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Abstracts

Random Matrix EurAsia 2022

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1 Alexander I. Aptekarev

Keldysh Institute of Applied Mathematics, Russia
Multiple orthogonal polynomials ensembles and determinantal processes

Abstract

Let $\mu(x) := (\mu_1(x), \ldots, \mu_d(x))$ be a vector of positive measures. For a given multiindex $n = (n_1, \ldots, n_d)$ we consider a polynomial $P_n(x)$ of degree $|n| := n_1 + \ldots + n_p$, which satisfies n_j orthogonality relations to the degrees of the scalar variable x with respect to the measure $\mu_j, \quad j = 1, \ldots, p$. Such polynomials always exist and they are called multiple orthogonal polynomials. For $p = 1$ we have usual orthogonal polynomials. We discuss several examples of ensembles of random matrices related to the multiple orthogonal polynomials (namely: random matrix model with external source, two matrix model and normal matrix model). An application to the Brownian bridges will be highlighted.

2 Zhigang Bao

Hong Kong University of Science and Technology, China
Phase transition of eigenvector for spiked random matrices

Abstract

In this talk, we will first review some recent results on the eigenvectors of random matrices under fixed-rank deformation, and then we will focus on

the limit distribution of the leading eigenvectors of the Gaussian Unitary Ensemble (GUE) with fixed-rank (aka spiked) external source, in the critical regime of the Baik-Ben Arous-Peche (BBP) phase transition. The distribution is given in terms of a determinantal point process with extended Airy kernel. Our result can be regarded as an eigenvector counterpart of the BBP eigenvalue phase transition. The derivation of the distribution makes use of the recently rediscovered eigenvector-eigenvalue identity, together with the determinantal point process representation of the GUE minor process with external source. This is a joint work with Dong Wang (UCAS).

3 Guillaume Barraquand

École Normale Supérieure, France
Pinning of directed polymers and the Baik-Ben Arous-Péché phase transition

Abstract

Models of directed polymers confined by a wall are expected to obey a phase transition as the strength of the noise along the wall increases. This was first predicted in 1985 in physics by Kardar. We will show that for an exactly sovable model, the log-gamma polymer, the partition function of polymers confined to an octant of the Z^2 lattice with free endpoint indeed satisfies a phase transition. Near the critical point, the statistics are asymptotically distributed as in the Baik-Ben Arous-Péché phase transition for outliers of Hermitian random matrices. The talk is based on a joint work with Shouda Wang, and will also touch on earlier works with Alexei Borodin and Ivan Corwin, with Ivan Corwin and Evgeni Dimitrov, and with Pierre Le Doussal.

4 Anirban Basak

Tata Institute of Fundamental Research, India
Spectral properties of random perturbations of non-self-adjoint operators

Abstract

Understanding spectral properties of non-self-adjoint operators are of significant importance as they arise in many problems such as scattering systems,

open or damped quantum systems, and the analysis of the stability of solutions to nonlinear PDEs. Absence of suitable methods (e.g. variational methods) renders the study of the spectrum of such operators to be difficult. On the other hand, its high sensitivity to small perturbations leads to serious numerical errors. Motivated by problems in different fields such as numerical analysis, semiclassical analysis, fluid dynamics, and mathematical physics, during the last fifteen years there have been several works in understanding the spectral properties of random perturbations of non-self-adjoint operators. In this talk, we will focus on random perturbations of large dimensional non-self-adjoint Toeplitz matrices, and discuss (i) Weyl type law for the empirical measure of its eigenvalues, (ii) limiting eigenvalue density inside the zone of spectral instability (i.e. limit law for outlier eigenvalues), and (iii) localization/delocalization of its eigenvectors, and the universality and non-universality of these features. I will also present some fun pictures and simulations. Based on joint works with Elliot Paquette, Martin Vogel, and Ofer Zeitouni.

5 Gordon Blower

Lancaster University, UK
Hashimoto frames and the Gibbs measure of NLS

Abstract

The talk interprets the cubic nonlinear Schroedinger equation as a Hamiltonian system with infinite dimensional phase space. There is a Gibbs measure which is invariant under the flow associated with the canonical equations of motion. The talk reviews logarithmic Sobolev and concentration of measure inequalities for the Gibbs measures, and applies them to the k-point correlation function. Recalling Hasimoto's method of moving frames, the talk illustrates the typical solutions of the NLSE via stochastic ODE in the matrix group SO(3).

6 Mattia Cafasso

Université d'Angers, France
Integrability of integro-differential Painlevé equation

Abstract

During my talk, I will discuss about two different Riemann -Hilbert approaches to the study of integro-differential Painlevé equations. The study of these equations is motivated by a few interesting physical applications, such as free fermions at finite temperature and stochastic growth models (Kardar-Parisi-Zhang equation). The results I will speak about have been obtained in collaboration with Thomas Bothner, Tom Claeys, Giulio Ruzza and Sofia Tarricone.

7 Prathapasinghe Dharmawansa

University of Moratuwa, Sri Lanka
The eigenvectors of single-spiked complex Wishart matrices: finite and asymptotic analyses

Abstract

Let $\mathbf{W} \in \mathbb{C}^{n\times n}$ be a *single-spiked* Wishart matrix[1] in the class $\mathbf{W} \sim \mathcal{CW}_n(m, \mathbf{I}_n + \theta\mathbf{v}\mathbf{v}^\dagger)$ with $m \geq n$, where $\mathbf{I}_n$ is the $n \times n$ identity matrix, $\mathbf{v} \in \mathbb{C}^{n\times 1}$ is an arbitrary vector with unit Euclidean norm, $\theta \geq 0$ is a non-random parameter, and $(\cdot)^\dagger$ represents the conjugate-transpose operator. Let $\mathbf{u}_1$ and $\mathbf{u}_n$ denote the eigenvectors corresponding to the smallest and the largest eigenvalues of $\mathbf{W}$, respectively. This talk focuses on the probability density function (p.d.f.) of the random quantity $Z_\ell^{(n)} = \left|\mathbf{v}^\dagger \mathbf{u}_\ell\right|^2 \in (0,1)$ for $\ell = 1, n$. In particular, we derive a finite dimensional closed-form p.d.f. for $Z_1^{(n)}$ which is amenable to asymptotic analysis as m, n diverges with $m - n$ fixed. It turns out that, in this asymptotic regime, the scaled random variable $nZ_1^{(n)}$ converges in distribution to $\chi_2^2/2(1+\theta)$, where χ_2^2 denotes a chi-squared random variable with two degrees of freedom. This reveals that $\mathbf{u}_1$ can be used to infer information

[1] These spikes arise in various practical settings in different scientific disciplines. For instance, they correspond to the first few dominant factors in factor models arising in financial economics, the number of clusters in gene expression data, and the number of signals in single detection and estimation.

about the spike. On the other hand, the finite dimensional p.d.f. of $Z_n^{(n)}$ is expressed as a double integral in which the integrand contains a determinant of a square matrix of dimension $(n-2)$. Although a simple solution to this double integral seems intractable, for special configurations of $n = 2, 3$, and 4, we obtain closed-form expressions.

We also also extend the former analytical framework to the real and complex singular Wishart cases as well. The main technical challenge pertaining to these extensions is that the corresponding p.d.f.s, in general, do not admit tractable simple closed-form expressions which are amenable to further analysis. However, as our detailed analysis reveals, they can be expressed in closed-form for a few special configurations of m and n.

8 Victor Didenko

Southern University of Science and Technology, China
Toeplitz and Toeplitz plus Hankel operators on l^p-spaces

Abstract

A new approach to Toeplitz and Toeplitz plus Hankel operators on l^p-spaces will be discussed. It is based on the celebrated Hausdorff-Young Theorem and allows to obtain new results even for Toeplitz operators.

9 Anton Dzhamay

University of Northern Colorado, USA
Orthogonal polynomials and discrete Painlevé equations

Abstract

In this talk we describe an algorithmic procedure on how to identify different recurrence relations appearing in the theory of orthogonal polynomials with discrete Painlevé equations. Our approach is based on the geometric theory of discrete Painlevé equations developed by H. Sakai. Using that we are able to produce explicit change of variables transforming these recurrences to some canonical form. This talk is based on a series of papers with Galina Filipuk and Alexander Stokes, and also Yang Chen, Jie Hu, Xing Li, Adam Ligeza, and Da-jun Zhang.

10 Galina Filipuk

University of Warsaw, Poland
Nonlinear differential equations and the geometric approach

Abstract

Nonlinear differential equations may have complicated singularities in the complex plane. Painleve equations are nonlinear second order differential equations solutions of which have no movable critical points. They have a lot of nice properties. The quasi-Painleve equations admit algebraic branch points. The geometric approach to the Painleve equations was developed in the works of K. Okamoto, H. Sakai and many others. In this talk I shall present some recent results using the geometric approach (including, if time permits, the general procedure of the reduction of recurrence coefficients of semi-classical orthogonal polynomials to the Painleve equations as an example of the solution of the so-called Painleve equivalence problem, equations with the algebro-Painleve property, equations with the quasi-Painleve property, Lienard type equations and so on). This talk is based on several published papers and preprints joint with A. Dzhamay, A.

11 Galina Filipuk

University of Warsaw, Poland
On the Painleve XXV $--$ Ermakov equation

Abstract

Starting from the Riccati equation and the second order element of the Riccati chain as the simplest examples of linearizable equations, by introducing a suitable change of variables, it is shown how the Schwarzian derivative represents a key tool in the construction of solutions of the Painleve XXV-Ermakov equation. Two families of Backlund transformations which link the linear and nonlinear equations under investigation are obtained. The talk is based on the joint work with S.Carillo, A. Chichurin and F. Zullo and is partially based on the arxiv preprint `https://arxiv.org/abs/2201.02267`.

12 Subhro Ghosh

National University of Singapore, Singapore
Stochastic geometry beyond independence and its applications

Abstract

The classical paradigm of randomness is the model of independent and identically distributed (i.i.d.) random variables, and venturing beyond i.i.d. is often considered a challenge to be overcome. In this talk, we will explore a different perspective, wherein stochastic systems with constraints in fact aid in understanding fundamental problems. Our constrained systems are well-motivated from statistical physics, including models like the random critical points and determinantal probability measures. These will be used to shed important light on natural questions of relevance in understanding data, including problems of likelihood maximization and dimensionality reduction. En route, we will explore connections to spiked random matrix models and novel asymptotics for the fluctuations of spectrally constrained random systems. Based on the joint works below.

References

[1] Gaussian determinantal processes: A new model for directionality in data, with P. Rigollet, Proceedings of the National Academy of Sciences, vol. 117, no. 24 (2020), pp. 13207–13213

[2] Fluctuation and Entropy in Spectrally Constrained random fields, with K. Adhikari, J.L. Lebowitz, Communications in Math. Physics, 386, 749–780 (2021).

[3] Maximum Likelihood under constraints: Degeneracies and Random Critical Points, with S. Chaudhuri, U. Gangopadhyay, submitted.

13 Alexey Glazyrin

University of Texas Rio Grande Valley, USA
Optimal measures for p-frame energies on spheres

Abstract

We study distributions of mass on spheres minimizing energies of pairwise interactions. In particular, we are interested in optimal measures for the p-frame energies, i.e., energies with the kernel given by the absolute value of the inner product raised to a positive power p. Application of linear programming methods in the setting of projective spaces allows for describing the minimizing measures in full in several cases: tight designs and the 600-cell for several ranges of p in different dimensions. Our methods apply to a much broader class of potential functions, namely, those which are absolutely monotonic up to a particular order.

We also prove that the support of any minimizer of the p-frame energy has empty interior whenever p is not an even integer. A similar effect is demonstrated for energies with analytic potentials which are not positive definite. In addition, we establish the existence of discrete minimizers for a large class of energies, which includes energies with polynomial potentials. This is a joint project with Dmitriy Bilyk, Ryan Matzke, Josiah Park, Oleksandr Vlasiuk.

14 Vadim Gorin

University of Wisconsin–Madison, USA
Addition of matrices at high and low temperatures

Abstract

Suppose that you are given self-adjoint matrices A and B with known eigenvalues and unknown eigenvectors. What can you say about eigenvalues of C=A+B? It took the entire 20th century to obtain deterministic characterizations of the eigenvalues in the work of Weyl, Horn, Klyachko, and Knutson-Tao. In the talk we will discuss the probabilistic version of the problem, in which A and B are random and an important role is played by the random matrix parameter Beta, that takes values 1, 2, or 4, depending on whether we deal with real, complex, or quaternionic matrices. I will explain how this parameter can be taken to be an arbitrary positive real number (identified with the inverse temperature in the terminology of the statistical mechanics) and outline a rich asymptotic theory as Beta tends to zero and to infinity.

15 Tamara Grava

University of Bristol, UK and SISSA, Italy

Gibbs ensemble for integrable systems, a case study: the discrete nonlinear Schrodinger equation

Abstract

We consider discrete integrable systems with random initial data and connect them with the theory of random matrices. In particular we consider the defocusing nonlinear Schrodinger equation in its integrable version, that is called Ablowitz Ladik lattice. In the random initial data setting the Lax matrix of the Ablowitz Ladik lattice turns into a random matrix that is related to the circular beta-ensemble at high temperature. We obtain the density of states of the random Lax matrix, when the size of the matrix goes to infinity, by establishing a mapping to the one-dimensional log-gas. The density of states is obtained via a particular solution of the double-confluent Heun equation. Joint work with Guido Mazzuca.

16 F. Alberto Grünbaum

The University of California, Berkeley, USA

Commuting integral and differential operators: ome extensions of the work of Slepian, Landau and Pollak and the master symmetries of KdV

Abstract

Since the early days of Random matrix theory the fact that certain integral operators commute with an appropriate differential one has been useful, as can be seen in the work of Mehta (sinc kernel) and later in the work of Tracy and Widom (the Bessel and Airy kernels) . The first two examples appear in the work of the Bell Labs group of Slepian, Landau and Pollak in giving quantitative answers to questions posed by Shannon.

The reason behind this miracle has been the subject of extensive work, and in recent papers with W. Casper, M. Yakimov and I. Zurrian a large class of new situations has been uncovered. In the case of appropriate deformations of the Bessel kernels the master symmetries of KdV play a crucial role.

17 Takashi Imamura

Chiba University, Japan

Exact analyses of the KPZ models by the periodic and free boundary Schur measures Part 2

Abstract

An important finding in integrable probability is determinantal structure in the KPZ models. The first breakthrough have appeared in the totally asymmetric simple exclusion process (TASEP) and related directed polymer models with zero temperature. In these models, one can find a connection with the Schur processes thanks to the pioneering work by Johansson using RSK correspondence. Recent developments in the integrable probability enable us to analyze one parameter generalizations of the TASEP such as q-(Push)TASEP, ASEP and related directed polymer models with finite temperature, which can be regarded as discretizations of the KPZ equation. Surprisingly some determinantal formulas can be obtained for these models by the techniques recently developed in the integrable probability such as Macdonald processes, Markov duality, and six vertex model. However to get the formulas, we have to go through complicated calculations and their scope of application is quite limited.

In this talk, I will report a new approach to analyze the KPZ models based on our recent work arXiv:2106.11922. In this preprint, we found two identities connecting the q-Whittaker measures with the periodic Schur measure and the half space variant of the q-Whittaker measure with the free boundary Schur measure. The marginals of the q-Whittaker measures have been known to have the same law of the KPZ models while the slightly deformed versions of the periodic and free boundary Schur measures are the typical models of the determinantal and Pfaffian point processes respectively. Thus by these identities, one can analyze the KPZ models through determinantal or Pfaffian point processes. From the first identity, we can obtain new determinantal formulas for the q-Push TASEP and Log-gamma polymer from which one can reproduce known results about universal distributions in the KPZ scaling limit. From the second identity, we obtain new Pfaffian formulas for the q-Push TASEP, Log-gamma polymer in half space. So far in the conventional approach in the integrable probability, we have to use some non rigorous treatments to analyze the limiting behavior of these models. Our

Pfaffian formulas allow us to obtain the limiting distributions in a rigorous way. This is the joint work with Matteo Mucciconi and Tomohiro Sasamoto. The identity between q-Whittaker and periodic Schur measure obtained in arXiv:2106.11922 will be explained in the talk by Tomohiro Sasamoto.

18 Eugene Kanzieper

Holon Institute of Technology, Israel
Power Spectrum of the Circular Unitary Ensemble

Abstract

We study the power spectrum of eigen-angles of random matrices drawn from the circular unitary ensemble (CUE) and show that it can be evaluated in terms of either a Fredholm determinant, or a Toeplitz determinant, or a sixth Painlevé function. In the limit of infinite-dimensional matrices, we derive a *concise* parameter-free formula for the power spectrum which involves a fifth Painlevé transcendent. Further, we discuss a universality of the predicted power spectrum law (in random-matrix-theory context and beyond), and present a fair evidence that a universal Painlevé V is observed in the power spectrum of nontrivial zeros of the Riemann zeta function.

19 Xiangdong Li

Chinese Academy of Sciences, China
On the Dyson Brownian motion in a general external potential

Abstract

In this talk, we will recall the background and history of the study of Dyson Brownian motion. Then we will use the optimal transportation theory to study the McKean-Vlasov equation which is the large N limit of the empirical measure of the Dyson Brownian motion with general external potential. In particular, we prove the Law of the Large Numbers and the Central Limit Theorem for the empirical measure processes of the Dyson Brownian motion with general external potential.

20 Dang-Zheng Liu

University of Science and Technology of China, China
Duality and phase transition in non-Hermitian random matrix theory

Abstract

Consider a random matrix of size N as an additive deformation of the complex Ginibre ensemble under a deterministic matrix X_0 with a finite rank, independent of N. When some eigenvalues of X_0 separate from the unit disk, outlier eigenvalues may appear asymptotically in the same locations, and their fluctuations exhibit surprising phenomena that highly depend on the Jordan canonical form of X_0. These findings are largely due to Benaych-Georges and Rochet, Bordenave and Capitaine, and Tao. When all eigenvalues of X_0 lie inside the unit disk, we prove that local eigenvalue statistics at the spectral edge form a new class of determinantal point processes, for which correlation kernels are characterized in terms of the repeated erfc integrals. This thus completes a non-Hermitian analogue of the BBP phase transition in Random Matrix Theory. Similar results hold for the deformed quaternion Ginibre ensemble. Duality formulae between different matrix ensembles play a key role. Joint work with Lu Zhang (USTC).

21 Zhipeng Liu

The University of Kansas, USA
One-point distribution of the geodesic in directed last passage percolation

Abstract

In the recent twenty years, there has been a huge development in understanding the universal law behind a family of 2d random growth models, the so-called Kardar-Parisi-Zhang (KPZ) universality class. Especially, limiting distributions of the height functions are identified for a number of models in this class. On the other hand, different from the height functions, the geodesics of these models are much less understood. There were studies on the qualitative properties of the geodesics in the KPZ universality class very recently, but the precise limiting distributions of the geodesic locations remained unknown.

In this talk, we will discuss our recent results on the one-point distribution of the geodesic of a representative model in the KPZ universality class, the directed last passage percolation with iid exponential weights. We will give an explicit formula of the one-point distribution of the geodesic location joint with the last passage times, and its limit when the parameters go to infinity under the KPZ scaling. The limiting distribution is believed to be universal for all the models in the KPZ universality class. We will further give some applications of our formulas.

22 Shulin Lyu

Qilu University of Technology (Shandong Academy of Sciences), China
Laguerre unitary ensembles with jump discontinuities, PDEs and the coupled Painlevé V system

Abstract

We study the Hankel determinant generated by the Laguerre weight with jump discontinuities at $t_k, k = 1, \cdots, m$. By employing the ladder operator approach to establish Riccati equations, we show that $\sigma_n(t_1, \cdots, t_m)$, the logarithmic derivative of the n-dimensional Hankel determinant, satisfies a generalization of the σ-from of Painlevé V equation. Through investigating the Riemann-Hilbert problem for the associated orthogonal polynomials and via Lax pair, we express σ_n in terms of solutions of a coupled Painlevé V system. We also build relations between the auxiliary quantities introduced in the above two methods, which provides connections between the Riccati equations and Lax pair. In addition, when each t_k tends to the hard edge of the spectrum and n goes to ∞, the scaled σ_n is shown to satisfy a generalized Painlevé III equation.

Joint work with Yang Chen and Shuai-Xia Xu

23 András Mészáros

University of Toronto Scarborough, Canada
The distribution of sandpile groups of random regular graphs

Abstract

We study the distribution of the sandpile group of random d-regular graphs. For the directed model, we prove that it follows the Cohen-Lenstra heuristics, that is, the limiting probability that the p-Sylow subgroup of the sandpile group is a given p-group P, is proportional to $|Aut(P)|^{-1}$. For finitely many primes, these events get independent in the limit. Similar results hold for undirected random regular graphs, where for odd primes the limiting distributions are the ones given by Clancy, Leake and Payne.
This answers an open question of Frieze and Vu whether the adjacency matrix of a random regular graph is invertible with high probability. Note this was independently proved by Huang.

24 Chao Min

Hua Qiao University, China

Hankel determinant and orthogonal polynomials for a perturbed Gaussian weight: from finite n to large n asymptotics

Abstract

We will talk about the monic polynomials orthogonal with respect to a symmetric perturbed Gaussian weight

$$w(x;t) := \mathrm{e}^{-x^2}\left(1 + t\,x^2\right)^{\lambda}, \qquad x \in \mathbb{R},$$

where $t > 0,\ \lambda \in \mathbb{R}$. This weight is related to the single-user MIMO systems in information theory. It is shown that the recurrence coefficient $\beta_n(t)$ is related to a particular Painlevé V transcendent, and the sub-leading coefficient $\mathrm{p}(n,t)$ satisfies the Jimbo-Miwa-Okamoto σ-form of the Painlevé V equation. Furthermore, we derive the second-order difference equations satisfied by $\beta_n(t)$ and $\mathrm{p}(n,t)$, respectively. This enables us to obtain the large n full asymptotic expansions for $\beta_n(t)$ and $\mathrm{p}(n,t)$ with the aid of Dyson's Coulomb fluid approach. We also consider the Hankel determinant $D_n(t)$, generated by the perturbed Gaussian weight. It is found that $H_n(t)$, a quantity allied to the logarithmic derivative of $D_n(t)$, can be expressed in terms of $\beta_n(t)$ and $\mathrm{p}(n,t)$. Based on this result, we obtain the large n asymptotic expansion of $H_n(t)$ and then that of the Hankel determinant $D_n(t)$. This is a joint work with Prof. Yang Chen.

25 Soumendu Sundar Mukherjee

Indian Statistical Institute, India
An O(n)-bit random matrix model for circular operators

Abstract

We describe an n-by-n matrix, with only O(n) bits of randomness, that converges in *-distribution to Voiculescu's circular operator. The matrix is constructed from a pair of independent Toeplitz and Hankel matrices by taking their Schur-Hadamard product. We allow the entries of these component matrices to form strongly multiplicative systems of random variables, satisfying a certain admissibility condition. Based on numerical simulations, we conjecture that the circular law is the limiting spectral measure of the proposed matrix ensemble.

26 Hoi Nguyen

Ohio State University, USA
A universality result for the cokernel of random integral matrices

Abstract

For a random matrix of entries sampled independently from a fairly general distribution in Z we discuss the probability that the cokernel is isomorphic to a given finite abelian group, or when it is cyclic. We will show that these statistics are asymptotically universal, given by precise formulas involving zeta values, and agree with distributions defined by Cohen and Lenstra. Based on joint work with M. M. Wood.

27 Maciej A. Nowak

Jagiellonian University, Poland
Eikonal formulation of large dynamical random matrix models

Abstract

The standard approach to dynamical random matrix models relies on the description of trajectories of eigenvalues. Using the analogy from optics, based on the duality between the Fermat principle (rays) and the Huygens principle (wavefronts), we formulate the Hamilton-Jacobi dynamics for large random matrix models. The resulting equations describe a broad class of random matrix models in a unified way, including normal (Hermitian or unitary) as well as strictly non-normal dynamics. This formalism applied to Brownian bridge dynamics allows one to calculate the asymptotics of the Harish-Chandra-Itzykson-Zuber integrals. The presentation is based on recent article by Grela, Nowak and Tarnowski, Phys. Rev. E 104 (2021)054111.

28 Sean O'Rourke

University of Colorado Boulder, USA
Random perturbations of non-normal matrices

Abstract

Consider the eigenvalues of a large, square non-normal matrix. For example, consider the matrix which is zero everywhere, except for ones on the entries directly above the diagonal. The eigenvalues of this matrix are all zero. However, due to floating point errors, certain numerical software packages compute the eigenvalues to be nearly equidistributed on the unit circle in the complex plane. In this talk, I will explain how one can model these errors using small, random perturbations, and I will give a probabilistic explanation for why the eigenvalues appear near the unit circle. More generally, I will discuss several theoretical results which describe the local, deterministic behavior of the eigenvalues of randomly perturbed non-normal matrices. This talk is based on joint work with Phillip Matchett Wood.

29 Zhijun Qiao

University of Texas Rio Grande Valley, USA
Integrable peakon models in scalar form

Abstract

In this talk, I will first track the history of integrable peakon system development in last 30 years, then introduce a new higher order model with peakons or pseudo-peakons we proposed recently [1]. Some open problems will also be addressed for discussion in the end.

References

[1] Enrique G. Reyes, Mingxuan Zhu and Zhijun Qiao, Pseudo-peakons and Cauchy analysis for an integrable fifth-order equation of Camassa-Holm type, arXiv:2111.07733.

30 Tomohiro Sasamoto

Tokyo Institute of Technology, Japan
Connecting the q-Whittaker measure to the periodic Schur measure by skew RSK dynamics

Abstract

The q-Whittaker measure plays an important role in integrable probability because of its relations to various discrete models in the Kardar-Parisi-Zhang (KPZ) universality class. Many results have been obtained but analysis often involves lengthy calculations since it is not directly associated with a determinant point process (DPP). On the other hand the periodic Schur measure is a periodic generalization of the Schur measure. It is associated with a DPP and various determinantal formulas are automatically available.

In this presentation, we explain the connection between the two measures discovered in [1,2]. This is achieved by showing a bijection between a pair of semi-standard tableaux (P,Q) and quadruple (V,W,kappa,nu) where V,W

are what we call vertically strict tableaux (equivalent to tensor products of KR crystals) and additional data. The bijection is constructed by the skew RSK dynamics which is an iterated skew RSK maps introduced by Sagan and Stanley in the late 1980s.

The talk is based on collaborations with Takashi Imamura and Matteo Mucciconi. Applications of this connection to KPZ models will be explained in a separate talk by T. Imamura.

References

[1] Takashi Imamura, Matteo Mucciconi, Tomohiro Sasamoto, Skew RSK dynamics: Greene invariants, affine crystals and applications to q-Whittaker polynomials, arXiv: 2106.11922

[2] Takashi Imamura, Matteo Mucciconi, Tomohiro Sasamoto, Identity between restricted Cauchy sums for the q-Whittaker and skew Schur polynomials, arXiv: 2106.11913

31 Gregory Schehr

Sorbonne Université, France

Non-intersecting Brownian bridges in the flat-to-flat geometry

Abstract

In this talk, I will discuss N non-intersecting Brownian bridges propagating from an initial configuration $\{a_1 < a_2 < \ldots < a_N\}$ at time $t = 0$ to a final configuration $\{b_1 < b_2 < \ldots < b_N\}$. I will first show that this problem can be mapped to a non-intersecting Dyson's Brownian bridges with Dyson index $\beta = 2$. For the latter I will derive an exact effective Langevin equation that allows to generate very efficiently the non-intersecting bridge configurations.

In particular, for the flat-to-flat configuration in the large N limit, where $a_i = b_i = (i-1)/N$, for $i = 1, \cdots, N$, I will use this effective Langevin equation to derive an exact Burgers' equation (in the inviscid limit) for the Green's function and solve this Burgers' equation for arbitrary time $0 \leq t \leq t_f$. Finally, I will discuss connections to some well known problems, such as the Chern-Simons model, the related Stieltjes-Wigert orthogonal polynomials and the Borodin-Muttalib ensemble of determinantal point processes.

32 Boris Shapiro

Stockholm University, Sweden
Rodrigues' descendants of a polynomial and Boutroux curves

Abstract

Motivated by the classical Rodrigues' formula, we study the root asymptotic of the polynomial sequence

$$R_{[\alpha n],n,P}(z) = \frac{d^{[\alpha n]}P^n(z)}{dz^{[\alpha n]}}, n = 0, 1, \ldots$$

where $P(z)$ is a fixed univariate polynomial, α is a fixed positive number smaller than deg P, and $[\alpha n]$ stands for the integer part of αn. Our description of this asymptotic is expressed in terms of an explicit harmonic function uniquely determined by the plane rational curve emerging from the application of the saddle point method to the integral representation of the latter polynomials using Cauchy's formula for higher derivatives. We show that this harmonic function is also associated with an abelian differential having only purely imaginary periods and the latter plane curve belongs to the class of Boutroux curves initially introduced by M.Bertola with coauthors.

33 Hao Shen

University of Wisconsin–Madison, USA
Lattice Yang-Mills and a dynamical approach

Abstract

The Lattice Yang-Mills model can be viewed as a model of a collection of random matrices. We will discuss the fundamental questions of this model and prove some results using a dynamical approach via stochastic analysis. Based on joint work with Scott Smith, Rongchan Zhu, Xiangchan Zhu.

34 Jack Silverstein

North Carolina State University, USA
On the eigenvectors of large dimensional sample covariance matrices

Abstract

Let $M_n = (1/N)V_nV_n^T$ where $V_n = (v_{ij})$ $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, N = N(n)$, the v_{ij}'s are i.i.d. standardized random variables, and $n/N \to c > 0$ as $n \to \infty$. M_n can be viewed as a sample covariance matrix formed from the N columns of V_n. When v_{11} is standard normal, the orthogonal matrix, O_n of eigenvalues of M_n is Haar distributed in the group of $n \times n$ orthogonal matrices. A review of past attempts to compare O_n with Haar measure when v_{11} is not Gaussian for n large will be given. Recent results will then be presented which extend an earlier result on eigenvector behavior.

35 Alexander Soshnikov

University of California, Davis, USA
On pair counting statistics in circular beta ensembles of random matrices

Abstract

I will talk about joint work with Ander Aguirre and Josh Sumpter. Motivated by Montgomery's classical result in Number Theory we study pair counting statistics in Circular Beta Ensembles in global, mesoscopic, and microscopic regimes.

36 Jacobus Johannes Maria Verbaarschot

Stony Brook University, USA
The integrable Sachdev-Ye-Kitaev model

Abstract

We discuss the many-body partition function of integrable Sachdev-Ye-Kitaev (SYK) model. The single particle energies of this model are given by the eigenvalues of a random model which make it possible to obtain exact analytical results in the limit of a large number of fermions. New results are obtained for the case of two coupled SYK models. In the limit of a large number of fermions, this model shows an infinite sequence of phase transitions which are determined by a random matrix model that was first introduced to describe chiral symmetry breaking in Quantum Chromo Dynamics.

37 Van Vu

Yale University, USA
Matrices with random perturbation

Abstract

I am going to talk about some new works concerning matrices with random perturbation (which can also be viewed as random matrices where entries have arbitrary means). This study is motivated by real life problems in data science and the theory of numerical computation, with many real-life applications.

38 Lun Zhang

Fudan University, China
Local universality in the Muttalib-Borodin ensembles

Abstract

In this talk, we consider the Muttalib-Borodin ensemble of Laguerre type, a determinantal point process on $[0, \infty)$ which depends on the varying weights $x^{\alpha}e^{-nV(x)}$, $\alpha > -1$, and a parameter θ. For θ being a positive integer, we derive asymptotics of the associated biorthogonal polynomials near the origin for a large class of potential functions V as $n \to \infty$. This further allows us to establish the hard edge scaling limit of the correlation kernel, which is previously only known in special cases and conjectured to be universal. Our proof is based on a Deift/Zhou nonlinear steepest descent analysis of two 1×2 vector-valued Riemann-Hilbert problems. Based on a joint work with Dong Wang.

39 Mengkun Zhu

Qilu University of Technology (Shandong Academy of Sciences, China

Asymptotics for a singularly perturbed GUE, Painlevé III and double-confluent Heun equations, small eigenvalues

Abstract

We discuss the recurrence coefficients of the three-term recurrence relation for the orthogonal polynomials with a singularly perturbed Gaussian weight $w(z) = |z|^{\alpha} \exp\left(-z^2 - t/z^2\right), z \in \mathbb{R},\ t > 0,\ \alpha > 1$. Based on the ladder operator approach, two auxiliary quantities are defined. We show that the auxiliary quantities and the recurrence coefficients satisfy some equations with the aid of three compatibility conditions, which will be used to derive the Riccati equations and Painlevé III. We show that the Hankel determinant has an integral representation involving a particular $\sigma-$form of Painlevé III. We study the asymptotics of the Hankel determinant under a suitable double scaling, i.e. $n \to \infty$ and $t \to 0$ such that $s = (2n + 1 + \lambda)t$ is fixed, where λ is a parameter with $\lambda := (\alpha \mp 1)/2$. The asymptotic behaviors of the Hankel determinant for large s and small s are obtained and the Dyson's constant is recovered here. They have generalized the results in the literature [C. Min et al, *Nucl. Phys. B*, **936** (2018) 169-188] where $\alpha = 0$. Together the Coulomb fluid method with the orthogonality principle, we obtain the asymptotic expansions of the recurrence coefficients, which are applied to derive the relationship between second order differential equations satisfied by our monic orthogonal polynomials and the double-confluent Heun equations, as well as to calculate the smallest eigenvalue of the large Hankel matrices generated by the above weight. Particularly, when $\alpha = t = 0$, the asymptotic behavior of the smallest eigenvalue for the classical Gaussian weight $\exp(-z^2)$ [G. Szegö, *Trans. Amer. Math. Soc.*, **40** (1936) 450-461] is recovered.