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Abstracts

Workshop I: Computation, Analysis and Applications of PDEs with Nonlocal and Singular Operators

7–11 February 2022

1 Gabriel Acosta

University of Buenos Aires, Argentina
Interpolation for weigthed Sobolev spaces

Abstract

Weighted and fractional Sobolev spaces associated to seminorms of the type

$$\int_\Omega \int_\Omega \frac{|f(x) - f(y)|^p}{|x-y|^{n+sp}} \, \delta^\beta(x,y) \, dy \, dx,$$

where $\delta(x,y) = \min\{d(x), d(y)\}$, have shown to be useful for establishing regularity results and for developing efficient numerical methods for nonlocal equations involving the fractional Laplace operator [1]. In this talk [3] we characterize the real interpolation space between L^p and a weighted Sobolev space (in arbitrary bounded domains in $\mathbb{R}^n$), with weights that are certain positive powers of the distance to the boundary. In particular,

$$(L^p(\Omega), W^{1,p}(\Omega, 1, d^p))_{s,p},$$

is characterized by means of the seminorm

$$\int_\Omega \int_{|x-y|<\frac{d(x)}{2}} \frac{|f(x) - f(y)|^p}{|x-y|^{n+sp}} \, dy \, d(x)^{sp} \, dx.$$

We discuss some related works and implications and generalizations of this result and its connection with the unweighted case treated in [2].

References

[1] Acosta, G.; Borthagaray, J. P. *A fractional Laplace equation: regularity of solutions and finite element approximations*. SIAM J. Numer. Anal. 55, (2017), no. 2, 472–495.

[2] Drelichman, I.; Durán, R. G. *On the interpolation space* $(L^p(\Omega), W^{1,p}(\Omega))_{s,p}$ *in non-smooth domains*. J. Math. Anal. Appl. 470 (2019), no. 1, 91–101.

[3] Acosta G.; Drelichman, I.; Durán, R. G. *Weighted fractional Sobolev spaces as interpolation spaces in bounded domains*, https://arxiv.org/pdf/2112.03416.pdf.

2 Xavier Antoine

Université de Lorraine, France

Design of perfectly matched layers for time-dependent space fractional PDEs

Abstract

Perfectly Matched Layers (PML) are proposed for time-dependent space fractional PDEs. Within this approach, widely used powerful Fourier solvers based on FFTs can be adapted without much effort to compute Initial Boundary Value Problems (IBVP) for well-posed fractional equations with absorbing boundary layers. We analyze mathematically the approach and propose some illustrating numerical experiments. This is a joint work with Emmanuel LORIN (Carleton University Canada).

References

X. Antoine and E. Lorin, Towards Perfectly Matched Layers for time-dependent space fractional PDEs, Journal of Computational Physics, 391, pp. 59-90, 2019.

3 Remi Carles

Centre national de la recherche scientifique, France
Logarithmic Schrödinger equationn with quadratic potential

Abstract

We consider the Schrödinger equation with a logarithmic nonlinearity, whose singularity at the origin causes important modifications on the dynamical properties. In the presence of a quadratic potential, we analyze the existence and stability of solitary waves, which may exhibit rather unusual features. The results presented are based on collaborations with Guillaume Ferriere and Chunmei Su, respectively.

4 José A. Carrillo

Oxford University, UK
Nonlocal aggregation-diffusion equations: entropies, gradient flows, phase transitions and applications

Abstract

This talk will be devoted to an overview of recent results understanding the bifurcation analysis of nonlinear Fokker-Planck equations arising in a myriad of applications such as consensus formation, optimization, granular media, swarming behavior, opinion dynamics and financial mathematics to name a few. We will present several results related to localized Cucker-Smale orientation dynamics, McKean-Vlasov equations, and nonlinear diffusion Keller-Segel type models in several settings. We will show the existence of continuous or discontinuous phase transitions on the torus under suitable assumptions on the Fourier modes of the interaction potential. The analysis is based on linear stability in the right functional space associated to the regularity of the problem at hand. While in the case of linear diffusion, one can work in the L2 framework, nonlinear diffusion needs the stronger Linfty topology to proceed with the analysis based on Crandall-Rabinowitz bifurcation analysis applied to the variation of the entropy functional. Explicit examples show that the global bifurcation branches can be very complicated. Stability of the solutions will be discussed based on numerical simulations with fully explicit energy decaying finite volume schemes specifically tailored

to the gradient flow structure of these problems. The theoretical analysis of the asymptotic stability of the different branches of solutions is a challenging open problem. This overview talk is based on several works in collaboration with R. Bailo, A. Barbaro, J. A. Canizo, X. Chen, P. Degond, R. Gvalani, J. Hu, G. Pavliotis, A. Schlichting, Q. Wang, Z. Wang, and L. Zhang. This research has been funded by EPSRC EP/P031587/1 and ERC Advanced Grant Nonlocal-CPD 883363.

5 Sheng Chen

Beijing Normal University, China

Log orthogonal functions in semi-infinite intervals: approximation results and applications

Abstract

We construct two new classes of log orthogonal functions in semi-infinite intervals, LOFs-II and GLOFs-II, by applying a suitable log mapping to Laguerre polynomials. We develop basic approximation theory for these new orthogonal functions, and show that they can provide uniformly good exponential convergence rates for problems in semi-infinite intervals with slow decay at infinity. We apply them to solve several linear and nonlinear differential equations whose solutions decay algebraically or exponentially with very slow rates, and present ample numerical results to show the effectiveness of the approximations by LOFs-II and GLOFs-II.

6 Jin Cheng

Fudan University, China
Inverse contact problems in elasticity: an non-local formulation

Abstract

In elasticity, the local stress and deformation of two contact objects under pressure are referred to as contact problems. The contact problem used to be a difficult problem for applied mathematical mechanics. In practical application, in some scenarios, it is difficult or impossible to observe some quantities, such as the stress distribution on the contact surface, so how to invert other information from the observed data on the elastic body has important theoretical and practical significance. In this talk, we propose a class of inverse problems for inversing the stress distributions of contact surfaces from boundary displacement data of half-space elastomer. The inverse problem is formulated as non-local fractional differential operator equation. It is proved that the observed data can uniquely invert the unknown function. We show that this kind of problem is a serious ill-posed problem. By using the method of analytic continuity, we prove that this problem has certain conditional stability, which provides a theoretical guarantee for the construction of stable numerical algorithms. At the same time, in order to obtain accurate measurement data, we put forward a method of using a large amount of data in exchange for measurement accuracy. Some numerical examples are presented.

Joint work with Chen Yu, Ke Yufei (Shanghai University of Finance and Economics, Shanghai, China)

7 Marta D'Elia

Sandia National Laboratories, USA
Nonlocal kernel network (NKN): a stable and resolution-independent deep neural network

Abstract

Neural operators have recently become popular tools for designing solution maps between function spaces in the form of neural networks. Differently

from classical scientific machine learning approaches that learn parameters of a known partial differential equation (PDE) for a single instance of the input parameters at a fixed resolution, neural operators approximate the solution map of a family of PDEs. Despite their success, the uses of neural operators are so far restricted to relatively shallow neural networks and confined to learning hidden governing laws. In this work, we propose a novel nonlocal neural operator, which we refer to as nonlocal kernel network (NKN), that is resolution independent, characterized by deep neural networks, and that is capable of handling a variety of tasks such as learning governing equations and classifying images.

Our NKN stems from the interpretation of the neural network as a discrete nonlocal diffusion reaction equation that, in the limit of infinite layers, is equivalent to a parabolic nonlocal equation, whose stability is analyzed via nonlocal vector calculus. The resemblance with integral forms of neural operators allows NKNs to capture long-range dependencies in the feature space, while the continuous treatment of node-to-node interactions makes NKNs resolution independent. The resemblance with neural ODEs, reinterpreted in a nonlocal sense, and the stable network dynamics between layers allow for generalization of NKN's optimal parameters from shallow to deep networks. This fact enables the use of shallow-to-deep initialization techniques.

Our tests show that NKNs outperform baseline methods in both learning governing equations and image classification tasks and generalize well to different resolutions and depths.

8 Weihua Deng

Lanzhou University, China
Probability perspective on nonlocal operators and nonlocal PDEs

Abstract

In this talk, we discuss the nonlocal PDEs in nonequilibrium statistical mechanics. We provide the probability points of view on nonlocal operators and nonlocal PDEs. Combining the methods of stochastic analyses and PDEs, we propose the numerical schemes and perform the numerical/wellposedness analyses for the nonlocal PDEs.

9 Jinqiao Duan

Illinois Institute of Technology, USA
Transition phenomena in non-Gaussian stochastic dynamical systems

Abstract

Dynamical systems under non-Gaussian Levy fluctuations manifest as nonlocality at a certain "macroscopic" level. Transition phenomena are special events for evolution from one metastable state to another, caused by the interaction between nonlinearity and uncertainty. Examples for such events are phase transition, pattern change, gene transcription, climate change, abrupt shifts, extreme transition, and other rare events. The most probable transition pathways are the maximal likely trajectory (in the sense of optimizing a probability or an action functional) between metastable states.

The speaker will present recent work (theory and machine learning methods) on the most probable transition pathways for stochastic dynamical systems, in the context of the Onsager-Machlup action functionals. This is joint work with Xiaoli Chen, Jianyu Chen, Jianyu HuWei Wei and Ting Gao.

10 Christian Glusa

Sandia National Laboratories, USA
Scalable methods for nonlocal models

Abstract

The naive discretization of nonlocal operators leads to matrices with significant density, as compared to classical PDE equations. This makes the efficient solution of nonlocal models a challenging task.

In this presentation, we will discuss on-going research into assembly and multilevel solution techniques that are suitable for nonlocal models.

11 Gerd Grubb

University of Copenhagen, Denmark
Dirichlet problems and evolution problems for the fractional Laplacian and generalizations

Abstract

We shall give an overview of recent results for the fractional Laplacian $(-\Delta)^a$ $(0 < a < 1)$ and related $2a$-order pseudodifferential operators P, in L_2-related spaces. In particular, we deal with boundary problems on sets $\Omega \subset R^n$ of limited smoothness $C^{1+\tau}$, including a LOCAL nonhomogeneous Dirichlet condition. Moreover, evolution problems (for $\partial_t + P$) are studied, both with homogeneous and nonhomogeneous Dirichlet conditions.

12 Hoang Viet Ha

Nanyang Technological University, Singapore

High dimensional finite elements for multiscale Maxwell wave equations

Abstract

We develop an essentially optimal numerical method for solving multiscale Maxwell wave equations in a domain D in $\mathbb{R}^d$. The problems depend on $n+1$ scales: one macroscopic scale and n microscopic scales. Solving the macroscopic multiscale homogenized problem, we obtain all the desired macroscopic and microscopic information. This problem depends on $n+1$ variables in $\mathbb{R}^d$, one for each scale that the original multiscale equation depends on, and is thus posed in a high dimensional tensorized domain. The straightforward full tensor product finite element (FE) method is exceedingly expensive. We develop the sparse tensor product FEs that solve this multiscale homogenized problem with essentially optimal number of degrees of freedom, that is essentially equal to that required for solving a problem posed in a domain in $\mathbb{R}^d$ only, for obtaining a required level of accuracy. The problems are more complicated than multiscale scalar wave equations due to the noncompact embedding of the space $H(curl, D)$ in $L^2(D)$ and the low regularity of the solution. Numerical correctors are constructed from the FE solution. For two sale problems, we show a rate of convergence for the numerical corrector in terms of the microscopic scale and the FE mesh width. Numerical examples confirm our analysis.

13 Lili Ju

University of South Carolina, USA

Unconditionally MBP-preserving exponential time differencing schemes for conservative Allen-Cahn equations

Abstract

In comparison with the Cahn-Hilliard equation, the classic Allen-Cahn equation satisfies the maximum bound principle (MBP) but fails to conserve the mass along the time. In this work, we study MBPs and corresponding MBP-preserving numerical schemes for two types of modified Allen-Cahn equations which can conserve the mass exactly. One is formed by introducing a nonlocal Lagrange multiplier to enforce the mass conservation, and the other is achieved through a Lagrange multiplier with both nonlocal and local effects. We propose first and second order stabilized exponential time differencing schemes (ETD) for solving these conservative Allen-Chan equations, which are linear and shown to be unconditionally MBP-preserving in the time discrete sense. Convergence of the two ETD schemes is then analyzed as well as their energy stability. Various numerical experiments in two and three dimensions are also carried out to validate the theoretical results and demonstrate the performance of the proposed schemes.

14 Honglin Liao

Nanjing University of Aeronautics and Astronautics, China

Energy stability of variable-step L1-type schemes for time-fractional Cahn-Hilliard model

Abstract

This talk concerns the positive definiteness of discrete time-fractional derivatives, which is fundamental to the numerical stability (in the energy sense) for time-fractional phase-field models. A novel technique is proposed to estimate the minimum eigenvalue of discrete convolution kernels generated by the nonuniform L1, half-grid based L1 and time-averaged L1 formulas of the fractional Caputo's derivative. The main discrete tools are the discrete orthogonal convolution kernels and discrete complementary convolution kernels. Certain variational energy dissipation laws at discrete levels of the variable-step L1-type methods are then established for time-fractional Cahn-Hilliard model. They are shown to be asymptotically compatible, in the fractional order limit $\alpha \to 1$, with the associated energy dissipation law for the classical Cahn-Hilliard equation. Numerical examples together with an

adaptive time-stepping procedure are provided to demonstrate the effectiveness of the proposed methods.

15 Emmanuel Lorin

Carleton University, Canada

Efficient computation of fractional linear algebraic systems

Abstract

In particular motivated by the approximation of fractional differential operators, we are interested in this talk, in the numerical computation of fractional algebraic linear systems [1]. Different strategies will be presented, including a double-preconditioning approach applied to Cauchy integrals and to ODE-based solvers [2,3]. If time permits, I will also talk about fractional linear systems involving several matrix power functions [4].

This is a joint work with X. Antoine.

References

[1] Numerical study of fractional linear system solvers. E. Lorin, S.Tian. Math. Mathematics and Computers in Simulations, 2021.

[2] ODE-based double-preconditioning for solving linear systems. X. Antoine, E. Lorin. Numerical Linear Algebra with Applications. 2021.

[3] Double Cauchy integral preconditioning for fractional linear systems. X. Antoine, E. Lorin. Submitted.

[4] Generalized fractional linear system solvers. X. Antoine, E. Lorin. To appear in Journal of Scientific Computing, 2022.

16 Zhiping Mao

Xiamen University, China

Spectral approximations of fractional Schrödinger equations and the ground states

Abstract

In this talk I shall present some results about the spectral approximations of fractional nonlinear Schrödinger equations and the ground states. First, I will show some results for the fractional nonlinear Schrödinger equations in unbounded domain using Hermite spectral method. Then I will show some results for the ground states of fractional nonlinear Schrödinger equations with spectral definition and verify the blowup solutions for the fractional nonlinear Schrödinger equations with different initial conditions.

17 Peter A. Markowich

King Abdullah University of Science and Technology, Saudi Arabia
Selection dynamics for deep neural networks

Abstract

We present a partial differential equation framework for deep residual neural networks and for the associated learning problem. This is done by carrying out the continuum limits of neural networks with respect to width and depth. We study the wellposedness, the large time solution behavior, and the characterization of the steady states of the forward problem. Several useful time-uniform estimates and stability/instability conditions are presented. We state and prove optimality conditions for the inverse deep learning problem, using standard variational calculus, the Hamilton-Jacobi-Bellmann equation and the Pontryagin maximum principle. This serves to establish a mathematical foundation for investigating the algorithmic and theoretical connections between neural networks, PDE theory, variational analysis, optimal control, and deep learning. (This is based on joint work with Hailiang Liu.)

18 William McLean

UNSW Sydney, Australia
Superconvergence for discontinuous Galerkin time stepping

Abstract

We describe a characteristic error profile for discontinuous Galerkin time stepping. Using piecewise polynomials of degree at most $r - 1$ in time,

with $r \geq 1$, the method is unconditionally stable and achieves an optimal convergence rate of order k^r, where k denotes the maximum time step. For classical parabolic PDEs, if $r \geq 2$ then the right-hand limit of the DG solution at each break point can be superconvergent of order up to k^{2r-1}, depending on the regularity of the solution. We show that the DG solution is also superconvergent of order k^{r+1} at the right Radau quadrature points in each subinterval. A simple postprocessing step then yields a *continuous* piecewise polynomial of degree at most $r \geq 2$ that achieves the optimal convergence rate k^{r+1} uniformly in time. In the nonlocal, subdiffusive case we conjecture that this k^{r+1} rate reduces to $k^{r+\alpha}$ with $0 < \alpha < 1$.

19 Kassem Mustapha

King Fahd University of Petroleum and Minerals, Saudi Arabia
A second-order accurate numerical scheme for a time-fractional Fokker-Planck equation with a general driving force

Abstract

A second-order accurate time-stepping scheme for solving a time fractional Fokker-Planck equation of order $\alpha \in (0, 1)$, with a general driving force, will be introduced. We state some regularity properties and briefly mention the challenges. Then, the focus will be on the α-robust stability bound of our time-stepping numerical scheme. Concerning the error analysis, we show an optimal second-order accurate estimate where time-graded mesh is employed to compensate for the singular behavior of the continuous solution near the origin. For the fully discrete scheme, our time-stepping scheme is combined with a standard spatial Galerkin finite element discretization. We discuss the accuracy of the obtained scheme and deliver some numerical tests to numerically support our theoretical contributions. For the current work, some interesting cases will be highlighted at the end.

20 Ricardo Nochetto

University of Maryland, USA
Fractional diffusion in Lipschitz domains: regularity and approximation

Abstract

This talk describes the formulation of linear fractional diffusion via the integral Dirichlet Laplacian, the regularity of solutions on bounded domains and the approximation by finite element methods. It emphasizes recent research about Besov regularity on Lipschitz domains, BPX preconditioning, and a priori error estimates in quasi-uniform and graded meshes. It also discusses extensions to quasi-linear fractional problems.

21 Xavier Ros-Oton

ICREA and University of Barcelona, Spain
The Neumann problem for the fractional Laplacian

Abstract

We study a Neumann problem for the fractional Laplacian. We start with some basic properties, such as its variational formulation, probabilistic interpretation, existence of solutions, and then turn our attention to their regularity. We will present our main new boundary regularity result, and list some questions that remain open. This is a joint work with A. Audrito and J.C. Felipe-Navarro.

22 Zhonghua Qiao

The Hong Kong Polytechnic University, Hong Kong, China
Stabilization parameter analysis of a second order linear numerical scheme

Abstract

A second order accurate (in time) and linear numerical scheme is proposed and analyzed for the nonlocal Cahn-Hilliard equation. The backward differentiation formula (BDF) is used as the temporal discretization, while an explicit extrapolation is applied to the nonlinear term and the concave expansive term. In addition, an $O(^2)$ artificial regularization term, in the form of $A\Delta_N(\phi^{n+1} - 2\phi^n + \phi^{n-1})$, is added for the sake of numerical stability. The resulting constant-coefficient linear scheme brings great numerical convenience; however, its theoretical analysis turns out to be very challenging, due to the lack of higher order diffusion in the nonlocal model. In fact, a rough energy stability analysis can be derived, where an assumption on the

ℓ^∞ bound of the numerical solution is required. To recover such an ℓ^∞ bound, an optimal rate convergence analysis has to be conducted, which combines a high order consistency analysis for the numerical system and the stability estimate for the error function. We adopt a novel test function for the error equation, so that a higher order temporal truncation error is derived to match the accuracy for discretizing the temporal derivative. Under the view that the numerical solution is actually a small perturbation of the exact solution, a uniform ℓ^∞ bound of the numerical solution can be obtained, by resorting to the error estimate under a moderate constraint of the time step size. Therefore, the result of the energy stability is restated with a new assumption on the stabilization parameter A. Some numerical experiments are carried out to display the behavior of the proposed second order scheme, including the convergence tests and long-time coarsening dynamics.

23 Martin Stynes

Beijing Computational Science Research Center, China
Variable-exponent Volterra integral equations (and variable-order fractional-derivative problems)

Abstract

Piecewise polynomial collocation of weakly singular Volterra integral equations (VIEs) of the second kind has been extensively studied in the literature, where integral kernels of the form $(t-s)^{-\alpha}$ for some constant $\alpha \in (0,1)$ are considered. Variable-order fractional-derivative differential equations currently attract much research interest, and in Zheng and Wang SIAM J. Numer. Anal. 2020 such a problem is transformed to a weakly singular VIE whose kernel has the above form with variable $\alpha = \alpha(t)$, then solved numerically by piecewise linear collocation, but it is unclear whether this analysis could be extended to more general problems or to polynomials of higher degree. In the present paper the general theory (existence, uniqueness, regularity of solutions) of variable-exponent weakly singular VIEs is developed, then used to underpin an analysis of collocation methods where piecewise polynomials of any degree can be used. The sharpness of the theoretical error bounds obtained for the collocation methods is demonstrated by numerical examples.

24 Chunmei Su

Tsinghua University, China

Regularized numerical methods and analysis for the Logarithmic Schrödinger equation

Abstract

We present some regularized models for the singular logarithmic Schrödinger equation (LogSE) and establish the error bounds. In order to suppress the round-off error and to avoid the blow-up of the logarithmic nonlinearity, some regularized logarithmic Schrödinger equations (RLogSE) are proposed with a small regularization parameter $\varepsilon \in (0,1]$ and linear convergence is established between the solutions of RLogSE and LogSE in terms of ε. Then we use the first-order splitting integrator to solve the regularized model and establish a nontrivial error bound $O(\tau^{1/2}\ln(\varepsilon^{-1}))$ with the time step τ, which implies an error bound at $O(\varepsilon + \tau^{1/2}\ln(\varepsilon^{-1}))$ for the LogSE by the Lie-Trotter splitting method. Numerical results are shown to confirm the error bounds and to demonstrate rich and complicated dynamics of the LogSE.

25 Hai-wei Sun

University of Macau, China

Strang's splitting method for spatial fractional Allen-Cahn equations

Abstract

In this talk, we study the numerical solutions of the spatial fractional Allen-Cahn equations. After semi-discretization for the spatial fractional Riesz derivative, a system of nonlinear ordinary differential equations with Toeplitz structure is obtained. For the sake of reducing the computational complexity, Strang splitting methods are proposed where the Toeplitz matrix in the system is split into the sum of a circulant matrix and a skew circulant matrix. Therefore, the proposed method can be quickly implemented by the fast Fourier transform, substituting to calculate the expensive Toeplitz matrix exponential. Theoretically, the discrete maximum principle of our method is unconditionally preserved. Moreover, the analysis of error in norms in Banach space with second-order accuracy is conducted in both time and space.

Finally, numerical tests are given to corroborate our theoretical conclusions and the efficiency of the proposed method.

It is a joint work with Yaoyuan Cai, Zhiwei Fang, and Hao Chen

26 Xiaochuan Tian

University of California San Diego, USA

Dyadic norm nonlocal function spaces with heterogeneous localization

Abstract

The study of nonlocal function spaces with heterogeneous localization was motivated by one of the nonlocal-local coupling techniques. In [Tian-Du, 2017] and [Du-Mengesha-Tian, 2021], mathematical properties have been shown for such nonlocal functions spaces which generalize the well-known trace theorems and Hardy's inequalities for the classical $W^{s,p}$ functions. In this talk, we will give a new class of nonlocal function spaces defined via dyadic norms. On one hand, this will largely simplify the previous proof of trace theorems for nonlocal function spaces (which contain $W^{s,p}$ as subspaces) of Sobolev-type. On the other hand, the dyadic norm definition also generalizes to a new class of Besov-type spaces on which trace theorems can be easily discussed.

27 Hong Wang

University of South Carolina, USA

An optimal control of a variable-order fractional PDE

Abstract

Optimal control of fractional diffusion PDEs demonstrates its competitive modeling abilities of challenging phenomena as anomalously diffusive transport and long-range interactions. In applications as bioclogging and oil/gas recovery, the structure of porous materials may evolve in time that leads to variable-order fractional FDEs via the Hurst index of the fractal dimension of the porous materials. The variable-order optimal control encounters mathematical and numerical issues that are not common in its integer-order and constant-order fractional analogues: (i) The adjoint state equation of the

variable-order Caputo time-fractional PDE turns out to be a different and more complex type of variable-order Riemann-Liouville time-fractional PDE. (ii) The coupling of the variable-order fractional state PDE and adjoint state PDE, and the variational inequality reduces the regularity of the solution to the optimal control. (iii) The numerical approximation to fractional optimal control model needs to be analyzed due to the low regularity and coupling of the model. We will prove the wellposedness and regularity of the model and an optimal-order error estimate to its numerical discretization.

28 Juncheng Wei

The University of British Columbia, Canada
On fractional Gierer-Meinhardt system

Abstract

We consider the one-dimensional fractional Gierer-Meinhardt system

$$u_t + (-\epsilon^2\Delta)^{s_1} u = -u + u^2/v$$

$$v_t + (-D_1\Delta)^{s_2} v = -v + u^2$$

with periodic boundary condition, $0 < s_1 < 1, 0 < s_2 < 1$. We show that the critical thereshold for the stability of spike solutions for the exponent s_1 is $s_1 = 0.25$. The effect of the second fractional exponent s_2 is more like a dimensional constant: when $s_2 > 1/2$ this system behaves like classical one-dimensional GM, and for $s_2 = 1/2$ it is like two-dimensional GM, and for $s_2 < 1/2$ phenomena similar to three-dimensional GM are found. It is quite interesting to see that all kinds of pattern formation phenomena can be found in a simple one-dimensional fractional reaction-diffusion system. (Joint work with D. Gomez and W. Yang.)

29 Masahiro Yamamoto

The University of Tokyo, Japan
Comparison principles and time fractional diffusion-wave equations

Abstract

We consider a time-fractional diffusion equation $(u(x,t) - a(x)) + Au(x,t) = F(x)$ for $0 < t < T$ and $x \in$, where is a bounded domain in ^{d} with $d = 1, 2, 3$ and , $0 < \alpha < 1$, is a Caputo type of fractional deriavtive in Sobolev spaces, and $-A$ is a uniform elliptic operator of the second order. We attach the homogeneous Neumann or the Robin boundary condition. First we prove the positivity: If $F \geq 0$ in $\times(0,T)$ and $a \geq 0$ in , then $u \geq 0$ in $\Omega \times (0,T)$. Next we apply the positivity to a semilinear time-fractional diffusion equation $(u(x,t) - a(x)) + Au(x,t) = f(x, u(x,t))$ for $(x,t) \in \Omega \times (0,T)$, and establish a monotone method by upper and lower solutions. As applications, we prove

a priori estimates or asymptotic behavior of solutions to a semilinear time-fractional diffusion equation.

This is a joint work with Professor Yury Luchko (Beuth Technical University of Applied Sciences Berlin).

References

[1] A. Kubica, K. Ryszewska, and M. Yamamoto, *Time-fractional Differential Equations, A Theoretical Introduction*, Springer Japan, Tokyo, 2020.

30 Jiwei Zhang

Wuhan University, China

On uniform second order nonlocal approximations to linear two-point boundary value problems

Abstract

In this talk, nonlocal approximations are considered for linear two-point boundary value problems (BVPs) with Dirichlet and mixed boundary conditions, respectively. These nonlocal formulations are constructed from nonlocal variational problems that are analogous to local problems. The well-posedness and regularity of the resulting nonlocal problems are established, along with the convergence to local problem as the nonlocal horizon parameter tends to 0. Uniform second order accuracy with respect to of the nonlocal approximation to the local solution, spatially in the pointwise sense, can be achieved under suitable conditions. Numerical simulations are carried out to examine the order of convergence rate, which also motivate further refined asymptotic estimates.

31 Yong Zhang

Tianjin University, China

A spectrally accurate numerical method for computing the Bogoliubov-de Gennes excitations of dipolar Bose-Einstein condensates

Abstract

In this paper, we propose an efficient and robust numerical method to study the ele-mentary excitation of dipolar Bose-Einstein condensates (BEC), which is governed by the Bogoliubov- de Gennes equations (BdGEs) with nonlocal dipole-dipole interaction, around the mean field ground state. Analytical properties of the BdGEs are investigated, which could serve as benchmarks for the numerical methods. To evaluate the nonlocal interactions accurately and efficiently, we propose a new Simple Fourier Spectral Convolution method (SFSC). Then, integrating SFSC with the standard Fourier spectral method for spatial discretization and Implicitly Restarted Arnoldi Methods (IRAM) for the eigenvalue problem, we derive an efficient and spectrally accurate method, named as SFSC-IRAM method, for the BdGEs. Ample numerical tests are provided to illustrate the accuracy and efficiency. Finally, we apply the new method to study systematically the excitation spectrum and Bogoliubov amplitudes around the ground state with different parameters in different spatial dimensions.

32 Zhimin Zhang

Beijing Computational Science Research Center, China
Efficient spectral methods and error analysis for nonlinear Hamiltonian systems

Abstract

We investigate efficient numerical methods for nonlinear Hamiltonian systems. Three polynomial spectral methods (including spectral Galerkin, Petrov-Galerkin, and collocation methods). Our main results include the energy and symplectic structure preserving properties and error estimates. We prove that the spectral Petrov-Galerkin method preserves the energy exactly and both the spectral Gauss collocation and spectral Galerkin methods are energy conserving up to spectral accuracy. While it is well known that collocation at Gauss points preserves symplectic structure, we prove that the Petrov-Galerkin method preserves the symplectic structure up to a Gauss quadrature error and the spectral Galerkin method preserves the symplectic structure to spectral accuracy. Furthermore, we prove that all three methods converge exponentially (with respect to the polynomial degree) under

sufficient regularity assumption. All these aforementioned properties make our methods possible to simulate the long time behavior of the Hamiltonian system. Numerical experiments indicate that our algorithms are efficient.