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Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil

— *A satellite conference of the virtual ICM 2022*

From 1 to 15 July 2022, the Institute hosted a workshop on "Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil". The organizers contributed this invited article to Imprints.

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TOMASZ PRZEBINDA *(The University of Oklahoma)*
ANGELA PASQUALE *(Université de Lorraine)*
BINYONG SUN *(Chinese Academy of Sciences)*

The theory of representations of reductive groups over local fields is a profound and active area of research.

Chi-Tat Chong

It has found important applications to other areas of mathematics like number theory, harmonic analysis and to mathematical physics. It is also a crucial part of the Langlands program.

Two central questions in representation theory are to classify all equivalence classes of irreducible (unitary) representations and to decompose a representation into irreducible ones. Harish-Chandra and the Gelfand school made fundamental progress in this area in the 1960's and 1970's. Characters and matrix coefficients are among the most important analytic invariants of representations. Harish-Chandra's description of discrete series representations is based on the construction of their characters. Another useful method in understanding representations is to study dual pair correspondences via the Segal-Shale-Weil representations and the minimal representations. Despite the immense progress, there are still many interesting open questions.

A two week long conference on representation theory was held at Institute for Mathematical Sciences (IMS) from 1 to 15 July 2022. The date of the conference overlapped with the International Congress for Mathematicians (ICM) 2022. Through the efforts of the management of IMS, the conference became an overlay satellite conference of the virtual 2022 ICM. As one of



(From L-R) Marie-France Vigneras, Roger Howe and Toshiyuki Kobayashi

the six events allowing ICM speakers to give their talks in front of a live audience, the conference received greater prominence and attracted in-person and online participants.

The first three days were dedicated to series of two to three-hour tutorial lectures by Professor Roger Howe (Yale University and Texas A&M University), Professor Toshiyuki Kobayashi (University of Tokyo) and Professor Wee Teck Gan (National University of Singapore). Each lecturer presented an overview of his area of expertise targeted at postdoctoral fellows and graduation students. Both Howe's and Kobayashi's lectures were named as IMS Distinguished Visitor Lecture Series. Roger Howe gave a survey of the history and major results in the Theta Correspondences. This is the celebrated research program which he pioneered in the 1970's. Kobayashi's lectures touched on recent topics on proper group actions and their relation to representation theory. In his lectures, Wee Teck Gan interpreted the theta correspondence from the view point of quantization and the relative Langlands program. Then he proposed a framework to relate the characters of two representations in theta correspondence.

The highlight of the conference were the ICM lectures held in the afternoons from 4 July to 7 July. These included the ICM Emmy Noether Lecture by Professor Marie-France Vignéras (Institut de mathématiques de Jussieu – Paris Rive Gauche) on the modular representations of p-adic groups and on the local Langlands correspondences over fields of nonzero characteristics. Her program gives rise to new research directions and is gaining much attention in the mathematical community. It is going to be a major research topic in representation theory in the coming years.

Most ICM invited lecturers of Section 3 (Number Theory) and Section 7 (Lie Theory and Generalizations) gave their ICM lectures at this conference. They were Raphaél Beuzart-Plessis (Aix-Marseille University), Atsushi Ichino (Kyoto University), Tasho Kaletha (University of Michigan), Sug Woo Shin (The University of California, Berkeley), Yiannis Sakellaridis (Johns Hopkins University), Chen-Bo Zhu (National University of Singapore) and Binyong Sun (Zhejiang University), and Weiqiang Wang (University of Virginia). Evgeny Feigin (HSE University) delivered an online lecture. These are among the best researchers in representation theory and number theory, and their lectures were excellent. They gave an overview of the latest developments in their research area, described their results and presented some open problems.

In addition to the local participants from the National University of Singapore, the institute hosted 37 overseas visitors. Besides the tutorial lectures and ICM talks, there were 31 research talks delivered at IMS and 11 online talks. While many talks were given by established scholars, it is heartening to note that more than half of the speakers were young and promising researchers. The lectures spanned a wide range of topics: the orbit method and nilpotent orbits, character formulas of supercuspidal representations, character expansions of representations of p-adic groups, higher branching laws, conformally invariant differential operations, Fourier transform for tempered unipotent representations of p-adic groups, harmonic analysis on GLn over finite fields, Hecke algebras and theta correspondence over local and finite fields. It was a wonderful experience for the participants to meet each other, to learn recent results and to exchange ideas. All the lectures were broadcasted live over the internet and there were more than 200 registered online participants to view them.

The conference created an opportunity for researchers in representation theory to meet and to report on recent developments and new results. The organizers are pleased and grateful for the enthusiastic participation of colleagues from around the world. We would like to thank IMS for the financial and administrative support. We hope that this conference will promote greater sharing of views and ideas, inspire new research and strengthen collaborations.

Statistical Methods in Genetic/ Genomic Studies

3–14 JAN 2022

CO-CHAIRS:
Jialiang Li | National University of Singapore
Jin Liu | National University of Singapore
Indranil Mukhopadhyay | Indian Statistical Institute

This two-week workshop had 60 talks which included two talks under the IMS Distinguished Lecture Series by Xihong Lin (Harvard University, USA) on 3 January 2022 and Mark van der Laan (University of California, Berkeley, USA) on 10 January 2022.

Many novel statistical approaches for genetic/genomic studies were shared in the talks, which involved inference in domains where the genetic covariance structure is unknown and needs to be estimated from the data. The workshop also illustrated recent developments in methods for biobank-scale data, multi-platform genomics data, multi-omics QTL, single cell RNA sequencing data, non-normal and noisy genomic data, and spatial transcriptomics data. The participants have agreed that there is a growing need for a confluence of modern theory and large-scale genetic applications.

There were more than 500 participants which included researchers from many countries including USA, China, Singapore, India, UK, Italy, Malaysia, Australia, New Zealand, Netherlands, Canada, Saudi Arab, and Bangladesh.

Modelling and Numerical Simulation of Non-Equilibrium Processes Part 2

17–28 JAN 2022

CO-CHAIRS:
Zhenning Cai | National University of Singapore
Ruo Li | Peking University
Yanli Wang | Beijing Computational Science Research Center

The second part of the program was planned with seminar talks and a one-week workshop. focuses mainly on more precise modelling and simulations using kinetic equations directly. Most talks in the workshop are of high quality. For example, the machine learning approach is applied to build kinetic models to obtain a much better accuracy; the method of low-rank decomposition has gained a number of new applications; coupling of microscopic and macroscopic models has been further explored to derive the Unified Gas-kinetic Wave-Particle methods and the General Synthetic Iterative Scheme; new ideas proposed to build simpler non-equilibrium models; and exciting findings in the finite element method for fluid simulations.

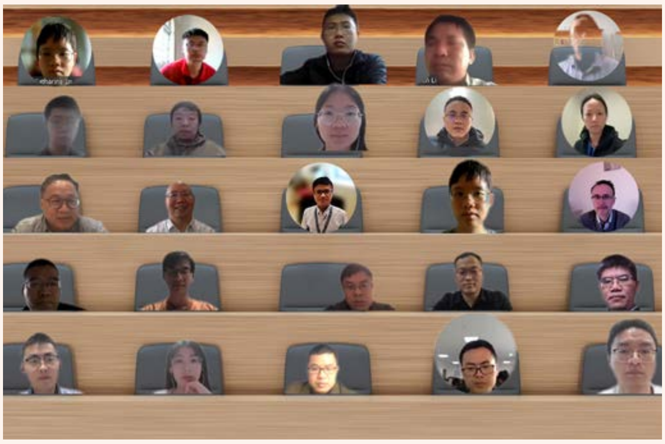
There were 150 participants, which included more than 70 graduate students.

Computation, Analysis and Applications of PDEs with Nonlocal and Singular Operators

4 FEB–4 MAR 2022

CO-CHAIRS:
Jie Shen | Purdue University
Li-Lian Wang | Nanyang Technological University

The program was planned with tutorial sessions and two workshops. The first workshop (7–11 February 2022) focused on progress and challenges for PDEs with nonlocal and singular operators, in particular on nonlocal modelling and simulation techniques for various problems arisen from engineering and science applications. A second workshop from 28 February to 4 March 2022 revolved around the recent advances and outlook of future directions. There were a total of 63 talks and more than 250 participants, which included close to 100 graduate students.



Interactions of Statistics and Geometry (ISAG)

14–18 FEB 2022

CO-CHAIRS:
Stephan Huckemann | Georg-August-Universität Göttingen
Ezra Miller | Nanyang Technological University
Zhigang Yao | National University of Singapore

The goal of the workshop was to develop statistical methods and theories capable of addressing contemporary challenges posed by real problems, particularly when it comes to fully exploiting combinatorial or algebraic or (stratified) differentiable structures. There were 22 talks over five days, each day of talks ending with a conversation session which allowed discussions between participants. There were more than 30 participants which included five PhD students.

Combinatorial Problems of Strings and Graphs and Their Applications in Bioinformatics Part 2

4–15 APR 2022

CO-CHAIRS:
Kwok Pui Choi | National University of Singapore
Sohel Rahman | Bangladesh University of Engineering & Technology
Vaibhav Rajan | National University of Singapore
Kunihiko Sadakane | The University of Tokyo
Wing-Kin Sung, Ken | National University of Singapore

Strings and graphs are fundamental data structures in mathematics and computer science and have many interesting mathematics properties. Sixteen experts in the domain of bioinformatics, mathematics and computer science gave talks on string data structures, genome assembly, phylogenetic, metagenomics and single-cell analysis. There were more than 30 participants which included more than 15 graduate students.

Random Matrix EurAsia 2022

18 APR–13 MAY 2022

CO-CHAIRS:
Yang Chen | University of Macau
Dong Wang | University of Chinese Academy of Sciences

This workshop has encompassed areas such as integrable systems, multivariate statistics, and probability. Problems related to multi-input-multi-output wireless communication were of interest to local NUS researchers. There were nine talks on 28 to 29 April 2022. The program then continued with tutorial lectures by the following speakers; Yang Chen (University of Macau, China), Anton Dzhamay (University of Northern Colorado, USA), András Mészáros (University of Toronto Scarborough, Canada), Sean O'Rourke (University of Colorado Boulder, USA) and Tomohiro Sasamoto (Tokyo Institute of Technology, Japan). Professor Van Vu (Yale University) gave a talk under the IMS Distinguished Visitor Lecture Series. There were in total 30 talks from 4 and 9–12 May 2022. There were more than 100 participants and more than 20 graduate students.

Interactive session with Oppenheim speaker Caucher Birkar (top left)

Oppenheim Lecture 2022 and Workshop on Recent Progress in Algebraic Geometry (13 May 2022)

Jointly organized with Department of Mathematics, NUS

The 2022 Oppenheim Lecture was delivered virtually by Professor Caucher Birkar (Tsinghua University, China). There were more than 70 participants. The workshop was held in conjunction with the Oppenheim Lecture and had three talks.

Algorithms and Foundations for Data Science

30 MAY–10 JUN 2022

CO-CHAIRS:
Yi Li | Nanyang Technological University
David P. Woodruff | Carnegie Mellon University

The workshop's theme focuses on the algorithmic foundations of fast data processing, which is crucialfor today's ever-increasing size of data. There were 23 talks, which presented new techniques for fundamental problems in data stream algorithms, and also addressed problems that arise in machine learning. This is the first workshop of its kind in Singapore, addressing the algorithms for massive data from the perspective of theoretical computer science. There were more than 80 participants which included close to 40 graduate students.

Research in Industrial Projects for Students (RIPS) 2022 - Singapore

30 MAY–29 JUL 2022

The industry sponsors were Google, Grab, Procter & Gamble (P&G) Singapore Innovation Center (SgIC) and the Ministry of Health (MOH) Office for Healthcare Transformation, Singapore. Fifteen undergraduate students were selected for the program.

Representations and Characters: Revisiting Some Aspects of the Works of Harish-Chandra and Weil

—A satellite conference of the virtual ICM 2022

1–15 JUL 2022

CO-CHAIRS:
Hung Yean Loke | National University of Singapore
Tomasz Przebinda | The University of Oklahoma
Angela Pasquale | Université de Lorraine
Binyong Sun | Chinese Academy of Sciences

This program was planned with three tutorial lectures (1, 2 and 4 July 2022) to make the talks more accessible to participants. Professor Marie-France Vignéras gave the ICM Emmy Noether Lecture on 4 July 2022. The conference was an satellite conference of the ICM 2022, and thus seven ICM talks (sections 3 number theory and section 7 lie theory and generalizations) were held at IMS from 4 to 7 July 2022.

Participants discussed on research in Representation Theory, as well as their interactions and applications in other domains, such as Number Theory, Analysis, Mathematical Physics. This meeting was devoted to Representation Theory of reductive groups over local fields and was focused on characters, matrix coefficients and branching rules. There were also 31 research talks, of which 11 were conducted via Zoom. There were more than 200 participants which included more than 50 graduate students.

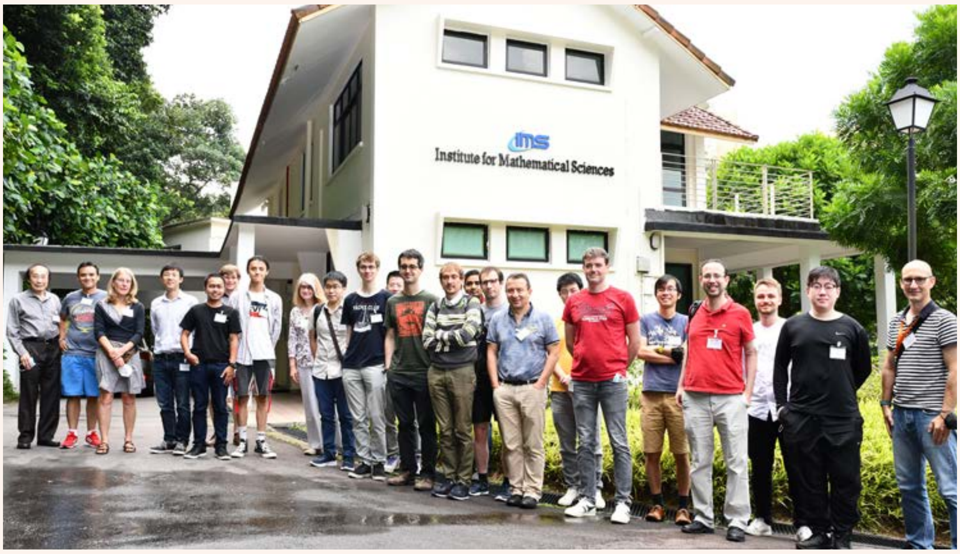
Stein's Method: The Golden Anniversary

13 JUN–8 JUL 2022

CO-CHAIRS:
Larry Goldstein | University of Southern California
Adrian Röllin | National University of Singapore

The program started with a workshop on stochastic analysis from 13 to 16 June 2022, and had nine talks. Activities continued from 22 to 24 June 2022 with a workshop on discrete probability and combinatorial structures and had eight talks. The third workshop on Statistical and Machine Learning (27 June to 1 July 2022) was planned with 17 talks.

The topics of the talks included number theory, analysis of neural networks, random graphs, Stein Variational Gradient Descent, Kernel Stein Discrepancy, high dimensional CLTs, stochastic geometry, and extreme value distributions. There were tutorials given by Murat A. Erdogdu (University of Toronto, Canada), Lester Mackey (Microsoft Research, USA), Nathan Ross (The University of Melbourne, Australia), Adil Salim (Microsoft Research, USA) and Matthias Schulte (Hamburg University of Technology, Germany). During the final open problem session week of the conference three research groups formed with the objective of making progress on problems that were chosen from a list of those proposed by the participants, a format inspired by the AIM style workshops. There were more than 100 participants.



A Tale of Rare Events — Symposium in Honour of Louis Chen on his 82nd Birthday

17–21 JUN 2022

CO-CHAIRS:
Adrian Röllin | National University of Singapore

The one-week symposium was a tribute to Professor Louis Chen on the occasion of his 82nd birthday and to celebrate his academic achievements. The one-week symposium was jointly organised by the Department of Mathematics, the Department of Statistics and Applied Probability, the Faculty of Science and the Institute for Mathematical Sciences.

IMS Graduate Summer School in Logic

4–20 JUL 2022

Tutorials were given by Ralf Schindler (University of Münster, Germany), Theodore A. Slaman (University of California, Berkeley, USA) and Isaac Goldbring (University of California, Irvine, USA)). Each speaker gave 12.5 hours of tutorials over five days. There were also six talks by the student participants. There were close to 50 participants.

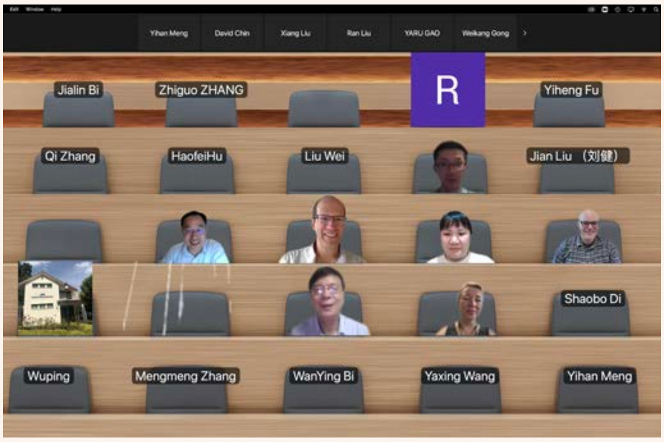
Applied Topology in Frontier Sciences

11–22 JUL 2022

CO-CHAIRS:
Jelena Grbic | University of Southampton
Fei Han | National University of Singapore
Jie Wu | Yanqi Lake Beijing Institute of Mathematical Sciences and Applications
Kelin Xia | Nanyang Technological University

The program started with a conference on Topology-based Learning, Biomolecular Topology and Related Topics from 11 to 15 July 2022, which had 23 talks. The first conference emphasized on applied topology, including DNA knot theory, mathematical virology, drug design, mathematical biology, etc.

The second week continued with a conference on Applied, Combinatorial and Toric Topology from 18 to 22 July 2022, which had 25 talks. This conference emphasized more on the topological theories or models that can be of great potential for application and invited speakers who are world-leading experts in topological data analysis (TDA), discrete topology, combinatorial topology, Toric topology, homological algebra, de Rhan-homology, etc. There were more than 100 participants which included more than 40 graduate students.



Asian School in Economic Theory 2022

25–29 JUL 2022

LOCAL ORGANIZER
Satoru Takahashi | National University of Singapore

This event was jointly organized with the Department of Economics, NUS and the Econometric Society. There were five lecturers; In-Koo Cho (Emory University), Jeff Ely (Northwestern University), Herve Moulin (University of Glasgow and Higher School of Economics, St. Petersburg), Marzena Rostek (University of Wisconsin-Madison) and Ariel Rubinstein (Tel Aviv University and New York University). Each lecturer delivered two 90 minutes lectures. Thirty-four students from overseas were selected for the summer school. There were seven sessions (90 minutes each) allocated for student presentations, which had a total of 21 talks.

Ng Kong Beng Public Lecture Series

Professor Jürgen Jost of the Max Planck Institute for Mathematics in the Sciences, Germany, delivered a public lecture entitled "Data and Mathematics, or Mathematics and Data" online via Zoom on 4 July 2022.

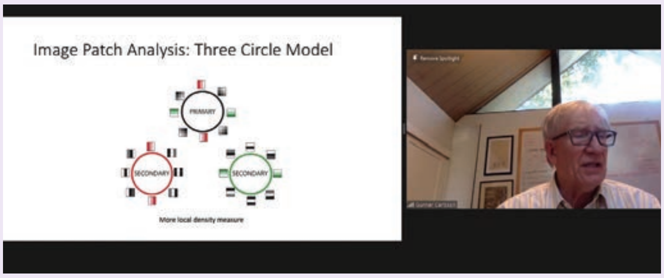
In his lecture, Professor Jost discussed the general picture of the relationship between data and mathematics, with a particular focus on the biological sciences. The talk started with a description of the use of algebraic structures for the classification of extensive collections of minerals. Throughout the lectures, borrowing examples from population genetics, biochemical kinetics, genetic sequences, phylogenies, network data, and 3D patterns, Professor Jost emphasized the role of Mathematics in the analysis of biological and other data. Common data structures such as intrinsic low-dimensionality, hierarchical organizations, or symmetries can all be exploited by modern Machine-Learning techniques for a better understanding of data. Professor Jost concluded his lecture by describing recent very successful applications of deep-learning techniques to the problem of predicting the 3D folding structures of proteins from their amino acid sequences.

A total of 167 people attended the lecture online.

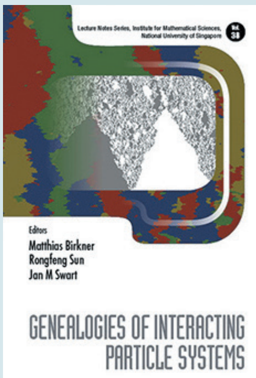
Gunnar Carlsson, Professor Emeritus and Ann and Bill Swindells Professor at Stanford University, delivered a public lecture entitled "Topological deep learning" on the 26th of July 2022 via Zoom.

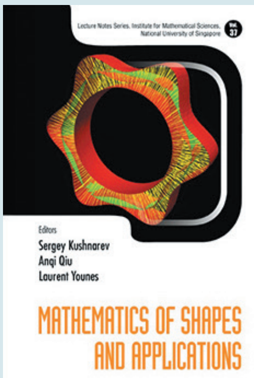
The overarching theme of Professor Carlsson's talk was the idea of leveraging topology to get some understanding of the internal working of deep neural networks. Although deep learning techniques have led to outstanding empirical successes for the analysis of complex data such as texts, videos, or molecular structures, issues such as adversarial examples and the general lack of transparency limit their usefulness in many heavily regulated industries such as finance and healthcare. By illustrating his points with a large number of applied problems, Professor Carlsson explained how the use of topological data analysis, an approach to the analysis of datasets using techniques from topology, can mitigate some of the drawbacks faced by standard deep-learning methodologies. In some challenging scenarios, this class of methods can lead to algorithms that are more accurate and can rely on a smaller amount of training data.

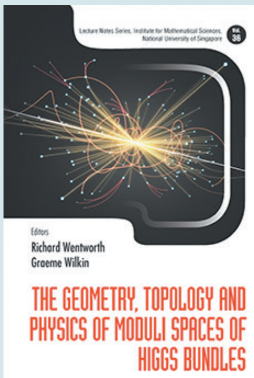
A total of 212 people attended the lecture online.

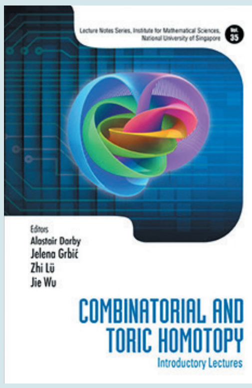


Lecture Note Series









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BẢO CHÂU NGÔ: FUNDAMENTAL LEMMA, SCIENTIFIC DISCOVERY 2009

Interview of Bao Chau Ngo by Y.K. Leong

Bao Chau Ngo[1] is well-known "for his proof of the Fundamental Lemma in the theory of automorphic forms through the introduction of new algebro-geometric methods."[2]

Ngo's rise to fame is a story of precocity coupled with the courage, determination and dedication of a budding mathematician from a developing country in the late 1980s. His personal story begins when his father who is a full professor of physics at the Vietnam National Institute of Mechanics enrolled him at age 15 in the special mathematics class at the High School for Gifted Students, Hanoi University of Science. His mathematical talent was immediately recognised when he became the first Vietnamese student (in grades 11 and 12) to bag two gold medals in the International Mathematical Olympiad in 1988 and 1989, with a perfect score of (42/42) in 1988.

Ngo was supposed to further his studies after high school in Hungary when political events of the early 1990s overtook his plans. The Soviet Union had disintegrated, and East European countries subsequently changed their political affiliation from communism to a more democratic system, thereby nullifying his scholarship agreement with Hungary. "When one door is closed, another door is opened", as Ngo said in this interview. At this point in time, the Secretary of the French Academy of Sciences, Paul Germain was visiting Hanoi and he arranged for a French government undergraduate scholarship for Ngo to study in Paris VI University. Then in 1992, Ngo moved to École Normale Supérieure to complete his BS in Mathematics and Computer Sciences and subsequently to Université Paris Sud where he obtained his PhD in 1997 under the supervision of Gérard Laumon, who gave Ngo a problem similar to the Fundamental Lemma (FL).

After his PhD, from 1998 to 2004, he was a Fellow of CNRS (*Centre National de la Recherche Scientifique*) at Université Paris Nord where he defended his habilitation degree in 2003, thus paving an academic pathway in France. In 2004 he became a professor at Université Paris Sud and also the youngest person to receive the title of professor in Vietnam at age 33. From 2007 to 2010, he was a long-term member of the Institute for Advanced Study at Princeton, where he finally completed the proof of the Fundamental Lemma in 2008 after an intensive period of 5-6 years of dedicated work on it, culminating in the award of the Fields medal in 2010 by the International Mathematical Union. For his achievement he was conferred French citizenship. In 2010 he joined the University of Chicago where he is now Francis and Rose Yuen Distinguished Service Professor. Since 2011 he has

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[1] Ngô Bảo Châu is the traditional way of writing his name in Vietnamese, "Ngô" being his family name and "Bảo Châu" his given name.
[2] Citation of the Fields Medal 2010

been concurrently the Scientific Director of the Vietnam Institute for Advanced Study in Mathematics in Hanoi, which was established in December 2011.

The mathematical story of Ngo's achievement can be traced back to 1967 when the Canadian mathematician Robert Phelan Langlands (Abel Prize 2015) put forward in a 17-page handwritten letter to the legendary mathematician André Weil[3], who was then a permanent member of the Institute for Advanced Study in Princeton, a bold vision for building a bridge connecting apparently unrelated concepts in number theory, algebraic geometry and the theory of automorphic forms. The contents of this letter were subsequently circulated among mathematicians in the 1960s and 1970s and came to be collectively known as the Langlands Program.

Then at the invitation of Marie-France Vignéras,[4] Langlands gave a lecture in 1976 on "*Les débuts d'une formule des traces stable*"[5] at the École Normale Supérieure. There Langlands sought to stabilise the Arthur-Selberg trace formula and he introduced the term "*Lemme Fondamental*" or the French term for "Fundamental Lemma" (FL) for a family of conjectural identities between orbital integrals on a reductive group over a local field and stable orbital integrals on the its endoscopic groups. Though FL is a cornerstone of his original program, Langlands had relegated its status to that of a "lemma" (lesser than that of a theorem), thinking that it would be resolved as a matter of course by some future graduate student. Meanwhile, more than two decades had passed and there was only partial progress on FL. On the other hand, FL has become pivotal to a host of results in number theory. So, there was a real bottleneck and roadblock in the progress of the Langlands Program despite the efforts of well-known mathematicians like Don Malcolm Blasius, Robert Mark Goresky, Thomas Hales, David Kazhdan, Robert Edward Kottwitz, Jean-Pierre Labesse, Robert Langlands, Gerard Laumon, George Lusztig, Robert Duncan MacPherson, Jonathan Rogawski,[6] Diana Shelstad, Jean-Loup Wadspurger, and Rainer Weissauer among others.

It was against this backdrop that Ngo found himself working on a problem related to FL for his doctoral dissertation. He tells us in the interview of the feeling that he had of his novel way of solving his PhD problem which might lead to a solution of FL. However, in the following 5-6 years after his PhD, the gregarious, unassuming, youthful looking Ngo worked on other more tractable problems in the Langlands Program, often in collaboration with others. However, still thinking about FL, together with his former PhD advisor, he proved in 2004 FL for unitary groups,[7] for which they were awarded the Clay Research Award. By then, Ngo had invested much effort in the FL problem. Having spent three years working on the problem from every possible angle, he realised that one step was missing. While the mathematical roadblock had caused him to be in a state of despair, he was undaunted by the magnitude of the problem. He trusted his intuition which was eventually vindicated by the final act in the drama of his discovery of the proof of FL.

The final act[8] was played out in 2006 in a serendipitous way when Ngo was at the Institute (IAS) conducting a seminar on FL during a three-month visit. He had asked Goresky something about perverse sheaves unrelated to FL, and what Goresky said immediately made him (Ngo) realize that it was the missing piece of the puzzle he was looking for. There was, however, a lot of details to be worked out after he returned to Paris-Sud from the Institute. Then in the fall of 2007, Ngo returned to IAS and presented his proof in a seminar. It was scrutinized by Pierre Deligne (Fields Medal 1978) and others, his paper going through six versions before it was made available online in 2008 and published in 2010.[9]

First, Ngo proved FL for Lie algebras and hence FL would follow from a result of Waldspurger that FL for Lie algebras implies FL for groups. It was a tour de force by Ngo in applying local and global methods in his interpretation of the orbital integrals in terms of the cohomology of the fibers of the Hitchin fibration.[10] The proof is based on a study of the decomposition of the l-adic cohomology of the Hitchin fibration into a direct sum of simple perverse sheaves.

Ngo is regarded as the person who "made one of the most important contributions to the theory of automorphic forms in the last few decades … by combining ideas from representation theory, arithmetic geometry, topology, singularity theory and mathematical physics."

As James Arthur has said in his laudatory address on the work of Ngo, "It will be clear that Ngo's proof is deep and difficult. What may be less clear is the enormous scope

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[3] André Weil (1915-1970)

[4] Interview, *Imprints* Issue 29, 18-23 (2017)

[5] Robert Langlands "*Les débuts d'une formule des traces stable*" *Publications de l'Université Paris 7*, 13 (1983)

[6] Jonathan Rogawski (1955 – 2011)

[7] Laumon, Gérard and Ngô Bao Chau (2004), *Le lemme fondamental pour les groupes unitaires*, arXiv:math/0404454. *Le leme fondamental pour les groupes unitaires*, *Annals of Math.* (2) 168 (2008), no. 2, 477–573.

[8] https://www.ias.edu/idea-tags/fundamental-lemma
Kelly Devine Thomas, "*The Fundamental Lemma: From Minor Irritant to Central Problem*", 2010.

[9] "*Le lemme fondamental pour les algèbres de Lie,*" by Bao Châu Ngô, *Publications Mathématiques de l'IHÉS 111* (2010), 1–169

of his methods. The many diverse geometric objects he introduces are all completely natural. That they so closely reflect objects from the trace formula and local harmonic analysis, and fit together so beautifully in Ngo's proof, is truly remarkable."[11]

Never in the history of mathematics has a mathematical "lemma" been considered so "fundamental" and important that Ngo's proof of FL is voted as one of the top ten scientific discoveries of the year 2009 by *Time* magazine, even though the statement of FL is only comprehensible to the specialist in the fields of algebraic geometry and number theory.

Ngo has been invited to give the following lectures: Distinguished Ordway Lectures (University of Minnesota), Erwin Schrödinger Lectures (University of Vienna), Takagi Lectures (Tokyo University), Oppenheim Lectures (National University of Singapore), Chern Lectures (University of California at Berkeley), Leonard da Vinci Lectures (University of Milan), Fields Symposium Lectures (Fields Institute, Toronto), Plenary Address (International Congress of Mathematicians in Hyderabad, 2010), Eilenberg Lectures (Columbia University), Hahn Lectures (Yale University) and a Sectional Address (International Congress of Mathematicians in Madrid, 2006).

In addition to the Fields Medal, his mathematical contributions have been recognised by the award of the Oberwolfach Prize and Sophie Germain Prize as well as election as member of the *Académie des Sciences de Paris* and American Academy of Arts and Sciences and Fellow of the American Mathematical Society. Ngo has been helming the direction of mathematical development in his home country since his appointment as Scientific Director of Vietnam's Institute for Advanced Study in Mathematics in March 2011. He has been actively engaged in outreach activities for high school students in Vietnam, and even in Singapore. For instance, in 2022 he was a speaker at this year's Global Young Scientists Summit (GYSS), which took place virtually from 17-21 January 2022, and was organised by Singapore's National Research Foundation (NRF). He has helped to promote awareness in mathematics by writing books and articles in Vietnamese at a popular level since 2012. He has actively participated in independent research groups in higher education. On the professional side, he has been on the editorial boards of *Mathematische Zeitschrift* and *Inventiones Mathematicae* and is currently an editor of *Compositio Mathematica*, *Grundlehren der mathematischen Wissenschaften*, *Acta Mathematica Vietnamica* and *Vietnam Journal of Mathematics*. He has also served on the Scientific Advisory Board of the Institute for Mathematical Sciences (IMS), NUS since 2019.

On 28 January 2015, Ngo was invited to give the Oppenheim Lecture at the NUS Department of Mathematics. In his talk "On the average rank of elliptic curve over function fields", he proved a similar result for function fields of some recent ground-breaking work of Manjul Bhargava[12] and Arul Shankar by using a geometric setup similar to FL. In conjunction with the lecture, a workshop on representation theory and automorphic forms was conducted from 27-29 January 2015, ending with a discussion with Professor Ngo from 3.00-5.00 pm. He was back in NUS from 6-18 January 2019 as Distinguished Visitor of the IMS program "On the Langlands Program: Endoscopy and Beyond" (17 December 2018–18 January 2019), and gave two talks "On the Braverman-Kazhdan program" (9 January 2019) and "On the Hitchin fibration for algebraic surfaces" (16 January 2019). It was during this visit that Y.K. Leong took the opportunity to interview him on 17 January 2019 on behalf of *Imprints*. The following is an edited and vetted version of the transcript of the interview, in which he talks about his early start in mathematics from the mathematical Olympiads to his formative years in Paris, the geometric intuition that underlies his research and some of his thoughts about the development of mathematics in Vietnam.

Acknowledgement. Y.K. Leong would like to thank Chee Whye Chin of the Department of Mathematics, NUS for preparing a raw copy of the transcript of the interview.

IMPRINTS I I believe you were originally supposed to study mathematics in Hungary, but destiny has it that you went to France instead. Also, you were offered a scholarship to study at l'Université de Paris VI but instead you went to the École Normale Supérieure and then to l'Université de Paris Sud. Could you tell us how this came about?

BẢO CHÂU NGÔ N It's a long and complicated story with a lot of unexpected turns. Before 1990 most students in Vietnam go to the Soviet Union or East European countries to study. I was supposed to go to Hungary, and I studied in Hungary for one year. But that was the year when the Berlin Wall collapsed and many countries in Eastern Europe changed their political direction and as a result, the agreement between Vietnam and Hungary was no longer valid. I lost my scholarship and I had no possibility to study in Hungary. But when

[10] The Hitchin fibration is an abstract geometric analogue of the Hitchin system of complex algebraic geometry, first used in mathematical physics by Nigel Hitchin.
[11] *Proceedings of the International Congress of Mathematicians*, Hyderabad, India, 2010
[12] Fields Medal 2014

one door is closed, another door is opened. There was a French professor (Professor Paul Germain) who came to Vietnam, and it just happened that they worked very hard to get me a scholarship to go study in France. After that, I studied in Université Paris VI for the first year because one of the professors who organised my going out of Vietnam was from Paris VI. So, I studied in Paris VI for one year. But then the professor from Paris VI thought that I was a good student and that I should go to study at École Normale Supérieure. They recommended me to École Normale. That's how I ended up at École Normale in my second year in France. And after studying two years in École Normale, the director of the math department, Professor Michel Broué advised me to go study with my PhD advisor Professor Gérard Laumon. That was how I ended up studying algebraic geometry and Langlands program in Orsay with Professor Laumon.

I **Your father is a physicist, and your work on the fundamental lemma uses an abstract geometric analogue of the Hitchin system in mathematical physics. Did you ever consider choosing to do research in physics?**

N Not really. I wasn't so much attracted by physics when I was younger. Then I became more and more interested in physics. My father is a physicist, but he's doing mechanics of fluids, not the same kind of physics. And it just so happened that this moduli space that I've been studying with arithmetic applications was first constructed and considered in mathematical physics by Professor [Nigel] Hitchin at Oxford University.

I **You won two gold medals at the International Mathematical Olympiad. And you once said that many Vietnamese mathematicians have been IMO medallists. Does that mean that training for the IMO plays an important role in the mathematical development of Vietnam?**

N I think it is, for good and bad reasons. It is a fact that most of the established Vietnamese mathematicians were former IMO students. I think the fact that having received this intensive training to perform in IMO keeps people interested in mathematics a long, long way. There are some not so good aspects because it took us, or it took me a while to move beyond elementary mathematics to real mathematics. But it does some good for somebody to be very persistent, to try very hard to solve problems. Yeah, the IMO plays an important role in the mathematical development in Vietnam; on one hand because it attracts these gifted kids who are interested in mathematics (my case, in particular). Also, it plays a role somehow in the public image of mathematics. So, the bad reason is that in Vietnamese society, IMO is a kind of sports in which it is doing well and therefore society should support mathematics more. Maybe this is less the case now, but it was so until ten years ago.

I **I remember that the Vietnamese team used to do very, very well [in the IMO].**

N They're still doing well. It's kind of random: In a good year, it can be in third position. In bad years, it can be in tenth position, but somehow it varies between 3 and 10. Very good for a country like Vietnam.

I **Do you think that the IMO training has given you the ability to solve problems?**

N I think so. Of course, I may have been gifted for doing that, but the training gave me more aptitude to solve problems.

I **I think you once mentioned that you were at first interested in combinatorics.**

N That's right. That was the influence of the Olympiad. The Olympiad was very much about combinatorics. That's why I was very much interested in combinatorics, and I wanted to go to study in Hungary which, you know, has some of the best combinatorists in the world

I **Before you were awarded the Fields Medal in 2010, a French mathematician Laurent Lafforgue was awarded the Fields' medal also for work in the Langlands program. Has there been some kind of collective effort in France to work on the Langlands program?**

N Yes and no. I mean, Laurent Lafforgue and I are very good friends. We have the same advisor; so, yeah, it's not a coincidence that we both have won Fields medals because we had an excellent advisor, Gérard Laumon. And also, you know, the Langlands program has been a big topic so far. There would be a seminar on Langlands program and we've done it every week for the last thirty years [in Paris]. And students would go there, listen to the talks. At the beginning, you don't understand much, but after a while it becomes very much in your consciousness.

I **Is this collective effort still going on in France?**

N Yes, pretty much so.

I **I believe [Roger] Godement[13] was . . .**

N Godement is the one who brought Langlands program to France. Godement was very influential with the young mathematicians at that time, people like my teachers and a lot of his friends. Maybe some of his

[13] Roger Godement (1921-2016)

students went on to study other things but they were very much influenced by Godement.

I **As I mentioned in an earlier interview with Cédric Villani,[14] who was also awarded the Fields Medal in 2010, the same year as you, France has produced a disproportionately large number of Fields medallists, second only to the United States. Is there any difference in the style or the way mathematical research is done in France and in the United States?**

N Probably, yes, I think so. France has this very singular system with CNRS (*Centre National de la Recherche Scientifique*) where very early on, mathematicians have permanent positions and no teaching duties. They could just stay there for as long as they want. Maybe the salary is not so great, but in mathematics you don't need a big salary, you just need to have complete freedom to do mathematics. And they have complete freedom to do the research they want. You don't have this pressure to publish as in the United States. So, that helps the young people. who are very ambitious to pursue something very ambitious or long projects, as opposed to (the situation) in the US, where you have to maybe choose some more modest goals in order to publish to keep their positions. In France, you don't have this pressure. The young people [in France] are more encouraged to pursue bigger goals than in the US.

I **In the US, it's either publish or perish.**

N In the US, you either publish or perish. After the PhD, you have a 3-year postdoc, you publish papers, you move on to assistant professorship, and you keep publishing for around 10 years. A lot of pressure. In France there is pressure too, but in a different way.

I **It must be very difficult to get selected by CNRS.**

N It is, it is, but there are positions. There are five positions every year.

I **It is quite surprising that France has produced something like 12 Fields medallists whereas the US has only 13, just one more according to my latest count. Even Russia is behind France. It's amazing! Were you already fluent in French before you went to France?**

N No, I learned French when I first come to France.

I **It must have been quite tough.**

N Yes, it was tough.

I **Many of the mathematical objects you used are basically geometric in nature. Would you consider yourself to be a geometer by disposition?**

N Yes, I do. Even though I worked in the Langlands program, my basic training and my basic way of thinking is how geometers think about space.

I **Do you think in terms of pictures?**

N Yeah, a little bit. I mean, it's very difficult to represent a picture in high dimensions, but I do think about pictures, and I do think about how some things move, how shapes move, deform, and so on. To get some kind of intuition before doing any calculation, I start with some kind of geometric intuition about what's going on.

I **I think in the Langlands program, there's a lot of interplay between geometry and algebra and analysis.**

N At the beginning it is very much about number theory and analysis, but more and more there's a lot of geometries. You know, there is a lot of structures that are governed by the peculiar geometries related to the Langlands program.

I **But the methods are all algebraic, mostly, I mean.**

N They can be algebraic, but not all of them. There's some analysis too.

I **In some sense, you started working on the fundamental lemma from the beginning of your PhD thesis until your proof of the fundamental lemma, for a period of more than 10 years. It seems that Lafforgue spent six years working on his solution of some conjectures in the Langlands program. How do you go about working on hard problems?**

N Yeah, yeah. It's true that for my PhD thesis, Gérard Laumont gave me a problem that is related to the fundamental lemma; it's kind of similar problem. At that time, I found a completely new way to address this kind of problems, and that gave me hope that maybe one day I could address this big fundamental lemma, which I did. But it's a very long [story]. I mean, the whole thing started from my PhD in 1993 and then maybe I finished [the proof of] the fundamental lemma in 2008-2009 (I don't remember well now), but it's some 16 years' span. But it's not continuous [work]. After my PhD, I spent something like four or five years doing another thing in the Langlands program, but not the fundamental lemma. And then after I had finished the other one, I came back to that problem [fundamental lemma] that I was dreaming about, and basically, I quit on the other projects to

[14] Interview, *Imprints* Issue 29, 10-17 (2017)

concentrate on this problem. It was a very conscious effort. I just informed my collaborators when I stopped working with them, and I just chose my own problem. There was a period of some five or six years of total concentration on this problem. And so did Laurent Lafforgue; he was working on his problem intensely for six years.

I think for a very hard problem, you need to start with some new idea… well, of course, it's some kind of feeling that maybe you have another new angle on top of that of other people, who are smarter than you. If you don't have a new angle to begin with, there's no chance for you to succeed where other people didn't succeed. You need to have a completely new angle. And one problem leads to another problem; you just need to think again and again about this every day … (for me, six years or more, every day). And also to constantly exchange ideas and get advice from other people. That helps a lot.

I Andrew Wiles spent a number of years on his own in isolation, but I think in your case, you actually had a lot of interactions with other people.

N I had a lot of interactions. Yeah. I work most of the time alone, but I like to talk with a lot of people. I don't keep it secret, and I keep my colleagues informed about my progress and then I got a lot of help.

I When you discovered the solution for the fundamental lemma, was it in the Institute for Advanced Studies in the US?

N Yes.

I Your proof of the fundamental level was considered by *Times* magazine to be one of the top 10 scientific discoveries of 2009. Would you agree to have your proof called a scientific discovery?

N I think so, yes, although the fundamental lemma itself is not a discovery. It's known before; Langlands knew about the statement 30 years before. But the proof [of the fundamental lemma] is itself a discovery – how everything is related to Hitchin fibrations, how the geometric Hitchin fibration gives us the hint and the argument to prove the fundamental lemma. I think this is very much a discovery.

I The Hitchin fibration was already considered by you even earlier than that, in nineteen something …

N No, no. It's done around 2003. Well, I mean, when I was doing my PhD thesis, I had some kind of vague strategy how to prove the fundamental lemma, but to apply it, I needed a space, some kind of space, a geometric construction, where I can draw on this strategy, but I didn't have that space then. But in 2003 after a while, I realized that the Hitchin fibration was the space I was looking for. That's why I could reformulate the whole conjecture in some entirely new way, in the geometry of Hitchin fibrations. And that way, it was telling me that I was maybe onto something big. And I quitted working on other things to work on this.

> **"MATHEMATICS AND THE LANGLANDS PROGRAM, IN PARTICULAR, HAS GREAT BEAUTY AND IT IS VERY SATISFYING TO WORK ON DIFFERENT PROBLEMS TO EXPAND HUMAN KNOWLEDGE WHICH, IN MY MIND, IS BEAUTIFUL TO UNCOVER."**

I You have mentioned that you're now working on some problems in physics.

N No, I'm interested in but I'm not working on any problem in physics.

I No? Not even some abstract form, mathematical form (of physics)?

N Well, I mean, there are many aspects of Langlands program related to physics: Hitchin fibration, the dual Hitchin fibration… There are many intersections with topics in physics but, you know, I don't have enough physics background to think in those terms. I'm aware of the connections with physics.

I I think somebody once compared the Langlands program to the Grand Unification Theory in physics.

N Yeah, yes. I don't know much about it, but I think it is a surprise that in the Langlands program the dual groups that are invented appear somehow in a different way in theoretical physics, in electromagnetic dualities.

I I think in the Grand Unification Theory of physics, one of the central problems is about how to connect gravity with quantum mechanics. Is there a similar kind of central problem in the Langlands program similar to this one?

N There may be, but I'm not aware of it.

I While you are a faculty in the University of Chicago, you are also the scientific director of the Vietnam Institute for Advanced Study in Mathematics. How do you chart the development of mathematics in Vietnam?

N I have been involved in the development of mathematics in Vietnam for a while since 2002-2004, giving lectures every year, and since my Fields Medal [in 2010] as the scientific director of the Vietnam Institute for Advanced Study in Mathematics. I think that the birth of this institute has been a great stimulus for mathematics in Vietnam. They get more funding, they get better organization, people organizing more research projects and getting more connected. There is more vitality. You know, if you just look at the number of publications [by Vietnamese mathematicians] in the ISI [International Scientific Indexing] journals, it has been multiplied by three times. It may not mean much, but to people in the [Vietnamese] Ministry, it means something. There's really good development. So, we manage to attract young people to come back to Vietnam. It's globally very positive, but on the other side, we are facing new problems. We have less students studying mathematics now.

I **Are there many Vietnamese students who go overseas nowadays?**

N There's a lot of Vietnamese students who go overseas, but you have less and less students who choose to study mathematics. It's a very serious problem. We have a very good high school system and a lot of them are very gifted in mathematics, but many of them go abroad to study. Those who stay in Vietnam do not study mathematics.

I **How many of them go back to Vietnam?**

N I don't have the numbers, but not many of them go back [to Vietnam].

I **Your achievements are undoubtedly a source of inspiration to students in Asia. What is your advice to students who wish to do research in mathematics in general and on problems in the Langlands program in particular?**

N Well, you know, I hate giving advice to people. I just want to say that mathematics and the Langlands program, in particular, has great beauty and it is very satisfying to work on different problems to expand human knowledge which, in my mind, is beautiful to uncover.

CALL FOR PROPOSALS

The Institute for Mathematical Sciences (IMS) of the National University of Singapore (NUS) invites submissions of proposals from researchers in academia and industry. The proposals are for organizing thematic programs or workshops to be held at IMS.

The IMS is particularly interested in receiving proposals of programs/workshops that focus on exciting new developments in the mathematical sciences. Proposals of interdisciplinary nature in areas that interface mathematics with science, social science or engineering are welcome.

A soft copy of the proposal, for the period of funding from **1 July 2024 to 30 June 2025**, should be sent to the Director of the Institute at imsdir@nus.edu.sg by **31 May 2022.**

The exposition of a proposal should be aimed at the non-specialist and will be evaluated by a scientific panel. Proposals of interdisciplinary programs/workshops should describe how the activity would benefit the intended audience with diverse backgrounds and facilitate research collaboration.

Information on the Institute and its activities, as well as a detailed format for the proposal are available on the IMS website ims.nus.edu.sg. Enquiries may be directed to imssec@nus.edu.sg.

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