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Abstracts

Workshop 1: Multiscale analysis and methods for PDEs: fluids and active matter dynamics

(6–10 Feb 2023)

1 Clarissa Astuto

King Abdullah University of Science and Technology,Saudi Arabia
Finite-differences scheme for a tensor PDE model of biological network formation and applications

Abstract

We propose a method for the numerical solution of a complex biological network. We refer to the Cai-Hu model, where they hypothesized that the topology of the leaf pattern is governed by an optimization of the global energy consumption, starting with a general framework and adding some hypothesis based on experimental results. The evolution in time of the fluid is governed by an elliptic-parabolic system of partial differential equations for the conductance tensor $\mathbb{C}$, and the pressure p. The parabolic equation is augmented by the derivative of a metabolic function, that describes a decay term, and by the gradient of the pressure, that is obtained by a Darcy's type equation. The conductance appears also in the elliptic equation, making it depending on time.

The method adopted to solve the system is based on a finite-difference scheme in a uniform Cartesian grid in a 2D domain. Since the system consists of three (the tensor $\mathbb{C}$ is symmetric) components for the conductivity tensor, plus one for the pressure, at the end we compute the solution of four linear systems at each time step. To accelerate the simulation we make use of a *symmetric*-ADI method, since, intrinsically, the Alternating Direction Implicit method is not symmetric. However, we will see that the stiffness in the system may ruin some property of the solution of the numerical scheme.

Many tests are provided, on varying the values of the parameters involved. There are also accuracy tests that prove the second order accuracy of the method.

At the end, as an application of the biological network, we explore the resulting graph and its structure, showing important structural differences when changing the parameters of the system.

2 Russel Caflisch

New York University, USA

An Adjoint Method for the Nonlinear Boltzmann Equation

Abstract

We present an adjoint method for the spatially homogeneous, nonlinear Boltzmann equation, based on the "discretize then optimize" approach. The discretization (in time and velocity) is the DSMC method, and adjoint equations are derived from an augmented Lagrangian. After a forward (in time) solution of DSMC, the adjoint variables are found by a backwards solver. The adjoint variable is equal to a velocity derivative of an objective function. Numerical tests show that this gives accurate velocity derivatives and can be used for optimization of the Boltzmann equation. This is joint work with Yunan Yang and Denis Silantyev.

3 Young-Pil Choi

Yonsei University, Korea

Critical thresholds in pressureless Euler-Poisson equations: a new method based on Lyapunov functions

Abstract

We investigate critical thresholds phenomena in pressureless Euler-Poisson equations with variable background states in one dimension. We propose a new method based on Lyapunov functions to construct the supercritical region with finite-time breakdown and the supercritical region with global regularity of $\mathcal{C}^1$ solutions. By employing our new method, we also provide the large time behavior of solutions with the initial data in the subcritical region.

4 Michele Coti Zelati

Imperial College London, UK
Orientation mixing in active suspensions

Abstract

We study a popular kinetic model introduced by Saintillan and Shelley for the dynamics of suspensions of active elongated particles. We focus on the linear analysis of incoherence, that is on the linearized equation around the uniform distribution, in the regime of parameters corresponding to spectral (neutral) stability. We show that in the absence of rotational diffusion, the suspension experiences a mixing phenomenon similar to Landau damping. We show that this phenomenon persists for small rotational diffusion, and is combined with an enhanced dissipation at time scale at a faster time scale than the diffusive one.

5 Ionut Danaila

Université de Rouen Normandie, France
Numerical models for coupling Navier-Stokes and Gross-Pitaevskii solvers for two-fluid quantum flows

Abstract

In quantum flows, like liquid helium II at intermediate temperatures between zero and 2.17 K, a normal fluid and a superfluid coexist with independent velocity fields. The most advanced existing models for such systems use the Navier-Stokes equations for the normal fluid and a simplified description of the superfluid, based on the dynamics of quantized vortex filaments, with ad hoc reconnection rules. There was a single attempt [1] to couple Navier-Stokes and Gross-Pitaevskii equations in a global model intended to describe the compressible two-fluid liquid helium II. We present in this contribution a new numerical model to couple a Navier-Stokes incompressible fluid with a Gross-Pitaevskii superfluid [2]. A numerical algorithm based on pseudo-spectral Fourier methods is presented for solving the coupled system of equations. The new numerical system is validated against well-known benchmarks for the evolution in a normal fluid of different types or arrangements of quantized vortices (vortex crystal, vortex dipole and vortex rings).

References

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[2] M. Brachet, G. Sadaka, Z. Zhang, V. Kalt and I. Danaila, Coupling Navier-Stokes and Gross-Pitaevskii equations for the numerical simulation of two-fluid quantum flows, arxiv 4598109, November 2022.

6 Noemi David

Université Claude Bernard Lyon 1, France

Incompressible limit and rate of convergence for tumor growth models with drift

Abstract

Both compressible and incompressible models of porous medium type have been used in the literature to describe the mechanical aspects of living tissues. Using a stiff pressure law, it is possible to build a bridge between these two different representations. In the stiff pressure limit, compressible models generate free boundary problems of Hele-Shaw type where saturation holds in the moving domain. In this talk, I will present the study of the incompressible limit for advection-porous medium equations motivated by tumor development. The derivation of the pressure equation in the limit was an open problem for which the strong compactness of the pressure gradient was needed. Then, I will discuss the convergence rate of solutions of the compressible model to solutions of the Hele-Shaw problem.

7 Bruno Després

Sorbonne Université, France

New results on the stability of thick spray equations

Abstract

Thick spray models are based on a coupling between a kinetic equation of Vlasov type (for the droplets or dust specks constituting the disperse phase)

and a system of (compressible) fluid equations for the gas, so that they belong to the class of coupled kinetic-(compressible) fluid models. Even if the theory of thin spray models now is becoming more mature, the theory of thick spray models is much less advanced. Actually it seems that there even a debate wether thick spray models can be well-posed. I will described a ensemble of new results on the linear and non linear stability of thick spray models, obtained with C. Buet, L. Desvillettes and V. Fournier (Phd).

8 Gissell Estrada-Rodriguez

University of Oxford, UK

From kinetic to nonlocal PDEs for individual and collective migration

Abstract

Motivated by the nonlocal movement of biological organisms such as the bacteria *E. coli* and T cells, we consider an interacting particle (robotic) system which combines superdiffusive random movement with emergent collective behaviour from local communication and alignment. We derived a fractional PDE from the movement strategies of the individuals, introducing long range interactions and alignment into the analysis. The resulting kinetic model is studied at short and long time scales.

Applications we study include targeting efficiency and optimal search strategies. We showed that the system allows efficient parameter studies for a search problem, addressing basic questions such as the optimal number of robots needed to cover an area.

Validation against concrete robotic simulations with *e-puck* robots are also included in collaboration with computer scientists from the Edinburgh Centre for Robotics.

9 Francisco Gancedo Garcia

Universidad de Sevilla, Spain

Global-in-time dynamics for Muskat and two-phase Stokes gravity waves

Abstract

In this talk we consider the evolution of an interface evolving by an incompressible flow. On the one hand, we study the one-phase Muskat problem,

where the fluid is filtered in a porous medium. In the gravity-stable case, we show that initial Lipschitz graphs of arbitrary size provide global-in-time well-posedness. On the other hand, we study the interface dynamics given by two fluids of different densities evolving by the linear Stokes law. We show stability to the flat stable case and exponential growth in the unstable regime.

10 Diogo Aguiar Gomes

King Abdullah University of Science and Technology, Saudi Arabia
Hessian Riemannian flows in mean-field games

Abstract

Hessian Riemannian flows are a powerful tool for the construction of numerical schemes for monotone mean-field games that have their origin in constrained optimization problems. In this talk, we discuss the general construction of these flows for monotone mean-field games, their existence and regularity properties, and their asymptotic convergence.

11 Helge Holden

Norwegian University of Science and Technology, Norway
On the stochastic Camassa–Holm and Hunter–Saxton equations with transport noise

Abstract

We will discuss recent result for the stochastic Camassa–Holm equation

$u_t + uu_x + P_x + \sigma u_x dW = 0, -P_{xx} + P = u^2 + u_x^2/2$

and the stochastic Hunter–Saxton equation

$q_t + (uq)_x + (\sigma q)_x dW = q^2/2, u_x = q$

with transport noise.

This is joint work with L Galimberti (NTNU), KH Karlsen (Oslo), PHC Pang (NTNU/Oslo).

12 In-Jee Jeong

Seoul National University, Korea
Instabilities in vortex ring dynamics

Abstract

We consider vortex ring dynamics for incompressible Euler equations, and explain how to obtain the following infinite norm growth results: (i) filamentation (formation of a long tail) from a traveling vortex ring, and (ii) vortex stretching from the "head-on collision" of two counter-rotating vortex rings. This is based on joint works with Kyudong Choi (UNIST).

13 Nadia Loy

Politecnico di Torino, Italy
Kinetic models for multi-agent systems with multiple microscopic states

Abstract

In this talk we present a class of kinetic models describing interactions among individuals having multiple microscopic states. We shall consider microscopic states evolving according to both stochastic dependent and independent processes. In particular, we shall consider interacting agents who are divided into multiple sub-populations. As such, the agents are not indistinguishable, as classically assumed in kinetic theory, within the whole population. A general framework allowing to describe binary interactions and transfers among different sub-groups by deriving the model from microscopic stochastic processes will be presented. We shall discuss formal results concerning existence, uniqueness and equilibria. Moreover, we shall illustrate applications to epidemic models and to wealth exchange models with migration.

14 Alex Kiselev

Duke University, USA
Suppression of chemotactic blow up by active scalar

Abstract

Chemotactic blow up in the context of the Patlak-Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that the presence of fluid advection can arrest singularity formation given that the fluid flow possesses mixing or diffusion enhancing properties and its amplitude is sufficiently strong - an effect that is conjectured to hold for more general classes of nonlinear PDE. In this talk, I will discuss some results on suppression of singularity formation in systems where Patlak-Keller-Segel equation is coupled with fluid flow via buoyancy force. The talk is based on joint work with Zhongtian Hu and Yao Yao.

15 Lisa Maria Kreusser

University of Bath, UK

Generalised eikonal equations on graphs with applications to semi-supervised learning

Abstract

Many computational methods for semi-supervised and unsupervised classification are based on variational models and PDEs. Since shortest path graph distances are widely used in data science and machine learning, it is natural to introduce the concept of information propagation to data classification and semi-supervised learning. The success of eikonal equations in the continuum setting motivates the development of similar tools on graphs. We propose and unify classes of different models for information propagation over graphs, and prove equivalences between them. Motivated by the connection between first arrival time model and the eikonal equation in the continuum setting, we derive mean field limits for graphs based on uniform grids in Euclidean space under grid refinement. For a specific parameter setting, we demonstrate that the solution on the grid approximates the Euclidean distance. Finally, we illustrate the use of front propagation on graphs to semi-supervised learning.

16 Chenyun Luo

The Chinese University of Hong Kong, China

Unified Theory for Gravity-Capillary Water Waves with Vorticity

Abstract

We survey some recent results concerning the motion of 3D water waves described by the compressible Euler equations in an unbounded domain with a moving top boundary and a fixed flat bottom. The water waves are influenced by gravity and surface tension, and the velocity is not assumed to be irrotational. Specifically, we exhibit a newly developed unified approach through which we can prove the local well-posedness, incompressible limit, and zero surface tension limit in one attempt.

17 Natasa Pavlovic

The University of Texas at Austin, USA
Two tales of a rigorousd derivation of the Hamiltonian structure

Abstract

Many mathematical works have focused on understanding the manner in which the dynamics of a nonlinear equation arises as an effective equation.

- For example, the cubic nonlinear Schrodinger equation (NLS) is an effective equation for a system of N bosons interacting pairwise via a delta or approximate delta potential. In this talk, we will advance a new perspective on deriving an effective equation, which focuses on structure. In particular, we will show how the Hamiltonian structure for the cubic NLS in any dimension arises from corresponding structure at the N-particle level.

- On the other hand, the Vlasov equation in any spatial dimension has long been known to be an infinite-dimensional Hamiltonian system whose bracket structure is of Lie-Poisson type. In parallel, it is classical that the Vlasov equation is a mean-field limit for a pairwise interacting Newtonian system. Motivated by this knowledge, we provide a rigorous derivation of the Hamiltonian structure of the Vlasov equation, both the Hamiltonian functional and Poisson bracket, directly from the many-body problem. This work settles a question of Marsden, Morrison, and Weinstein on providing a "statistical basis" for the bracket structure of the Vlasov equation.

The talk is based on joint works with Dana Mendelson, Joseph Miller, Andrea Nahmod, Matthew Rosenzweig and Gigliola Staffilani.

18 Lorenzo Pareschi

University of Ferrara, Italy

Stochastic Galerkin particle methods for multiscale collisional plasmas with uncertainties

Abstract

The study of uncertainty propagation is of fundamental importance in plasma physics simulations. However, the construction of numerical methods is challenging due to the high-dimensionality of the problem, the presence of multiple space-time scales, and the constraints imposed by the need to preserve certain relevant physical properties. In the present talk we present a novel class of methods based on a stochastic-Galerkin (sG) approximation of the uncertain particles' position and velocity. We describe the method in the case of the Vlasov-Poisson system with a BGK term describing plasma collisions and for space homogeneous problems where collisions are characterized by the realistic Landau operator. We show that the sG particle method preserves the main physical properties of the problem, such as conservations and positivity of the solution, while achieving spectral accuracy for smooth solutions in the random space. Furthermore, in the fluid limit we discuss how to design the sG particle solver in order to possess the asymptotic-preserving property, thus avoiding the loss of hyperbolicity typical of conventional sG methods based on finite differences or finite volumes.

References

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[2] L. Pareschi, M. Zanella, Monte Carlo stochastic Galerkin methods for the Boltzmann equation with uncertainties: space-homogeneous case, J. Comp. Phys. 423, (2020), 109822

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[4] A. Medaglia, L. Pareschi, M. Zanella, Stochastic Galerkin direct simulation Monte Carlo schemes for the space-homogeneous Landau equation with uncertainties, preprint 2023

19 Benoît Perthame

Sorbonne Université, France
PDEs for neural assemblies; analysis, simulations and behaviour

Abstract

Neurons exchange informations via discharges, propagated by membrane potential, which trigger firing of the many connected neurons. How to describe large assemblies of such neurons? What are the properties of these mean-field equations? How can such a network generate a spontaneous activity? Such questions can be tackled using nonlinear integro-differential equations. These are now classically used in the neuroscience community to describe neuronal assemblies. Among them, the best known is certainly Wilson-Cowan's equation which describe spiking rates arising in different brain locations.

Another classical model is the integrate-and-fire equation that describes neurons through their voltage using a particular type of Fokker-Planck equations. Several mathematical results will be presented concerning existence, blow-up, convergence to steady state, for the excitatory and inhibitory neurons, with or without refractory states. Conditions for the transition to spontaneous activity (periodic solutions) will be discussed.

One can also describe directly the spike time distribution which seems to encode more directly the neuronal information. This leads to a structured population equation that describes at time t the probability to find a neuron with time s elapsed since its last discharge.

20 Luigi Preziosi

Politecnico di Torino, Italy
Modelling Cell Reorientation under Stretch

Abstract

The active response of cells to mechanical cues due to their interaction with the environment has been of increasing interest, since it is involved in many

physiological phenomena, pathologies, and in tissue engineering. In particular, several experiments have shown that, if a substrate with overlying cells is cyclically stretched, they will reorient to reach a well-defined angle between their major axis and the main stretching direction. The aim of this talk will be to investigate the interplay between mechanics and cell organization. It will be shown that cells organize their internal structure to minimize an elastic energy that then drives this reorientation process. Viscoelastic effects will then be included to explain the dependence of the appearance of the phenomenon as a function of oscillation frequency. Finally, randomness is taken into account and discussed on the basis of a Fokker-Plack equation.

21 Lenya Ryzhik

Stanford University, USA
Pushmi-pullyu fronts

Abstract

It is well-known that reaction-diffusion equations admit two types of front solutions: pulled and pushed. A typical example of the former are equations of the Fisher-KPP type, while the Allen-Cahn equation is an example of the latter. We will discuss some special properties of equations that lie at the transition from the pulled to the pushed type. This is a joint work with J. An and C. Henderson.

22 Markus Schmidtchen

Technische Universität Dresden, Germany
Excluded volume and order in systems of Brownian needles

Abstract

We consider a system of nonoverlapping Brownian needles in two dimensions. Unlike point particles, the needles' size and shape influence the system's evolution. We explore the effects of excluded volume and anisotropy at the population level. Since needles exclude less volume if aligned, can excluded-volume effects alone induce order in the system? Starting from the stochastic particle system, we derive a nonlocal nonlinear partial differential equation

for the population density using the methods of matched asymptotic expansions and conformal mapping. Finally, we show some numerical simulations and two model reductions.

23 Masahiro Suzuki

Nagoya Institute of Technology, Japan
Stability and instability of plasma boundary layers

Abstract

We investigate mathematically a plasma boundary layer near the surface of materials immersed in a plasma, called a sheath. From a kinetic point of view, Boyd–Thompson proposed a kinetic Bohm criterion which is required for the formation of sheaths. Then Riemann pointed out (although without a rigorous proof) that the criterion is a necessary condition for the solvability of the stationary Vlasov–Poisson system. Recently, Suzuki–Takayama analyzed rigorously the solvability of the stationary Vlasov–Poisson system, and clarified in all possible cases whether or not there is a stationary solution. It was concluded that the Bohm criterion is necessary but not sufficient for the solvability. In this talk, we study the nonlinear stability and instability of the stationary solutions of the Vlasov–Poisson system. The location of the support of the initial data is a major factor leading to stability/instability. This talk is based on a joint work with Professor M. Takayama (Keio Univ.) and Professor K. Z. Zhang (New York Univ.).

24 Eitan Tadmor

University of Maryland, USA
Swarm-Based Gradient Descent Method for Non-Convex Optimization

Abstract

We introduce a new swarm-based gradient descent (SBGD) method for non-convex optimization. The swarm consists of agents, identified with positions x and masses m. The key to their dynamics is transition of mass from high to lower ground, and a time stepping protocol, h(x,m), which decreases with m. The interplay between positions and masses leads to dynamic distinction

between 'leaders' and 'explorers'. Heavier agents lead the swarm near local minima with small time steps. Lighter agents, which explore the landscape by taking large time steps, are expected to encounter improved position for the swarm; if they do, then they assume the role of heavy swarm leaders and so on. Convergence analysis and numerical simulations demonstrate the effectiveness of SBGD method as a global optimizer.

25 Ariane Trescases

CNRS and Université Paul Sabatier, France
Models for chemotaxis with local sensing

Abstract

Chemotaxis is a process that drives cellular motility, which is ubiquitous in Biology. We present a class of parabolic systems that model chemotaxis with local sensing, that is, the cells respond to a certain concentration of chemoattractant perceived locally (as opposed to gradient sensing, when the cells are able to perceive a gradient of concentration). We study the well-posedness and long-time behaviour of these models.

26 Oliver Tse

Eindhoven University of Technology, Netherlands
Quantified overdamped limit for Kinetic Vlasov-Fokker-Planck equations

Abstract

The study of the overdamped limit for the kinetic Fokker-Planck equation has been of interest since the seminal work of Kramers in 1940, where he formally discussed the convergence by introducing a coarse-graining map. This talk is based on recent work with Young-Pil Choi, where we provide a framework to establish quantitative estimates for the overdamped limit of the kinetic Vlasov-Fokker-Planck with singular interaction in terms of the 2-Wasserstein distance.

27 Chiara Villa

Sorbonne Université, France
Evolutionary dynamics of glucose-deprived cancer cells: insights from experimentally-informed mathematical modelling

Abstract

Glucose is a primary energy source for cancer cells. Several lines of evidence support the idea that monocarboxylate transporters, such as MCT1, elicit metabolic reprogramming of cancer cells in glucose-poor environments, allowing them to reuse lactate, a byproduct of glucose metabolism, as an alternative energy source with serious consequences for disease progression. A more in-depth theoretical understanding of the evolutionary processes at the root of cancer cell adaptation to glucose deprivation can be achieved through analysis and numerical simulation of structured-population models. The focus of this talk is on an experimentally-informed mathematical model comprising a partial integro-differential equation for the dynamics of a population of cancer cells structured by the level of MCT1 expression. The results of the model calibrated with the experimental data used to inform it shed light on the mechanisms underlying the increase in MCT1 expression observed in glucose-deprived aggressive cancer cells during in vitro experiments.

28 Yao Yao

National University of Singapore, Singapore
Small scale formation for the 2D Boussinesq equation

Abstract

In this talk, we consider the 2D incompressible Boussinesq equation without thermal diffusion, and aim to construct rigorous examples of small scale formations as time goes to infinity. In the viscous case, we construct examples of global-in-time smooth solutions where the H^1 norm of density grows to infinity algebraically in time. For the inviscid equation in the strip, we construct examples whose vorticity grows at least like t^3 and gradient of density grows at least like t^2 during the existence of a smooth solution. These growth results work for a broad class of initial data, where we only require

certain symmetry and sign conditions. As an application, we also construct solutions to the 3D axisymmetric Euler equation whose velocity has infinite-in-time growth. This is a joint work with Alexander Kiselev and Jaemin Park.