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Abstracts

Tutorials on Multiscale Analysis and Methods
for Quantum and Kinetic Problems

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1 Blair Blakie

University of Otago, New Zealand
Introduction to dipolar quantum gases

Abstract

An introduction to the manybody physics of quantum degenerate atomic and molecular gases with long-ranged dipole-dipole interactions. This tutorial will discuss the basic theoretical tools used to describe the these systems and motivating experimental developments. The focus will be on bosonic gases, where Bose-Einstein condensation can occur. However, some comparisons to fermionic gases will also be examined. We will also explore the beyond-meanfield physics that can occur in this system leading to the emergence of quantum droplet states and supersolid states.

2 Yongyong Cai

Beijing Normal University, China
Numerical analysis for dispersive equations: from classical regime to oscillatory regime

Abstract

We shall discuss the common techniques for numerical analysis applied to the dispersive equations, including the nonlinear Schrödinger equation, nonlinear

Klein-Gordon equation and Dirac equation as typical examples. We will focus on the finite difference and the Fourier spectral discretization in space, while time-splitting, finite-difference and exponential integrators will be considered for time discretization. After presenting the analysis for typical equations in the classical regime, we will pay attention to the special regimes where the solutions exhibit oscillatory behavior, such as the non-relativistic limit of the Dirac equation and the long time dynamics of the weakly nonlinear Schrödinger equation.

3 Benoît Perthame

Sorbonne Université, France
Mathematical analysis of models for living tissues and free boundary problems

Abstract

Mechanical models of tissue growth are now well settled with continuous inputs from medicine, biology, physics, mechanics and mathematics, They contain several levels of complexity, both in terms of the biomedical content and mathematical description, from ordinary differential equations to sophisticated partial differential equations. They serve to predict the evolution of cancers in medical treatments, to understand the biological effects that permit tumor growth and control by treatment, in some cases, their implication in therapies failure.

Based on the mechanical point of view that a living tissue behaves as a porous media, this course aims at deriving, incompressible, free boundary problems departing from compressible models.

The specific questions that will be addressed are

- Aspects of tumor growth modeleld by differential equations
- Mechanical models of tissue growth
- The incompressible limit and the free boundary problems
- Models with multiple species, with surface tension

4 Alexander Ostermann

Universität Innsbruck, Austria
TBC

Abstract

5 Chunmei Su

Tsinghua University, China
Error estimates of splitting methods for the nonlinear Schrödinger equation

Abstract

Time splitting methods are very attractive and have been widely employed for solving the nonlinear Schrödinger equation since the linear kinetic flow can be solved (approximately) efficiently in phase space while the nonlinear flow can be solved exactly in physical space. Furthermore, high-order methods can be easily constructed. In recent years, the convergence of splitting methods has been investigated extensively. We shall give a tutorial introduction to the error estimates of splitting methods for the nonlinear Schrödinger equation.

This tutorial mainly consists of three parts:

1. We review some classical results and introduce some basic ideas to establish the error estimates based on Lady Windermere's fan.

2. To establish the error estimates of splitting methods with initial data of low regularity, we introduce the filtered Lie splitting and establish its convergence with the aid of discrete Strichartz type estimates.

3. Finally, we prove uniform in time error estimates for the filtered Lie splitting method for the nonlinear Schrödinger equation with a defocusing nonlinearity which is mass-supercritical and energy-subcritical. This uniformity in time is obtained thanks to a vector field which provides time decay estimates for the exact and numerical solutions.

6 Eitan Tadmor

University of Maryland, USA
Multiscale Analysis in Active Matter

Abstract

In this four-tutorial lecture series, we discuss the multi-scale phenomena which arise in collective dynamics, aggregation and chemo-tactic dynamics.

1. **Multi-flocks**. We study the multiscale description of large-time collective behavior of agents driven by alignment. The resulting multi-flock dynamics arises naturally with realistic initial configurations consisting of multiple spatial scaling, which peak at different time scales. We derive a master-equation which describes the formation of multi-flocks and the related dynamics of multi-species.

2. **Multi-species**. We study the hydrodynamics of multi-species driven by alignment. What distinguishes the different species is the protocol of their interaction with the rest of the crowd: different species employ different communication kernels with members of other species. We show that flocking of the overall crowd emerges provided the communication array between species forms a connected graph. The same methodology applies to multi-species aggregation dynamics governed by first-order alignment: connectivity implies concentration around an emerging consensus.

3. **p-alignment with pressure**. We study the swarming behavior of hydrodynamic p-alignment, based on 2p-graph Laplacians and weighted by a general family of symmetric communication kernels. This extends the classical alignment model corresponding to p=1. The main new aspect here is the long time emergence behavior for a general class of pressure tensors <u>without</u> a closure assumption, beyond the mere requirement that they form an energy dissipative process. We refer to such pressure laws as 'entropic', and prove the flocking of p-alignment hydrodynamics, driven by singular kernels with general class of entropic pressure tensors.

4. **Multi-species with pressure**. We extend these findings to systems of multi-species, proving their long-time flocking behavior for connected arrays of multi-species, with self-interactions governed by entropic pressure laws and driven by fractional p-alignment.

7 Yao Yao

National University of Singapore, Singapore
Symmetry and uniqueness via a variational approach

Abstract

For some nonlocal PDEs, its steady states can be seen as critical points of an associated energy functional. Therefore, if one can construct perturbations around a function such that the energy decreases to first order along the perturbation, this function cannot be a steady state.

In this series of tutorials, I will explain how this simple variational approach has led to some progress in the following equations in mathematical biology and fluid dynamics, where the key is to carefully construct a suitable perturbation. I will also discuss some remaining open questions in these directions.

The first application is the aggregation-diffusion equation, which is a nonlocal PDE driven by two competing effects: nonlinear diffusion and long-range attraction. We show that all steady states are radially symmetric up to a translation (joint with Carrillo, Hittmeir and Volzone), and give some criteria on the uniqueness/non-uniqueness of steady states within the radial class (joint with Delgadino and Yan).

The second application is the 2D Euler equation, where we aim to understand under what condition must a stationary/uniformly-rotating solution be radially symmetric. Using a variational approach, we settle some open questions on the radial symmetry of rotating patches, and also show that any smooth stationary solution with compactly supported and nonnegative vorticity must be radial (joint with Gómez-Serrano, Park and Shi).

8 Luca Alasio

Sorbonne Université, France
Towards a new mathematical model of the visual cycle

Abstract

The visual cycle is a fundamental bio-chemical process in the retina: it allows photoreceptors to convert light into electrical signals (phototransduction) and subsequently to return to the dark state. George Wald obtained the Nobel Prize in 1967 for his pioneering studies on this process and it has been an active field of research in Ophthalmology ever since. I will discuss the key aspects of the visual cycle in photoreceptor cells and present a new mathematical model for the visual cycle in rod cells. The model consists of a system of coupled ODEs and PDEs for the concentrations of relevant molecules and proceins in rod outer segments. The goal is to give a quantitative description of the kinetics of the main photo-sensitive molecules after exposure to light. I will explain how the model can be extended in order to account for the accumulation of toxic byproducts in the eye in connection to degenerative retinal diseases.

9 Cyril Tain

Université de Rouen Normandie, France

A boundary element method for the Time Dependent Ginzburg Landau 3D model

Abstract

We solve the Time Dependent Ginzburg Landau in 3D using a boundary element method (BEM); it is implemented with the free software FreeFEM interfaced with two libraries BEMTool and HTool; boundary integral operators are evaluated via a matrix compression technique. 3D experiments in the cube and the sphere are shown : we compare the BEM method with a mixed finite element scheme using uniform boundary conditions.

10 Teng Zhang

Beijing Computational Science Research Center (CSRC), China

Numerical methods for the biharmonic nonlinear Schrödinger equation

Abstract

In this talk, I will introduce several numerical methods for the biharmonic nonlinear Schrödinger equation (BNLS) and give the corresponding error estimates, including finite difference methods and spectral time splitting methods. BNLS is frequently used in the model of nonlinear optics, since the biharmonic operator can provide extra stability for soliton solutions, which provides a model for optical fibers with strong nonlinearity. The high dispersion term from the biharmonic operator brings out numerical burdens that require either large computational domain or high accuracy method. I will discuss the error estimates and properties of Crank-Nicolson finite difference method, semi-implicit finite difference method, and sine spectral time splitting method and then compare their advantages and disadvantages. Numerical examples to illustrate the dispersion relation and simulations on soliton collisions and 2D problems are also given.