebra. By applying the quantum duality of cluster algebras, we show that the quantum group admits a cluster canonical basis Θ whose structural coefficients are in $\mathbb{N}[q^{\frac{1}{2}}, q^{-\frac{1}{2}}]$. The basis

Θ satisfies an invariance property under Lusztig's braid group action, the Dynkin automorphisms, and the star anti-involution. Based on a recent preprint arXiv: 2209.06258

13 Eric Vasserot

Université Paris Cité, France
Critical convolution algebras and quantum loop groups

Abstract

We introduce a new family of algebras attached to quivers with potentials, using critical K- theory and critical Borel-Moore homology, which generalize the convolution algebras attached to quivers defined by Nakajima. We give some applications to cohomological and K-theoretical Hall algebras, to shifted quantum loop groups, and to Kirillov-Reshetikhin and prefundamental representations.

14 Weiqiang Wang

University of Virginia, USA
Relative braid group actions on $\imath$quantum groups and modules

Abstract

Lusztig's braid group symmetries are a basic tool in understanding the structures of Drinfeld-Jimbo quantum groups. $\imath$Quantum groups are coideal subalgebras of quantum groups arising from quantum symmetric pairs, which can be viewed as a generalization of quantum groups. In this talk, we will present a construction of relative braid group symmetries (associated to relative Weyl groups of symmetric pairs) on $\imath$quantum groups, which admit several favorable properties similar to Lusztig's. These $\imath$symmetries have further led to compatible relative braid group actions on modules with explicit rank one formulas. This is joint work with Weinan Zhang (Virginia).

15 Ting Xue

University of Melbourne, Australia
Character sheaves for graded Lie algebras

Abstract

We discuss character sheaves in the setting of graded Lie algebras, focusing on the construction of cuspidal character sheaves. Irreducible representations of Hecke algebras of complex reflection groups at roots of unity enter the picture. Recent work of Lusztig and Yun relates Fourier transforms of character sheaves to irreducible representations of trigonometric double affine Hecke algebras. We will explain the connection between the work of Lusztig-Yun and our work, and discuss some conjectures arising from this connection. This is based on joint work with Kari Vilonen and partly with Misha Grinberg.

16 Ben Webster

University of Waterloo, Canada
The noncommutative Springer resolution of type A and KLRW algebras

Abstract

The theory of Coulomb branch algebras, based on work of Braverman, Finkelberg and Nakajima, has shed new light on many interesting algebras in representation theory. One of the most notable is the universal enveloping algebra of gl_n. I'll explain how this theory shows a close relationship between the characteristic p and characteristic 0 representation of gl_n and the cylindrical and planar KLRW algebras, in particular, giving a new, combinatorial description of the non-commutative Springer resolution of type A.

17 Milen Yakimov

Northeastern University, USA
Poisson geometry and representation theory of cluster algebras

Abstract

In the area of cluster algebras, there are two general constructions: the Gekhtman-Shapiro-Vainshtein Poisson structures on cluster algebras and the associated root of unity quantum cluster algebras. We will prove that the spectrum of each of the former algebras has an explicit Zariski open torus orbit of symplectic leaves, which is a far-reaching generalization of the complement of the Richardson divisor of Schubert cells in Lie theory. We will then show that the algebras in the latter class have canonical Cayley–Hamilton structures in the sense of Procesi. Based on these two methods, we will describe explicitly the fully Azumaya loci of all root of unity quantum cluster algebras. This classifies their irreducible representations of maximal dimension. This is a joint work with Shengnan Huang, Thang Le, Greg Muller, Bach Nguyen and Kurt Trampel.

18 Yaping Yang

University of Melbourne, Australia
Higher dimensional loop Grassmannians via fusion

Abstract

I will discuss a version of factorization/local spaces over the Hilbert scheme of points on an algebraic surface M, which can be viewed as a 2-dimensional generalization of Beilinson-Drinfeld factorization spaces over an algebraic curve. An example of the factorization space will be given, as a quote scheme of torsion free sheaves on M. The factorization structure is constructed based on the work of Haiman on Hilbert schemes, as well as the work of Feigin-Loktev and Chari-Pressley on local Weyl modules. Global sections of the tautological line bundle on this factorization space yield a local Weyl module of the toroidal algebra, whose characters are given by Macdonald polynomials.

This is based on my joint work in progress with Ivan Mirkovic and Gufang Zhao.