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Abstracts

Workshop II: Nonlocal and Singular Problems:
Recent Advances and Outlook

28 February–4 March 2022

1 Christopher Angstmann

UNSW Sydney, Australia

Compartment models with non-local operators and related stochastic processes

Abstract

Compartment models have a long history of use in mathematical modelling, underpinning many areas of practice including epidemiology, pharmacokinetics, and mathematical biology. These models consider the flow of objects (or people or energy) between different states that are referred to as compartments. There is continued interest in the generalisation of compartment models with the inclusion of fractional derivatives of particular interest. The direct replacement of integer order derivatives with fractional derivatives can lead to issues such as the loss of conservation of mass and the difficulty of interpretation of many of the parameters in the resulting models. I will present a method of constructing compartment models with fractional derivatives, or other non-local operators, that overcome these issues. The construction of models in a physically constant manner is achieved by first considering a non-Markovian stochastic process consisting of particles that are jumping between compartments. The non-local equations arise upon consideration of the mean of the process. Examples will be given of fractional order models as well as some simple delay equations.

2 Mejdi Azaiez

Bienvenue à Bordeaux INP, France

High order approximation for Müntz and Müntz-logarithmic polynomials using empirical interpolation method

Abstract

Traditional spectral method is essentially discretization method using polynomials for approximating solutions of partial-differential equations. The most attractive property of this method may be that when the solution of the problem is infinitely smooth, its convergence is exponential. On one side, the spectral method has achieved great success and become popular in the scientific computing community. On the other side, the method using traditional polynomials meets some limits, such as loss of accuracy for problems having non-smooth/singular solutions. Such problems can be found, for example, in integro-differential equations with singular kernels, fractional partial differential equations, etc. Singular behaviors can also be present in traditional equations at domain corners or points where the boundary condition changes type, and/or with singular coefficients.

Considerable progress has been made in recent years for treating such problems. Basically, in order to construct accurate spectral methods for problems with non-smooth solutions, one has to choose suitable functions which allow to capture the singularity of the underlying problem. Nevertheless, despite this progress, there is still a need for improving the capability of the spectral methods facing various problems. The idea is to make use of a larger class of function systems to more efficiently approximate solutions, which may have complex singularity structure. In this framework, choosing good interpolation points is in general an open problem. The classical Lagrange interpolation polynomial to approximate the singular function is not suitable. Besides, we don't have gauss quadrature to approximate singular function by Müntz logarithmic polynomials.

In this talk, we introduce a quadrature formula based on Empirical Interpolation Method (EIM). We will explain the method and the way to compute the so-called magic points and weights. We will define an interpolation operator based on these points. We will give several numerical results to assess the efficiency of the approach to interpolate and to numerically integrate some smooth functions and singular functions. We end our presentation by given an example of solving parameterized elliptic problems using our new

quadrature formula.

3 Juan Pablo Borthagaray

Universidad de la República del Uruguay, Uruguay

Regularity and approximation of fractional quasi-linear operators on Lipschitz domains

Abstract

In this talk, we discuss the formulation, regularity and finite element approximation of certain fractional quasi-linear operators on Lipschitz domains. As a model, we consider the fractional p-Laplacian of order s, where $p \in (1, \infty)$ and $s \in (0, 1)$ and which in case $p = 2$ reduces to the integral fractional Laplacian. We discuss recent results about Besov regularity on Lipschitz domains and a priori error estimates for finite element discretizations.

4 Yongyong Cai

Beijing Normal University, China

Numerical methods for computing ground states of spinor Bose-Einstein condensates

Abstract

The remarkable experimental achievement of Bose-Einstein condensation (BEC) in 1995 has drawn significant research interests in understanding the ground states and dynamics of trapped cold atoms. Different from the single component BEC, spinor BEC possesses the spin degree of freedom and exhibits rich phenomenon. In the talk, we will present some recent work for computing ground states of general spin-F BECs.

5 Zhenning Cai

National University of Singapore, Singapore
Numerical solver for the Boltzmann equation with self-adaptive collision operators

Abstract

In the numerical solver of the Boltzmann equation, the collision term is the most expensive part due to its binary integral form. To reduce the computational cost, we build a sequence of modeling collision operators that separate the distribution function into a low-frequency part and a high-frequency part and treat the high-frequency part only linearly as our simplification. The idea of such simplification can be considered as extensions of classical BGK and Shakhov models, and it requires a parameter controlling the ratio of the low-frequency part and the high-frequency part. In our work, this parameter is selected adaptively on every grid cell at every time step. This self-adaptation is based on an error indicator describing the difference between the model collision term and the original binary collision term. The indicator is derived by controlling the quadratic terms in the modeling error with linear operators. Our numerical experiments show that such an error indicator is effective and computationally affordable.

6 Eric Cances

Ecole des Ponts ParisTech, France
Electronic transport in materials: the singularity of graphene

Abstract

When a crystalline material is submitted to an external electric field, extremely different behaviors are observed depending on its atomic structure. In this talk, I will first show that the simple quantum Liouville equation for non-interacting electrons is able to explain qualitatively not only the insulating or metallic character of usual materials, but also the conductance quantization observed for some recently discovered topological materials. In particular the exotic conductivity properties of graphene result from the presence of singularities (Dirac cones) in the band spectrum of the Hamiltonian. If time allows, I will briefly present more advanced models and numerical

methods for computing quantitatively the electronic conductivity of materials from the first principles of quantum mechanics.

7 Kai Diethelm

University of Applied Sciences Würzburg-Schweinfurt - FHWS, Germany
Numerical aspects of the infinite state representation of fractional differential operators

Abstract

The infinite state representation, also known as the diffusive representation, is a way to express a fractional differential operator in the form of an integral, typically over the positive half-line, whose integrand can be written as the solution to a relatively simple initial value problem for a first order differential equation. As such, it permits to approximately compute fractional derivatives by the application of a numerical integration method to an integrand obtained by numerically solving the first order initial value problem. Compared to traditional methods, applying this approach as the underlying discretization of the fractional derivative in a solver for fractional differential equations has significant advantages with respect to run time and memory requirements of the algorithm. However, the algorithms obtained in this way depend on a number of parameters that are not trivial to interpret and whose influence on the accuracy of the final result is often unclear. In this talk, we will present an investigation of these dependencies and present some guidelines on how to choose the parameters.

8 Ling Guo

Shanghai Normal University, China
PINNs for solving forward and inverse problems governed by stochastic-fractional PDEs

Abstract

We consider solutions of forward and inverse problems governed by stochastic-fractional PDEs (SFPDEs), and in particular we target long-time integration. To this end, we develop a Physics-Informed Neural Network (PINN), and

propose a new extension for SFPDEs, which incorporates the bi-orthogonal constraints of stochastic modeling into the loss function with an implicit form. This approach can overcome some of the drawbacks of the original bi-orthogonal methods for time dependent SDEs. We will demonstrate the efficiency of the new network via several numerical examples.

9 Zhongyi Huang

Tsinghua University, China
Variational principle based method for image processing

Abstract

In this talk, we focus on image restoration/segmentation/super-resolution. We propose some new models for image restoration/segmentation/super-resolution. We then employ the tailored-finite-point method (TFPM), to solve the associated equation based on variational principle. Numerical experiments are presented to demonstrate the effectiveness of the proposed models and its features.

10 Fukeng Huang

National University of Singapore, Singapore
A new SAV approach for general dissipative systems

Abstract

In this talk, I will talk about some essential improvements on the original scalar auxiliary variable(SAV) approach. The new approach enjoys the following remarkable properties: 1. The computational cost is half of the original SAV approach. 2. It can be used to construct high-order unconditionally stable schemes with rigorous error analysis. 3. The new approach is not limited to the gradient flow systems, it can be applied to general dissipative systems. I will talk about the numerical schemes and error analysis for the new approach and numerical examples are provided to show the advantages of the new approach.

11 Shidong Jiang

Flatiron Institute, Simons Foundation, USA

A universal method for solving elliptic PDEs with singular boundary data on non-smooth domains

Abstract

A numerical scheme is presented for the solution of Fredholm second-kind boundary integral equations with right-hand sides that are singular at a finite set of boundary points. The boundaries themselves may be non-smooth. The scheme, which builds on recursively compressed inverse preconditioning (RCIP), is universal as it is independent of the nature of the singularities. Strong right-hand side singularities, such as $1/|r|^{\alpha}$ with α close to 1, can be treated in full machine precision. Adaptive refinement is used only in the recursive construction of the preconditioner, leading to an optimal number of discretization points and superior stability in the solve phase. The performance of the scheme is illustrated via several numerical examples, including an application to an integral equation derived from the linearized BGKW kinetic equation for the steady Couette flow.

This is joint work with Johan Helsing at Lund University, Sweden.

12 Bangti Jin

University College London, UK
Discovering the subdiffusion model in an unknown medium

Abstract

The subdiffusion phenomenon is now widely recognized in many engineering and physical applications. The mathematical models for subdiffusion involve several parameters, e.g., diffusion coefficient, potential, initial and boundary conditions along with the order of derivation. Sometimes some of these parameters are not readily available, but one can measure additional information about the solution. Then one natural question is how much we can say about the mathematical model. In this talk, I will discuss several theoretical and computational results on determining the order of derivation and other parameters when the other problem data are not fully specified. The talk is based on joint works with Yavar Kian and Zhi Zhou.

13 Wenyu Lei

Scuola Internazionale Superiore di Studi Avanzati, Italy
Approximation of the spectral fractional Laplace-Beltrami operator and its application to Gaussian fields on surfaces

Abstract

In this talk, we consider numerical approximation of fractional powers of Laplace-Beltrami operator on closed surfaces. The proposed numerical algorithms rely on their Balakrishnan integral representation and consists a sinc quadrature coupled with standard finite element methods for parametric surfaces. Possibly up to a log term, optimal rate of convergence are observed and derived analytically when the discrepancies between the exact solution and its numerical approximations are measured in both L^2 and H^1 norms. As an application, we will illustrate how we use the proposed numerical scheme to approximate Gaussian random fields on closed surfaces. Convergence tests as well as numerical simulations for Gaussian random fields are provided in this talk.

14 Jichun Li

University of Nevada, Las Vegas, USA

Finite element analysis and simulation for wave propagation in the Cole-Cole medium

Abstract

In this talk, I'll first introduce the Maxwell's equations in the Cole-Cole medium, which couples the standard Maxwell's equations with a time fractional constitutive equation. Then I will present some finite element methods developed in recent years for solving this model. Numerical results will be present to justify the theoretical analysis.

15 Changpin Li

Shanghai University, China

Logarithmic asymptotics: analysis and computation

Abstract

This report includes a series of work with Drs. Enyu Fan, Zhiqiang Li and Zhen Wang. The so-called logarithmic asymptotics indicates that the solution to the evolution equation has algebraic asymptotics in the sense of logarithmic function. It is often used to characterize the ultra slow process, such as creep in igneous rocks. In this talk, we mainly introduce the logarithmic asymptotics and regularity of the solution to the Caputo-Hadamard fractional evolution equation. Based on these theoretical analysis, we construct the reliable numerical algorithms to numerically solve it, where various kinds of numerical algorithms for Caputo-Hadamard derivatives are also displayed.

16 Huiyuan Li

Institute of Software Chinese Academy of Sciences, China
Novel spectral methods for Schrödinger equations with inverse-power potentials

Abstract

In the talk, we propose and analyze some spectral/spectral element methods for the eigenvalue problems of the Schrödinger equations with inverse-power potentials. Owing to its importance in quantum mechanics, it is of fundamental and practical interest to search for the explicit eigen-functions of the Schrödinger operator with a more general potential or some variance of the operator. However, such an inverse-power potential generally causes strong singularities in the eigenfunctions such that the classic spectral methods fail and have a limited accuracy.

Firstly, efficient spectral methods with an exponential convergence rate are proposed for the Schrödinger equation with an inverse square potential $V(x) = Z|x|^{-2}$ on a unit ball and the whole space in any dimensions together with an arbitrary planer sector. A mortar spectral element method is also developed for the Schrödinger equation in other geometries.

Next, novel spectral methods are proposed for Schrödinger eigenvalue problems with a more general potential including the Coulomb potential $V(x) = Z|x|^{-\alpha}$ and the hyper-singular potentials $V(x) = Z|x|^{-\alpha}$ $(\alpha > 2)$. Numerical analysis is then given to explain in detail the exponential order of convergence of these spectral methods.

Numerical experiments show that our spectral/spectral element methods are perfectly competent not only for problems with an inverse-power potential, but also for those with a singularity caused by reentrant corners.

17 Dong Liang

York University, Canada
Energy-preserving high-order difference methods for nonlocal wave equations

Abstract

Nonlinear wave equations are extensively used to describe nonlinear physical phenomena such as Josephson junction oscillators, the interaction of solitons,

dislocations in metals. Recently, for describing the wave propagations in complex media, where coefficient may changes frequently and highly depend on media, the confined wave propagations can be modeled by space-fractional wave equations. The nonlocal fractional derivatives naturally take into account the frequency variability of some dynamic coefficients of the medium like tortuosity and compressibility. As an important research filed, fractional partial differential equations (FPDEs) have recently attracted increasing attention due to their wide application in physics, chemistry, rheology, bioengineering and finance and so on and it has been shown that FPDEs can provide relatively accurate descriptions of some challenging phenomena such as anomalously diffusive transport, long-range interaction and memory effect. In this talk, I will present our recent development and analysis of time fourth-order energy-preserving average vector field (AVF) finite difference method for the nonlinear fractional wave equations with Riesz space-fractional derivative. The Hamiltonian system of the nonlinear fractional wave equations is first introduced. The fourth-order AVF scheme combining with the weighted and shifted Lubich difference operator is then developed. Energy conservation in the discrete form and unique solvability of the scheme are proved. Error estimates of the scheme are further analyzed in the discrete L2-norm. Numerical experiments are given to show the performance of the high-order scheme.

18 J. Markus Melenk

Technische Universität Wien, Austria

Weighted analytic regularity for the integral fractional Laplacian in polygons and application to *hp*-FEM

Abstract

We study the Dirichlet problem for the integral fractional Laplacian in a polygon Ω with analytic right-hand side. We show the solution to be in a weighted analyticity class that captures both the analyticity of the solution in Ω and the singular behavior near the boundary. Near the boundary the solution has an anisotropic behavior: near edges but away from the corners, the solution is smooth in tangential direction and higher order derivatives in normal direction are singular at edges. At the corners, also higher order tangential derivatives are singular. This behavior is captured in terms of weights

that are are products of powers of the distance from edges and corners.

The proof of the regularity assertions is based on the Caffarelli-Silvestre extension, which realizes the non-local fractional Laplacian as a Dirichlet-to-Neumann map of a (degenerate) elliptic boundary problem.

Our weighted analytic regularity can be used to show exponential convergence of high order finite element methods (*hp*-FEM) on suitably refined meshes. The refinement is towards both the edges and the corners with the refinement towards edges being anisotropic away corners.

This work is joint with Markus Faustmann (TU Wien), Carlo Marcati (ETHZ), and Christoph Schwab (ETHZ).

19 Sheehan Olver

Imperial College London, UK
Computing equilibrium distributions with power law interactions

Abstract

When particles interact, say by attracting or repulsing, they tend to form nice distributions as the number of particles become large. Examples include both physical (electrons in a potential well, space dust) and biological (flocking birds, bacteria). Naïve simulation via differential equations proves insufficient, with computational cost becoming prohibitively expensive in more than one dimensions. Instead, we will introduce techniques based on a measure minimisation reformulation using expansions in weighted orthogonal polynomials to approximate the measures, whereby incorporating the correct singularities of the distributions we can rapidly and accurately compute many such distributions in arbitrary dimensions. This leads to high accuracy confirmation of open conjectures on gap formation (imagine a flock of birds forming a ring, with no density in the middle). These techniques involve new recurrence relationship of power law kernels applied to weighted orthogonal polynomials.

20 Sihong Shao

Peking University, China

An efficient 6-D deterministic solver for the Wigner-Coulomb dynamics

Abstract

As a permanent goal and a tireless direction of computational mathematics, developing an accurate and stable 6-D solver has been attracting more and more attentions in recent years due to the urgent need in e.g., quantum science and high energy density physics. In this talk, we will report our effort in using grid-based numerical methods to solve the 6-D Wigner-Coulomb system, in which both the locality of spatial advection and the non-locality of quantum interaction are intrinsically built. More precisely, we propose a massively parallel solver, termed the characteristic-spectral-mixed scheme (CHASM), which utilizes the locally distributed cubic B-spline basis to interpolate the local spatial advection and the truncated kernel method to approximate the pseudodifferential operator with weakly singular symbol under the Coulomb interaction. Several typical numerical experiments demonstrate the accuracy and efficiency of CHASM, as well as its scalability up to 16000 cores. This is a joint effort with Yunfeng Xiong at Peking University and Yong Zhang at Tianjin University.

21 Jie Shen

Purdue University, USA

Efficient space-time methods for a class of time dependent problems with applications to nonlocal and singular problems

Abstract

We present efficient and accurate space-time methods for solving a class of linear and nonlinear time dependent problems. The method is based on a dual-Petrov spectral method for the time variable with a generic spatial discretization, and is shown to be spectrally accurate in the time variable. While the method leads to a coupled systems in space and time, we show that for linear problems, it can be reduced to a sequence of spatial discretization problems, and for nonlinear problems, one can use a Jacobean-free Newton Krylov iteration with a suitable linear solver as the preconditioner. We will

apply the methods to solve fractional time PDEs and present an effective approach to deal with the singularity at the initial time.

22 Changtao Sheng

Shanghai University of Finance and Economics, China
Fast implementation of FEMs for nonlocal models in multiple dimensions

Abstract

We introduce a fast and accurate semi-analytic computation of the stiffness matrix associated with the integral fractional Laplacian operator. We show that for the rectangular or L-shaped domains, each entry of the FEM stiffness matrix associated with the tensorial rectangular elements can be expressed explicitly by some one-dimensional integrals, which can be evaluated accurately. The key is to implement the FEM in the Fourier transformed space. Moreover, we extended the proposed method for the Peridynamic models. Hence, we shall also report our recent attempts towards FEM implementation of nonlocal models.

23 Qinglin Tang

Sichuan University, China
An efficient numerical method to compute the ground state of rotating dipolar Bose-Einstein Condensates

Abstract

In this talk, we will present an efficient numerical method for computing the ground state of the rotating dipolar Bose-Einstein Condensates (BEC). The method consists two main merits: (i) efficient and accurate numerical methods will be proposed to evaluate the nonlocal dipole-dipole interaction. (ii). a nonlinear conjugate gradient method, accelerated by some well-adapted preconditioners, will be developed to compute the ground states. This work is realized in collaboration with Xavier ANTOINE (IECL, Lorraine, France), Antoine LEVITT (Inria, Paris, France) and Yong ZHANG (Tianjin University, Tianjin, China).

24 Chong Wang

Washington and Lee University, USA
Periodic minimizers of a ternary nonlocal isoperimetric problem

Abstract

We study a two-dimensional ternary inhibitory system. The free energy functional combines an interface energy favoring micro-domain growth with a Coulomb-type long range interaction energy which prevents micro-domains from unlimited spreading. Here we consider a limit in which two species are vanishingly small, but interactions are correspondingly large to maintain a nontrivial limit. In this limit two energy levels are distinguished: the highest order limit encodes information on the geometry of local structures as a two-component isoperimetric problem, while the second level describes the spatial distribution of components in global minimizers. We provide a sharp rigorous derivation of the asymptotic limit, both for minimizers and in the context of Gamma-convergence. Geometrical descriptions of limit configurations are derived. The main difficulties are hidden in the optimal solution of two-component isoperimetric problem: compared to binary systems, not only it lacks an explicit formula, but, more crucially, it can be neither concave nor convex on parts of its domain.

25 Bo Wang

Hunan Normal University, China
Fast multipole method in layered media

Abstract

A fast multipole method (FMM) is presented for the computation of long-range interactions of sources embedded in 3-D layered media. The layered media Green's functions, which satisfies the transmission conditions at material interfaces, is decomposed into free space components and four types of reaction field components from reflections and transmissions through the layered media. The proposed algorithm is a combination of the classic FMM for the free space components and FMMs specifically designed for the four types reaction components, made possible by new multipole expansions (MEs) and local expansions (LEs) as well as the multipole-to-local translation (M2L)

operators for the reaction field components. The FMMs for the reaction components, which are implemented with the target particles and equivalent polarization sources associated with the reaction field components, are found to be much more efficient than that for the free space component due to the fact that the equivalent polarization sources and the target particles are always separated by a material interface. As a result, the FMM algorithm developed for layered media has a similar computational cost as that for the free space. Numerical results validate the fast convergence of the MEs and the $O(N)$ complexity of the FMM for N particle problem in 3-D layered media.

26 Boyi Wang

National University of Singapore, Singapore
Dissipation functionals and energy stability of numerical schemes for the time-fractional Allen-Cahn/Cahn-Hilliard equations

Abstract

Recently, the time-fractional Allen-Cahn/Cahn-Hilliard equations have been introduced to model the memory effect. It is natural to investigate the dissipation property of those equations. In this talk, two positive dissipation functionals for the time-fractional Allen-Cahn equation and the time-fractional Cahn-Hilliard equation shall be introduced respectively and be proven to be decreasing with respect to time. Based on those dissipation functionals, a new energy dissipation law for numerical schemes is be obtained; in addition, stabilized schemes preserving this energy dissipation shall be proposed and compared. Finally, some numerical results will be presented.

27 Chuanju Xu

Xiamen University, China
Regularization methods for inverse problems of the sub-diffusion equation

Abstract

In this talk I will discuss about inverse problems for the time-fractional diffusion equation, which have been known to be ill-posed problems. Several

regularization methods are presented and compared. In particular, a regularized problem with L2 and TV regularization is proposed and analyzed for the inverse source problem. The regularized problem is then approximated by a fully discrete scheme. The convergence rate of the discrete solution with respect to the target source and the convergence of the regularized solution with respect to the noise level will be provided. A series of numerical examples will be given to demonstrate the accuracy of the proposed methods.

28 Mohsen Zayernouri

Michigan State University, USA
Nonlocal subgrid-scale modeling for turbulent flows

Abstract

Turbulence remembers and is fundamentally nonlocal. Such a longing portrait of turbulence originates from the delineation of coherent structures/ motions, being spatially spotty, giving rise to interestingly anomalous spatio-temporal fluctuating signals. The statistical anomalies in such stochastic fields emerge as: sharp peaks, heavy-skirts of power-law form, long-range correlations, and skewed distributions, which scientifically manifest the non-Markovian/non-Fickian nature of turbulence at small scales. Such physical-statistical evidence highlights that 'nonlocal features' and 'global inertial interactions' cannot be ruled out in turbulence physics. This urges the development of new LES modeling paradigms in addition to novel statistical measures that can meticulously extract, pin-down, and highlight the nonlocal character of turbulence (even in the most canonical flows e.g., homogeneous isotropic turbulence) and their absence in the common/classic turbulence modeling practice.

We start from the filtered Boltzmann kinetic transport equations and model the corresponding equilibrium distribution functions (for both the fluid and scalar particles) with stable heavy-tailed distributions to address and incorporate the anomalous features at small scales. Next, we derive a new class of fractional-order and tempered Laplacian models for the divergence of subgrid-scale stresses, naturally emerging as the underlying subgrid-scale (SGS) LES models. We subsequently carry out the corresponding a priori and a posteriori tests to examine the performance of each fractional SGS model. Our proposed dynamic LES modeling approach exhibits promising

capabilities to effectively model and incorporate nonlocalities on the fly in the very LES (VLES) as well as the LES inertial sub-ranges. This novel LES modeling paradigm can be imperative for cost-efficient nonlocal turbulence modeling e.g., in meteorological and environmental applications.

29 Yanzhi Zhang

Missouri University of Science and Technology, USA
Numerical methods for nonlocal problems with the fractional Laplacian

Abstract

Recently, the fractional Laplacian has received great attention in modeling complex phenomena that involve long-range interactions. However, its nonlocality introduces considerable challenges in both mathematical analysis and numerical simulations. So far, numerical methods for the fractional Laplacian still remain limited. It is well-known that the fractional Laplacian can be defined either in a pseudodifferential form via the Fourier transforms or in a hypersingular integral form. In this talk, I will discuss two types of numerical methods to discretize the fractional Laplacian. First, I will introduce the finite difference methods based on the hypersingular integral form of the fractional Laplacian. In the second method, we combine both pseudodifferential and hypersingular integral forms of the fractional Laplacian and introduce meshfree methods with radial basis functions. The properties of these methods will be discussed, and some applications of nonlocal problems with the fractional Laplacian will also be demonstrated.

30 Zhongqiang Zhang

Worcester Polytechnic Institute, USA
Towards high-order methods for fractional advection-diffusion-reaction equations in smooth domains

Abstract

The fractional Laplacian is a promising mathematical tool due to its ability to capture the anomalous diffusion and model the complex physical phenomenon with long range interaction. One of important applications of frac-

tional Laplacian is a turbulence intermittency model of fractional Navier-Stokes equation. However, efficient computation on bounded domains is challenging as highly accurate and efficient numerical methods are not yet available, due to the intrinsic singularity and nonlocal nature of the fractional Laplacian. In this talk, we will discuss spectral methods and finite difference methods for elliptic equations with integral fractional Laplacian. The talk is based on our developments in these methods, including regularity theory and fast algorithms in 1D and 2D.

31 Tao Zhou

The State Key Laboratory of Scientific and Engineering Computing (LSEC), China

Monte Carlo PINN: deep learning approaches for fractional PDEs

Abstract

We present a deep learning approache for fractional PDEs by combining the PINN and a Monte Carlo type sampling scheme for the nonlocal part, and this results in a reduced computational complexity compared to exsiting deep learning approaches such as F-PINN.

32 Zhi Zhou

The Hong Kong Polytechnic University, Hong Kong, China

Inverse potential problem for fractional subdiffusion from terminal observation

Abstract

In this talk, we study the inverse problem of recovering a potential coefficient in the subdiffusion model, which involves a Caputo derivative of order $\alpha \in (0,1)$ in time, from the terminal observational data. We shall present several conditional stability estimates in Sobolev spaces, and these stability estimates further inspire proper numerical algorithm and relevant error analysis in different scenarios. The analysis relies on refined properties of solution operators involving two-parameter Mittag–Leffler functions. The efficiency and accuracy of the proposed algorithms are illustrated with several numerical examples.

References

[1] Z. Zhang and Z. Zhou. Recovering the potential term in a fractional diffusion equation. IMA J. Appl. Math. 82 (2017), no. 3, 579–600.

[2] B. Jin and Z. Zhou. An inverse potential problem for subdiffusion: stability and reconstruction. Inverse Problems 37 (2021), no. 1, 015006.

[3] Z. Zhang, Z. Zhang and Z. Zhou. Identification of potential in diffusion equations from terminal observation: analysis and discrete approximation. (2022) arXiv:2201.01563.

33 Jiang Yang

Southern University of Science and Technology, China
How to define energy dissipations for time-fractional phase-field equations

Abstract

There exists a well defined energy for classical phase-field equations under which the dissipation law is satisfied, i.e., the energy is non-increasing with respect to time. However, it is not clear how to extend the energy definition to time-fractional phase-field equations so that the corresponding dissipation law is still satisfied. In this work, we will try to settle this problem for phase-field equations with Caputo timefractional derivative, by defining a nonlocal energy as an averaging of the classical energy with a time-dependent weight function. As the governing equation exhibits both nonlocal and nonlinear behavior, the dissipation analysis is challenging. To deal with this, we propose a new theorem on judging the positive definiteness of a symmetric function, that is derived from a special Cholesky decomposition. Then, the nonlocal energy is proved to be dissipative under a simple restriction of the weight function. Within the same framework, the time fractional derivative of classical energy for timefractional phase-field models can be proved to be always nonpositive.