

## Speakers

- 1 Blair Blakie
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# Abstracts

## Tutorials on Multiscale Analysis and Methods for Quantum and Kinetic Problems

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### 1 Blair Blakie

*University of Otago, New Zealand*

[Introduction to dipolar quantum gases](#)

Abstract

An introduction to the manybody physics of quantum degenerate atomic and molecular gases with long-ranged dipole-dipole interactions. This tutorial will discuss the basic theoretical tools used to describe these systems and motivating experimental developments. The focus will be on bosonic gases, where Bose-Einstein condensation can occur. However, some comparisons to fermionic gases will also be examined. We will also explore the beyond-meanfield physics that can occur in this system leading to the emergence of quantum droplet states and supersolid states.

### 2 Yongyong Cai

*Beijing Normal University, China*

[Numerical analysis for dispersive equations: from classical regime to oscillatory regime](#)

Abstract

We shall discuss the common techniques for numerical analysis applied to the dispersive equations, including the nonlinear Schrödinger equation, nonlinear

Klein-Gordon equation and Dirac equation as typical examples. We will focus on the finite difference and the Fourier spectral discretization in space, while time-splitting, finite-difference and exponential integrators will be considered for time discretization. After presenting the analysis for typical equations in the classical regime, we will pay attention to the special regimes where the solutions exhibit oscillatory behavior, such as the non-relativistic limit of the Dirac equation and the long time dynamics of the weakly nonlinear Schrödinger equation.

### 3 Benoît Perthame

*Sorbonne Université, France*

[Mathematical analysis of models for living tissues and free boundary problems](#)

#### Abstract

Mechanical models of tissue growth are now well settled with continuous inputs from medicine, biology, physics, mechanics and mathematics, They contain several levels of complexity, both in terms of the biomedical content and mathematical description, from ordinary differential equations to sophisticated partial differential equations. They serve to predict the evolution of cancers in medical treatments, to understand the biological effects that permit tumor growth and control by treatment, in some cases, their implication in therapies failure.

Based on the mechanical point of view that a living tissue behaves as a porous media, this course aims at deriving, incompressible, free boundary problems departing from compressible models.

The specific questions that will be addressed are

- Aspects of tumor growth modeled by differential equations
- Mechanical models of tissue growth
- The incompressible limit and the free boundary problems
- Models with multiple species, with surface tension

## 4 Alexander Ostermann

*Universität Innsbruck, Austria*

[TBC](#)

Abstract

## 5 Chunmei Su

*Tsinghua University, China*

[Error estimates of splitting methods for the nonlinear Schrödinger equation](#)

Abstract

Time splitting methods are very attractive and have been widely employed for solving the nonlinear Schrödinger equation since the linear kinetic flow can be solved (approximately) efficiently in phase space while the nonlinear flow can be solved exactly in physical space. Furthermore, high-order methods can be easily constructed. In recent years, the convergence of splitting methods has been investigated extensively. We shall give a tutorial introduction to the error estimates of splitting methods for the nonlinear Schrödinger equation.

This tutorial mainly consists of three parts:

1. We review some classical results and introduce some basic ideas to establish the error estimates based on Lady Windermere's fan.
2. To establish the error estimates of splitting methods with initial data of low regularity, we introduce the filtered Lie splitting and establish its convergence with the aid of discrete Strichartz type estimates.
3. Finally, we prove uniform in time error estimates for the filtered Lie splitting method for the nonlinear Schrödinger equation with a defocusing nonlinearity which is mass-supercritical and energy-subcritical. This uniformity in time is obtained thanks to a vector field which provides time decay estimates for the exact and numerical solutions.

## 6 Eitan Tadmor

*University of Maryland, USA*

[Multiscale Analysis in Active Matter](#)

### Abstract

In this four-tutorial lecture series, we discuss the multi-scale phenomena which arise in collective dynamics, aggregation and chemo-tactic dynamics.

1. **Multi-flocks.** We study the multiscale description of large-time collective behavior of agents driven by alignment. The resulting multi-flock dynamics arises naturally with realistic initial configurations consisting of multiple spatial scaling, which peak at different time scales. We derive a master-equation which describes the formation of multi-flocks and the related dynamics of multi-species.
2. **Multi-species.** We study the hydrodynamics of multi-species driven by alignment. What distinguishes the different species is the protocol of their interaction with the rest of the crowd: different species employ different communication kernels with members of other species. We show that flocking of the overall crowd emerges provided the communication array between species forms a connected graph. The same methodology applies to multi-species aggregation dynamics governed by first-order alignment: connectivity implies concentration around an emerging consensus.
3. **p-alignment with pressure.** We study the swarming behavior of hydrodynamic p-alignment, based on 2p-graph Laplacians and weighted by a general family of symmetric communication kernels. This extends the classical alignment model corresponding to  $p=1$ . The main new aspect here is the long time emergence behavior for a general class of pressure tensors without a closure assumption, beyond the mere requirement that they form an energy dissipative process. We refer to such pressure laws as ‘entropic’, and prove the flocking of p-alignment hydrodynamics, driven by singular kernels with general class of entropic pressure tensors.

4. **Multi-species with pressure.** We extend these findings to systems of multi-species, proving their long-time flocking behavior for connected arrays of multi-species, with self-interactions governed by entropic pressure laws and driven by fractional p-alignment.

## 7 Yao Yao

*National University of Singapore, Singapore*

[Symmetry and uniqueness via a variational approach](#)

### Abstract

For some nonlocal PDEs, its steady states can be seen as critical points of an associated energy functional. Therefore, if one can construct perturbations around a function such that the energy decreases to first order along the perturbation, this function cannot be a steady state.

In this series of tutorials, I will explain how this simple variational approach has led to some progress in the following equations in mathematical biology and fluid dynamics, where the key is to carefully construct a suitable perturbation. I will also discuss some remaining open questions in these directions.

The first application is the aggregation-diffusion equation, which is a non-local PDE driven by two competing effects: nonlinear diffusion and long-range attraction. We show that all steady states are radially symmetric up to a translation (joint with Carrillo, Hittmeir and Volzone), and give some criteria on the uniqueness/non-uniqueness of steady states within the radial class (joint with Delgadino and Yan).

The second application is the 2D Euler equation, where we aim to understand under what condition must a stationary/uniformly-rotating solution be radially symmetric. Using a variational approach, we settle some open questions on the radial symmetry of rotating patches, and also show that any smooth stationary solution with compactly supported and nonnegative vorticity must be radial (joint with Gómez-Serrano, Park and Shi).