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Abstracts

Workshop on representation theory of symmetric groups and related algebras

(12–16 Dec 2022)

1 Susumu Ariki

Osaka University, Japan

Tau-tilting finite block algebras of Hecke algebras

Abstract

Tau-tilting finite algebras are those finite dimensional algebras which enjoy the properties such as the number of isomorphism classes of Schurian modules is finite, the number of wide subcategories of its module category is finite, etc. Representation-finite algebras are tau-tilting finite, but the converse does not hold in general. After explaining the tau-tilting finite algebras and the chamber structure, we examine which block algebras of Hecke algebras are tau-tilting finite. Work by Ariki-Speyer and Lyle-Speyer give a complete answer for block algebras of type A Hecke algebras under the assumption that the quantum characteristic is greater than or equal to three, and I will report some partial results for block algebras of type B Hecke algebras, which are obtained in an on-going project with Liron Speyer and Qi Wang.

2 Christopher Bowman

University of York, UK

What has p-Kazhdan—Lusztig theory done for us?

Abstract

We will discuss recent results in modular Lie theory obtained in the setting of "diagrammatic Hecke categories". We will discuss the Kazhdan—Lusztig

positivity conjecture, the counter examples to Lusztig's conjecture, and how p-Kazhdan—Lusztig polynomials can be seen to control the representation theory of braid groups, cyclotomic Hecke algebras, and algebraic groups. We also discuss when these polynomials can and cannot be explicitly calculated.

3 Stephen Donkin

University of York, UK

On the conjugation action for quantum general linear groups.

Abstract

The quantum general linear groups are related to the Hecke algebras of type A in the same way that the general linear groups are related to the symmetric group algebras. Here we study the action of the general linear group G(n) on its coordinate algebra k[G(n)], at a non-zero parameter q. We give analogues of theorems of Kostant and Richardson (and the speaker) obtained in the classical case (q=1) describing the module structure over the algebra of invariants. We also consider, more generally, the invariants of the coordinate algebra of a quantum group G for the conjugation action by a (quantum) subgroup H. This is approached via finite dimensional subcoalgebras of k[G] and the theory of quasi-hereditary algebras.

4 Stephen Doty

Loyola University of Chicago, USA

Symmetric groups, increasing and decreasing subsequences, and canonical bases for centralizer algebras

Abstract

I will survey recent results linking the ideas in the title. The centralizer algebras in question are relative to the action of (a suitable form of) the enveloping algebra of the general linear Lie algebra and the partition algebra. The canonical bases consist of (images of) permutations satisfying combinatorial conditions. The Robinson–Schensted correspondence and the Kazhdan–Lusztig bases of Iwahori–Hecke algebra are lurking in the background.

5 Jie Du

University of New South Wales, Australia

The i-quantum groups $\mathbf{U}^\jmath(n)$ and $\mathbf{U}^\imath(n)$

Abstract

When I. Schur used the symmetric group representations to determine polynomial representations of the complex general linear group $GL_n(\mathbb{C})$, certain finite-dimensional algebras, known as Schur algebras, played a bridging role between the two. The well-known Schur duality summarizes the relation between the representations of $GL_n(\mathbb{C})$ and $\mathfrak{S}_r$. Over almost a hundred years, this duality has profoundly influenced representation theory and has evolved in various forms such as the Schur-Weyl duality, Schur-Weyl-Brauer duality, Schur-Weyl-Sergeev duality, and so on. In this talk, I will discuss a latest development, which I call the Schur-Weyl-Hecke duality, by Huanchen Bao and Weiqiang Wang.

Based on joint work with Yadi Wu, I will focus on the investigation of the i-quantum groups $U^\jmath(n)$ and $U^\imath(n)$ and their associated q-Schur algebras $\mathcal{S}^\jmath(n,r)$ and $\mathcal{S}^\imath(n,r)$ of types B and C, respectively. This includes short (element) multiplication formulas, long (element) multiplication formulas, and triangular relations in $\mathcal{S}^\jmath(n,r)$ and $\mathcal{S}^\imath(n,r)$. We will also give realisations of Beilinson–Lusztig–MacPherson type for both $U^\jmath(n)$ and $U^\imath(n)$ and discuss their Lusztig forms. This allows us to link representations of $U^\jmath(n)$ and $U^\imath(n)$ with those of finite orthogonal and symplectic groups.

6 Haralampos Geranios

University of York, UK

On self-extensions of irreducible modules for symmetric groups

Abstract

We work in the context of the modular representation theory of the symmetric groups. A long-standing conjecture, from the late 80s, suggests that there are no (non-trivial) self-extensions of irreducible modules over fields of odd characteristic. In this talk we will highlight several new positive results on this conjecture. This is a joint work with S. Kleshchev and L. Morotti.

7 Adam Higgins

University of York, UK
Endomorphisms of Specht Modules in Characteristic 2

Abstract

We will start with a survey of the (in)decomposability of Specht modules in general characteristic, and in particular the important role that endomorphisms play, thereby highlighting why this remains an open problem in characteristic 2. Then, taking inspiration from the representation theory of algebraic groups, we will present a new identification of the endomorphism algebra of a Specht module, along with some combinatorial tools that may assist with applying this identification. Finally, as a proof of concept, we discuss how this identification allows us to show that a certain broad infinite family of Specht modules have trivial endomorphism algebras, and so are indecomposable. This is joint work with Haralampos Geranios.

8 Berta Hudak

Okinawa Institute of Science and Technology, Japan
Representation type of level 1 KLR algebras $R^{\Lambda_k}(\beta)$ in type $C_\ell^{(1)}$

Abstract

Building on the work of Ariki and Park, we determine the representation type of the quiver Hecke algebras R^{Λ_k} for all k in type $C_\ell^{(1)}$. This is joint work with Chris Chung.

9 Stacey Law

University of Cambridge, UK
Sylow branching coefficients for symmetric groups

Abstract

One of the key questions in the representation theory of finite groups is to understand the relationship between the characters of a finite group G and its local subgroups. Sylow branching coefficients describe the restriction

of irreducible characters of G to a Sylow subgroup P of G, and have been recently shown to characterise structural properties such as the normality of P in G. In this talk, we will discuss and present some new results on Sylow branching coefficients for symmetric groups.

10 Stuart Martin

University of Cambridge, UK
Gram determinants of type A_n webs

Abstract

The category of representations of $\mathrm{U}_q(\mathfrak{sl}_n)$ can be realised diagrammatically by the 'web' or 'spider' category. This has a cellular structure, so we can study the cell modules. Such modules are equipped with a bilinear form, and it is the determinant of this form that we will compute in this talk. This relates to a 'triple clasp' conjecture of Ben Elias, a recent proof of which we will also mention. The techniques used are diagrammatic and should find applications to other monoidal cellular categories. This is joint work with Robert Spencer.

11 Eoghan McDowell

Okinawa Institute of Science and Technology, Japan
Determination of characters by their values on p'-classes

Abstract

Does knowing its values on p'-classes suffice to identify an ordinary irreducible character of a group? Here, a p'-class means a conjugacy class of elements of order not divisible by a prime p. Equivalently, are the rows of the p-modular decomposition matrix distinct? Wildon showed in 2008 that the answer for symmetric groups is "yes" except for conjugate pairs when p=2. In this talk I will discuss my recent work on these questions for the alternating groups. I show, for p not 3, the answer is again "yes" – that is, all characters of the alternating groups are uniquely determined by their values on the p'-classes, and the rows of the decomposition matrix are distinct – except for the pairs labelled by self-conjugate partitions with a diagonal hook length divisible

by p. I show similar results for the double covers of the symmetric and alternating groups. I also demonstrate that the claim does not hold when p=3 by exhibiting two additional infinite families of pairs of characters which agree on 3'-classes.

12 Hyohe Miyachi

Osaka Metropolitan University, Japan
On two reciprocities on Hecke algebras

Abstract

I would like to talk about two reciprocities on Hecke algebras. One is about (1) Iwahori-Hecke algebras and their Kazhdan-Lusztig left cells. I would like to report that Mackey formula holds for left cells as group elements. The other is about (2) a generalization of Robinson reciprocity in finite groups to Hecke algebras, especially on cyclotomic quiver Hecke algebras. Robinson found a reciprocity on the projective summand multiplicities between induced and restricted simple modules. I would like to report that its graded analogue holds.

13 Lucia Morotti

Heinrich Heine University Düsseldorf, Germany
Decomposition numbers of spin RoCK blocks of symmetric groups

Abstract

Formulas for decomposition numbers for RoCK blocks of symmetric groups have been obtained by Chuang and Tan in abelian defect and by Turner in the general case. In this talk I will show that similar formulas hold for decomposition numbers of supermodules of spin RoCK blocks of abelian defect. I will also present some partial results for the general case, in particular showing that the corresponding part of the decomposition matrix is triangular.

This is joint work with Matt Fayers and Sasha Kleshchev.

14 Robert Muth

Duquesne University, USA
Cyclotomic wreath-zigzag algebras and RoCK blocks in higher levels

Abstract

Evseev showed that idempotent truncations of RoCK blocks of the Iwahori-Hecke algebra are Morita equivalent to wreath-zigzag algebras, thereby generalizing work of Chuang and Kessar, and establishing a conjecture of Turner's. In joint work with Liron Speyer and Louise Sutton, we extend Evseev's result to higher levels, showing that idempotent truncations of certain level-ℓ, weight-$d\ell$ blocks of cyclotomic Hecke algebras are Morita equivalent to level-ℓ cyclotomic quotients of rank-d affine wreath-zigzag algebras. This yields some partial information about the structure of RoCK blocks of cyclotomic Hecke algebras, recently defined by Lyle.

15 Alison Parker

University of Leeds, UK
Some representation theory of Kadar-Martin-Yu algebras

Abstract

Kadar-Martin-Yu introduced a new chain of subalgebras of the Brauer algebra. These algebras start with Temperley-Lieb and end with the Brauer algebra and build in representation theoretic intensity. This gives a new tool to tackle the long standing problem of understanding the representation theory of the Brauer algebra. We present an introduction to these new algebras and some results about their representation theory. This is joint work with my PhD student N. M. Alraddadi.

16 Arun Ram

University of Melbourne, Australia
Murphys, Casimirs, Transvections and Hecke algebras

Abstract

One way to discover Murphy elements in the group algebra of the symmetric group is to push Casimir elements across the Schur-Weyl duality. I will review this construction and then explain how a similar construction allows one to push the conjugacy class of transvections in $GL_n(F_q)$ across a type of Schur-Weyl duality to obtain "Murphy elements" in Hecke algebras. In fact, the construction works for all Lie types and for conjugacy classes generalising the conjugacy class of transvections. These elements have been used to analyse a Markov chain on the symmetric group coming from double cosets. This is a report on joint works with Persi Diaconis, Mackenzie Simper and James Parkinson.

17 Liron Speyer

Okinawa Institute of Science and Technology, Japan
Graded decomposition matrices for type C KLR algebras

Abstract

Outside of type A, the cyclotomic KLR algebras are still quite mysterious, and have not been studied in great detail. They have been gaining attention in type C recently, and we now have several useful tools with which we may study their representation theory. These include a theory of Specht modules, underpinned by combinatorics of multipartitions and tableaux, analogous to the type A situation, and Evseev and Mathas's recent proof that these algebras are graded cellular. Building on these ideas, in joint work with Chris Chung and Andrew Mathas, we have calculated graded decomposition matrices in small ranks, and computed structures of many Specht modules. We will explain the techniques used in this endeavour, and point to some interesting examples whose behaviour look very different to what we see in type A.

18 Louise Sutton

Okinawa Institute of Science and Technology, Japan
TBA

Abstract

19 Daniel Tubbenhauer

University of Sydney, Australia
From crystals to cellularity of KLR algebras

Abstract

Weighted KLRW=wKLRW algebras are diagram algebras that depend on continuous parameters, generalizing KLR and Websters tensor product algebras. For certain Dynkin types they admit cellular bases. In particular, in finite types black magic happens and bases for these algebras can be constructed from paths in crystal graphs.

This talk is a user friendly introduction explaining these algebras, and their the connection to crystals and cellularity.

Based on joint work with A. Mathas.

20 Jialin Wang

Nanyang Technological University, Singapore
The rank varieties of some simple modules for symmetric groups

Abstract

For a field F of characteristic p¿0, Carlson defined the rank varieties of modules for elementary abelian p-groups over F. For a module M of any finite group G over F, one can also define the rank variety of M restricted to some elementary abelian p-subgroup of G. In particular, such rank varieties detect the projectivity and the corresponding dimensions are closely related to the complexities. In this talk, I will examine some specific simple modules for symmetric groups and determine the corresponding rank varieties and complexities.

21 Mark Wildon

Royal Holloway, University of London, UK
Stability of plethysms of symmetric functions

Abstract

Composition of polynomial representations of the general linear group corresponds to the plethysm product on symmetric functions. A typical example is $\mathrm{Sym}^n\mathrm{Sym}^m E$, where E is the d-dimensional natural representation of $\mathrm{GL}_d(\mathbb{C})$, corresponding to the plethysm of Schur functions $s_{(n)} \circ s_{(m)}$. The multiplicity of the irreducible Weyl module $\Delta^\lambda(E)$ in this representation is $\langle s_{(n)} \circ s_{(m)}, s_\lambda \rangle$. It is known that if γ is a partition of r then the multiplicity $\langle s_{(n)} \circ s_{(m)}, s_{(mn-|\gamma|;\gamma)} \rangle$ is constant for m, $n \geq r$. This result has been proved by Carré–Thibon (using vertex operators), Brion (using geometric methods), Manivel (again using geometric methods), and Bowman–Paget (using the partition algebra); it implies a stable version of Foulkes' Conjecture. In my talk I will outline a new approach using the combinatorics of plethystic semistandard tableaux that gives simple proofs of this and other stability results, most of them new. This is joint work with Rowena Paget (University of Kent).