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IMS DISTINGUISHED VISITOR LECTURE SERIES

The Theta Correspondence - Origins, Results, and Ramifications

Around 1830, C. G. Jacobi wrote down his original theta function $\theta(z)$ as a Fourier series, and used it to give a striking new proof of the theorem of Lagrange, from the late 18th century, that every whole number is the sum of four perfect squares. It is easy to see that number of ways to express a given number as a sum of squares of four integers is given by the Fourier coefficients of θ^4, and Jacobi was able to give an explicit formula for these. Both θ and θ^4 are *modular forms*, functions on the complex upper half plane defined by certain transformation conditions. Later in the 19th century, the identification by G. Eisenstein of θ^4 as a special type of modular form (now known as an *Eisenstein series*) shed further light in Jacobi's formula. The strong connection between modular forms and number theory, as well as the geometry of Riemann surfaces, led to further investigations by Hecke, Maass, Siegel, Shimura and others into modular forms, and analogous functions of many variables, called *automorphic forms*.

Also in the first half of the 19th century, investigations into the solutions of algebraic equations led to the study of groups of transformations, and of *invariants* - functions that were not changed by relevant transformations. This developed into a huge field, especially in England, and eventually led to the development of abstract algebra, especially by Hilbert and Noether. In the first half of the 20th century, interest in invariant theory subsided, but was partly preserved and extended by Weyl, in his book *The Classical Groups*, which established two basic results, which he called the *First (resp. Second) Fundamental Theorem* of invariant theory.

Partly through the above work, but also through developments in mathematical physics, especially quantum mechanics, attention to the way functions behave under transformations led to the development of *representation theory*, which describes how a given group can act on a vector space. At first, the vector space was presumed to be finite dimensional, but under the influence of physics, infinite dimensional representations also came under study.

All of these paths helped lead to *theta correspondences*, which tie many examples from all these fields together into tight packages.

These talks will to survey some of the history, describe some of the results and the techniques used to establish them, and some of the implications they have for these disparate lines of research.

Professor Roger Howe

Texas A&M University, USA

Professor Roger E. Howe is currently the Curtis D. Robert Endowed Chair in Mathematics Education at Texas A&M University. Prior to this appointment, he had been William R. Kenan, Jr. Professor of Mathematics at Yale University. Professor Howe received his PhD from the University of California, Berkeley. Before Yale, he spent several years at the State University of New York in Stony Brook, IHe has also held visiting positions at numerous universities and research institutes in the United States, Europe, and Asia. He served as the chair of the Scientific Advisory Board of the Institute for Mathematical Sciences at the National University of Singapore from 2000–2010.

Professor Howe is Fellow of the American Academy of Arts and Sciences (1993), Member of the US National Academy of Sciences (1994) and Fellow of the American Mathematical Society (2013). His awards include the American Mathematical Society Award for Distinguished Public Service (2006) and the Award for Interdisciplinary Excellence in Mathematics Education from the Texas A&M University (2015).

Professor Howe's research interests are in applications of symmetry, to harmonic analysis, representation theory, automorphic forms and invariant theory.

Part I
1 July 2022 (Friday)
2–3pm (GMT+8, Singapore)

Part II
2 July 2022 (Saturday)
1–2pm (GMT+8, Singapore)

Venue
IMS Auditorium
National University of Singapore
3 Prince George's Park Singapore 118402

Registration
https://tinyurl.com/ImsRepresentationsReg

The talks are part of the program on *Representations and Characters: Revisiting the Works of Harish-Chandra and André Weil — A satellite conference of the virtual ICM 2022*
https://tinyurl.com/harishjuly2022

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