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Abstracts

Workshop 1: Fast Optimization Algorithms in the Big Data Era

(5–9 Dec 2022)

1 Ahmet Alacaoglu

University of Wisconsin–Madison, USA

[Randomized first-order algorithms for min-max problems](#)

Abstract

In this talk, we focus on two classes of randomized first order algorithms for solving min-max problems in the finite sum form. When the coupling between primal and dual variables is bilinear, we use random coordinate updates with first order primal-dual algorithms and derive complexity results improving the deterministic baselines. We also showcase the adaptivity of these algorithms to problem structures such as sparsity, both in theory and practice. When the coupling between primal and dual variables is nonbilinear, we present a variance reduced algorithm that comes with improved theoretical and practical properties compared to baselines such as the extra-gradient algorithm.

2 Krishna Balasubramanian

University of California at Davis, USA

[High-dimensional Inference with Stochastic Approximation Algorithms](#)

Abstract

Stochastic approximation algorithms like SGD are widely used in modern data science. In order to perform statistical inference with such algorithms, it

is important to not only obtain point estimates but also to obtain confidence regions quantifying the associated uncertainty. In this talk, I will discuss high-dimensional central limit theorems for linear functionals of online SGD iterates for solving over-parameterized linear regression. Such results could be used for example to obtain prediction confidence intervals or entry-wise estimation confidence intervals. Our results hold in particular when the dimensionality grows exponentially in terms of the number of iterations of SGD (or equivalently the number of observations used). We will discuss such results for both the last SGD iterate and the average of the SGD iterates and highlight the similarities and differences between them.

3 Kabir Chandrasekher

Stanford University, USA

[Alternating minimization for generalized rank one matrix sensing: Sharp predictions from a random initialization](#)

Abstract

We consider the problem of estimating the factors of a rank-1 matrix with i.i.d. Gaussian, rank-1 measurements that are nonlinearly transformed and corrupted by noise. Considering two prototypical choices for the nonlinearity, we study the convergence properties of a natural alternating update rule for this nonconvex optimization problem starting from a random initialization. We show sharp convergence guarantees for a sample-split version of the algorithm by deriving a deterministic recursion that is accurate even in high-dimensional problems. Notably, while the infinite-sample population update is uninformative and suggests exact recovery in a single step, the algorithm—and our deterministic prediction—converges geometrically fast from a random initialization. Our sharp, non-asymptotic analysis also exposes several other fine-grained properties of this problem, including how the nonlinearity and noise level affect convergence behavior. On a technical level, our results are enabled by showing that the empirical error recursion can be predicted by our deterministic sequence within fluctuations of the order $n^{-1/2}$ when each iteration is run with n observations. Our technique leverages leave-one-out tools originating in the literature on high-dimensional M -estimation and provides an avenue for sharply analyzing higher-order iterative algorithms from a random initialization in other high-dimensional

optimization problems with random data.

4 Robert Freund

MIT Sloan School of Management, USA

[Recent Advances in First-Order Methods for Linear Programming](#)

Abstract

Linear programming (LP) is an essential optimization problem, either by itself or as a critical subproblem of a mixed integer program. In the last several years, as the scale of data and LPs has increased enormously, first-order methods (FOMs) have been developed for LP with the goal of solving huge-scale applications for which classic algorithms fail (namely simplex and interior-point methods). In this talk we present some new theoretical and computational results for FOMs for LP. We introduce new approaches to analyzing convergence guarantees that are more consistent with observed practical performance. We identify key bottlenecks of some existing methods, and we present new methods that resolve these bottlenecks and improve on current algorithms. This is joint work with Zikai Xiong, an MIT OR Center doctoral student.

5 Michael Gastner

Yale-NUS College, Singapore

[Accelerating the calculation of optimally smooth pycnophylactic interpolations](#)

Abstract

Many quantitative geospatial data are collected and aggregated in discrete enumeration units (e.g. countries or provinces). Smooth pycnophylactic interpolation aims to find a smooth, non-negative function such that the area integral over each enumeration unit is equal to the aggregated data. Conventionally, smooth pycnophylactic interpolation is achieved by a cellular automaton algorithm that converts a piecewise constant function into an approximately smooth function defined on a grid of coordinates on a geographic equal-area map projection. Although many software packages include implementations of this algorithm, its convergence is slow. A faster method is to

construct a density-equalising map projection in which areas of enumeration units are proportional to the aggregated data. A pycnophylactic interpolation can be efficiently obtained from the Jacobian of this map projection, but solutions do not necessarily achieve optimal smoothness. In this talk, I present ongoing work that aims to further accelerate the density-equalising algorithm without sacrificing the near-optimality achieved by the conventional cellular-automaton method.

6 Geovanni Grapiglia

UC Louvain, Belgium

[Derivative-Free Optimization Methods based on Finite-Differences](#)

Abstract

In many practical optimization problems, the gradients of the functions involved are not readily available. Examples include the tuning of algorithmic parameters, model calibration, and also the design of black-box attacks on Deep Neural Networks. These problems can be addressed with Derivative-Free Optimization (DFO) methods, which are methods that rely only on function evaluations. Very often, the evaluation of the objective functions is computationally expensive. Therefore, one of the main concerns in DFO is the development of methods with a low worst-case complexity in terms of function evaluations. In this talk, I will discuss some recent worst-case complexity results obtained for DFO methods based on finite-difference gradient approximations.

7 Serge Gratton

ENSEEIHT and Université de Toulouse, France

[Complexity and performance for two classes of noise-tolerant first-order algorithms](#)

Abstract

Two classes of algorithms for optimization in the presence of noise are presented, that do not require the evaluation of the objective function. The first generalizes the well-known Adagrad method. Its complexity is then analyzed as a function of its parameters, and it is shown that some methods of

the class enjoy a better asymptotic convergence rate than previously known. A second class of algorithms is then derived whose complexity is at least as good as that of the first class. Initial numerical experiments on finite-sum problems arising from deep-learning applications suggest that that methods of the second class often outperform those of the first.

8 Patrice Koehl

University of California, Davis, USA

[Light speed computation of exact solutions to generic and to degenerate assignment problems](#)

Abstract

The linear assignment problem is a fundamental problem in combinatorial optimization with a wide range of applications, from operational research to data sciences. It consists of assigning “agents” to “tasks” on a one-to-one basis, while minimizing the total cost associated with the assignment. While many exact algorithms have been developed to identify such an optimal assignment, most of these methods are computationally prohibitive for large size problems. In this talk, I will describe a novel approach to solving the assignment problem using techniques adapted from statistical physics. In particular I will derive a strongly concave effective free energy function that captures the constraints of the assignment problem at a finite temperature. This free energy decreases monotonically as a function of beta, the inverse of temperature, to the optimal assignment cost, providing a robust framework for temperature annealing. For large enough beta values the exact solution to the generic assignment problem can be derived using a simple round-off to the nearest integer of the elements of the computed assignment matrix. I will also describe a provably convergent method to handle degenerate assignment problems. Finally, I will describe computer implementations of this framework that are optimized for parallel architectures, one based on CPU, the other based on GPU. These implementations enable solving large assignment problems (of the orders of a few 10000s) in computing clock times of the orders of minutes.

9 Donghwan Kim

Korea Advanced Institute of Science Technology, Korea

[Semi-Anchored Multi-Step Gradient Method for Nonconvex-Nonconcave Minimax Optimization](#)

Abstract

Minimax problems, such as generative adversarial network, adversarial training, and fair training, are widely solved by a multi-step gradient descent ascent (MGDA) method in practice. However, its convergence guarantee is limited. In this work, inspired by the primal-dual hybrid gradient method, we propose a new semi-anchoring (SA) technique for the MGDA method. This makes the MGDA method find a stationary point of a structured nonconvex-nonconcave composite minimax problem. The resulting method, named SA-MGDA, is built upon the Bregman proximal point method.

10 Guanghui Lan

Georgia Institute of Technology, USA

[Fast Convergence of Novel Policy Gradient Methods](#)

Abstract

We present a few novel policy gradient methods for solving challenging Markov decision processes and reinforcement learning problems. These methods involve policy improvement, policy evaluation and function approximations. We show that these methods converge fast under quite general settings.

11 Ching-pei Lee

Academia Sinica, Taipei

[Solution Structure Utilization for Efficient Optimization and Large-scale Machine Learning](#)

Abstract

Regularized optimization that adds a regularizer to the objective function for minimization is widely used in numerous applications in machine learning

and signal processing to induce desired structures in an optimal solution. In this talk, I will first describe how to find such a structure in an approximate solution, without obtaining the optimal solution that is usually only the limit point of an iterative algorithm, in different scenarios and discuss why this matters.

I will then show how we can utilize such an optimal structure to devise more efficient optimization algorithms. Examples of our algorithms designed for specific regularizers used in various tasks in machine learning and signal processing to either identify such optimal structures for better performance or utilize them to accelerate the optimization will be discussed, including training structured neural network models, matrix completion, federated learning, and the best subset selection problem.

This is joint work with Yu-Sheng Li, Wei-Lin Chiang, Zih-Syuan Huang, Jan Harold Alcantara, Ling Liang, Tianyun Tang, Kim-Chuan Toh.

12 Yin Tat Lee

University of Washington, USA
[From Robustness to Efficiency](#)

Abstract

Traditionally, optimization methods like (stochastic) gradient descent take time at least linear to the number of parameters. For problems with many parameters, each step is too expensive.

In this talk, I will discuss how designing robust algorithms can lead to faster algorithms. In particular, I will explain the robust interior point method. Its robustness allows us to speed up its iteration cost via data structures. This results in theoretically faster algorithms for many important problems such as linear programming, semidefinite programming, and the maximum flow problem.

13 Zhaosong Lu

University of Minnesota, USA
[First-order methods for convex optimization and monotone inclusions under local Lipschitz conditions](#)

Abstract

In this talk, we first consider convex optimization whose smooth components have a locally Lipschitz continuous gradient and propose a first-order method for finding an epsilon-KKT solution. We then consider monotone inclusion in which the point-valued operator is locally Lipschitz continuous and propose a primal-dual extrapolation method for finding an epsilon-residual solution. All the proposed methods are parameter free with a verifiable termination criterion and also enjoy a nearly optimal complexity. This is joint work with Sanyou Mei (University of Minnesota).

14 Laura Palagi

Sapienza University of Rome, Italy

[Convergence under Lipschitz smoothness of ease-controlled Random Reshuffling gradient Algorithms](#)

Abstract

We consider minimizing the average of a very large number of smooth and possibly non-convex functions. This optimization problem has deserved much attention in the past years due to the many applications in different fields, the most challenging being training Machine Learning models. Widely used approaches for solving this problem are mini-batch (online) gradient methods which, at each iteration, update the decision vector moving along the gradient of a mini-batch selection of the component functions. Depending on the selection rule for the mini-batch different methods have been defined. We consider the Incremental Gradient (IG) and the Random reshuffling (RR) methods which proceed in cycles, picking a fixed or uniformly random order (permutation), respectively, and processing the component functions according to this order. Convergence properties of these schemes have been proved under different assumptions, usually quite strong. We aim to define ease-controlled modifications of the IG/RR schemes, which require a light additional computational effort and can be proved to converge under very weak and standard assumptions. In particular, we define two algorithmic schemes in which the IG/RR iteration is controlled by using a watchdog rule and a derivative-free line search that activates only sporadically to guarantee convergence. The two schemes differ in the watchdog and the line search,

which are performed using either a monotonic or a non-monotonic rule. The two schemes also allow controlling the updating of the learning rate used in the main IG/RR iteration, avoiding the use of preset rules that may drive it to zero too fast, thus overcoming another tricky aspect in implementing online methods, which is the updating rule of the stepsize. We proved convergence under the lonely assumption of Lipschitz continuity of the gradients of the component functions. We performed an extensive computational test using different Deep Neural Architectures and a benchmark of varying size datasets. We compare our implementation with both full batch gradient methods and online standard implementation of IG/RR methods, proving that the computational effort is comparable with the corresponding online methods and that the control on the learning rate may allow faster decrease.

15 Peter Richtarik

King Abdullah University of Science and Technology, Saudi Arabia

[ProxSkip: Local gradient steps provably lead to communication acceleration](#)

Abstract

In this talk I will introduce ProxSkip [1] - a surprisingly simple and provably efficient method for minimizing the sum of a smooth (f) and an expensive nonsmooth proximable (ψ) function. The canonical approach to solving such problems is via the proximal gradient descent (ProxGD) algorithm, which is based on the evaluation of the gradient of f and the prox operator of ψ in each iteration. In this work we are specifically interested in the regime in which the evaluation of prox is costly relative to the evaluation of the gradient, which is the case in many applications. ProxSkip allows for the expensive prox operator to be skipped in most iterations: while its iteration complexity is $O(\kappa \log 1/\epsilon)$, where κ is the condition number of f , the number of prox evaluations is $O(\sqrt{\kappa} \log 1/\epsilon)$ only. Our main motivation comes from federated learning, where evaluation of the gradient operator corresponds to taking a local GD step independently on all devices, and evaluation of prox corresponds to (expensive) communication in the form of gradient averaging. In this context, ProxSkip offers an effective acceleration of communication complexity. Unlike other local gradient-type methods, such as FedAvg [2], SCAFFOLD [3], S-Local-GD [4] and FedLin [5], whose theoretical communication complexity is worse than, or at best matching, that of vanilla GD in the heterogeneous data regime, we obtain a provable and large improvement without any heterogeneity-bounding assumptions.

Time permitting, I will mention several subsequent extensions, generalizations and improvements [6, 7, 8].

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16 Clément W. Royer

Université Paris Dauphine-PSL, France

[Optimization methods for highly nonconvex data science tasks](#)

Abstract

Nonconvex optimization formulations arise not only in deep learning, but also in data analysis and robust statistics. Although such problems typically come with a great deal of structure, certain formulations are quite challenging from an algorithmic perspective, as they induce complex optimization landscapes and numerous critical points. Designing fast algorithms for such problems is thus a challenging endeavor. On one hand, theoretical guarantees such as

complexity results (or convergence rates), that characterize how fast a given method can be in the worst case, often fail at representing the behavior of an algorithm over more specific instances. On the other hand, general purpose nonlinear optimization schemes may prove quite efficient on such problems, often without convergence rate guarantees or principled ways of exploiting the problems' structure.

In this talk, we investigate two highly nonconvex optimization problems from data science. For both problems, our goal is to propose a method that is not also theoretically fast in a complexity sense but also practical for the task at hand. The first part of the talk will discuss nonconvex formulations of robust linear regression. On such instances, we observe that the theoretically fastest methods are outperformed by standard schemes such as nonlinear conjugate gradient. We then propose an algorithm that is both equipped with complexity guarantees and of practical interest for this task. In the second part of the talk, we consider a nonconvex optimization problem resulting from the training of a discretized neural ordinary differential equation model. We leverage recent results from the landscape of shallow networks to understand that of our problem, thereby illustrating its highly nonconvex nature. Given such a landscape, we consider several optimization schemes with complexity guarantees, and explore both their convergence rates and their ability to avoid spurious critical points in practice.

17 Katya Scheinberg

Cornell University, USA

[Stochastic oracles and how to define them](#)

Abstract

Continuous optimization is a mature field, which has recently undergone major expansion and change. One of the key new directions is the development of methods that do not require exact information about the objective function.

Nevertheless, the majority of these methods, from stochastic gradient descent to “zero-th order” methods use some kind of approximate first order information.

We will introduce a general definition of a stochastic oracle and show how this definition applies in a variety of familiar settings, including simple

stochastic gradient via sampling, traditional and randomized finite difference methods and more. We will overview several stochastic methods and how the general definition extends to the oracles used by these methods.

18 Gesualdo Scutari

Purdue University, USA

[Bringing Statistical Thinking in Distributed Optimization. Vignettes from statistical inference over Networks](#)

Abstract

There is growing interest in solving large-scale statistical machine learning problems over decentralized networks, where data are distributed across the nodes of the network and no centralized coordination is present (we termed these systems “mesh” networks). Modern massive datasets create a fundamental problem at the intersection of the computational and statistical sciences: how to provide guarantees on the quality of statistical inference given bounds on computational resources, such as time and communication efforts? While statistical-computation tradeoffs have been largely explored in the centralized setting, our understanding over mesh networks is limited: (i) distributed schemes, designed and performing well in the classical low-dimensional regime, can break down in the high-dimensional case; and (ii) existing convergence studies may fail to predict algorithmic behaviors; some are in fact confuted by experiments. This is mainly due to the fact that the majority of distributed algorithms have been designed and studied only from the optimization perspective, lacking the statistical dimension. This talk will discuss some vignettes from high-dimensional statistical inference suggesting new analyses aiming at bringing statistical thinking in distributed optimization.

19 Philippe Toint

Université de Namur, Belgium

[Objective-Function-Free Optimization, Part II:
Complexity of Adaptive Regularization and Numerical Experiments](#)

Abstract

An adaptive regularization algorithm for unconstrained nonconvex optimization is presented in which the objective function is never evaluated, but only derivatives are used. This algorithm belongs to the class of adaptive regularization methods, for which optimal worst-case complexity results are known for the standard framework where the objective function is evaluated. It is shown in this paper that these excellent complexity bounds are also valid for the new algorithm, despite the fact that significantly less information is used. In particular, it is shown that, if derivatives of degree one to p are used, the algorithm will find a ϵ_1 -approximate first-order minimizer in at most $(\epsilon_1^{-(p+1)/p})$ iterations, and an (ϵ_1, ϵ_2) -approximate second-order minimizer in at most $(\max[\epsilon_1^{-(p+1)/p}, \epsilon_2^{-(p+1)/(p-1)}])$ iterations. As a special case, the new algorithm using first and second derivatives, when applied to functions with Lipschitz continuous Hessian, will find an iterate x_k at which the gradient's norm is less than ϵ_1 in at most $(\epsilon_1^{-3/2})$ iterations.

Numerical experiments will finally be presented showing the surprisingly good performance and robustness of “flexible” variants of the new methods (Adagrad-like and adaptive regularization).

20 Stephen Wright

University of Wisconsin–Madison, USA

[Optimization in theory and practice](#)

Abstract

Complexity analysis in optimization seeks upper bounds on the amount of work required to find approximate solutions of problems in a given class with a given algorithm, and also lower bounds, usually in the form of a worst-case example from a given problem class. The relationship between theoretical complexity bounds and practical performance of algorithms on “typical” problems varies widely across problem and algorithm classes, and relative

interest among researchers between the theoretical and practical aspects of algorithm design and analysis has waxed and waned over the years. This talk surveys complexity analysis and its relationship to practice in optimization, with an emphasis on linear programming and convex and nonconvex nonlinear optimization, providing historical (and cultural) perspectives on research in these areas.

21 Stephen Wright

University of Wisconsin–Madison, USA

[Primal-dual optimization methods for robust machine learning](#)

Abstract

We consider a convex-concave primal-dual optimization framework in which the coupling between primal and dual variables is bilinear. This framework admits linearly constrained optimization together with a variety of interesting problems in machine learning, including (linear) empirical risk minimization with various regularization terms. It also includes a formulation that we term “generalized linear programming” (GLP) in which regularization terms and constraints are added to the traditional linear programming formulation, provided they admit efficient prox operations. Problems from differentially robust optimization (DRO), using either f-divergence metrics or Wasserstein metrics, can be formulated as GLPs.

We describe algorithms for our framework that take prox-gradient steps alternately in the primal and dual variables, but incorporate such additional features as coordinate descent, variance reduction, dual averaging, importance sampling, and iterate averaging. Our methods can also exploit sparsity in the matrix that couples primal and dual variables. Our methods match or improve on the best known worst-case complexity bounds in various settings. Computational experiments indicate that our methods also have good practical performance.

The talk represents joint work with Ahmet Alacaoglu, Jelena Diakonikolas, Chaobing Song, Eric Lin, and Volkan Cevher

22 Yancheng Yuan

The Hong Kong Polytechnic University, Hong Kong

Abstract

23 Junyu Zhang

National University of Singapore, Singapore

[A Unified Primal-Dual Algorithm Framework for Inequality Constrained Problems](#)

Abstract

In this paper, we propose a unified primal-dual algorithm framework based on the augmented Lagrangian function for composite convex problems with conic inequality constraints. The new framework is highly versatile. First, it not only covers many existing algorithms such as PDHG, Chambolle-Pock (CP), GDA, OGDA and linearized ALM, but also guides us to design a new efficient algorithm called Simi-OGDA (SOGDA). Second, it enables us to study the role of the augmented penalty term in the convergence analysis. Interestingly, a properly selected penalty not only improves the numerical performance of the above methods, but also theoretically enables the convergence of algorithms like PDHG and SOGDA. Under properly designed step sizes and penalty term, our unified framework preserves the $\mathcal{O}(1/N)$ ergodic convergence while not requiring any prior knowledge about the magnitude of the optimal Lagrangian multiplier. Linear convergence rate for affine equality constrained problem is also obtained given appropriate conditions. Finally, numerical experiments on linear programming, ℓ_1 minimization problem, and multi-block basis pursuit problem demonstrate the efficiency of our methods.

24 Yangjing Zhang

Chinese Academy of Sciences, China

[On Efficient and Scalable Computation of the Nonparametric Maximum Likelihood Estimator in Mixture Models](#)

Abstract

The nonparametric maximum likelihood estimation (NPMLE) is a classic and important method to estimate the mixture models from finite observations. The discretization of the infinite dimensional probability measure with a fixed support in the NPMLE leads to a finite dimensional convex optimization problem. Although can be solved by off-the-shelf interior point based solvers, the algorithm does not scale well with the number of grid points (denoted as m) and the number of observed data points (denoted as n). In this talk, leveraging the observation that the solution of the finite dimensional NPMLE is usually sparse, we propose an efficient and scalable semismooth Newton based augmented Lagrangian method (ALM). By carefully exploring the structure of the ALM subproblem, we show that the computational cost of the generalized Hessian (second order information) is independent of the number of grid points. Hence, compared with the recent work Kim et al. (2020,JCGS) that using the active set based sequential quadratic programming to solve the NPMLE whose Hessian evaluation scales quadratically in m , our proposed algorithm is in particular suitable for the multivariate mixture models where a large number of grid points is needed in the multi-dimensional space. Extensive numerical experiments are conducted to show the effectiveness of our approach. In particular, we are able to get highly accurate solutions for the multivariate synthetic data with $n=100,000$ and $m=10,000$ within one minute. For the astronomy data set Gaia TGAS that contains 1.4 million dereddened stars with 10,000 grid points in the 2-dimensional space, our algorithm successfully solve it in about seven hours.