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Abstracts

Conference on Index Theory and Related Topics

23–27 August 2021

1 Pierre Albin

University of Illinois Urbana-Champaign, USA

[Dirac-type operators on stratified spaces](#)

Abstract

Stratified spaces arise naturally even when studying smooth objects, e.g., as algebraic varieties, orbit spaces of smooth group actions, and many moduli spaces. There has recently been a lot of activity developing analysis on these spaces and studying topological invariants such as the signature. I will report on work with Jesse Gell-Redman on the index of Dirac-type operators satisfying a ‘Witt’ condition and hence having a natural choice of Fredholm domain and then describe work in progress with Gell-Redman and Paolo Piazza that goes beyond the Witt condition.

2 Jean-Michel Bismut

Université Paris-Saclay, France

[Coherent sheaves, Chern character, and RRG](#)

Abstract

Let X be a compact complex manifold. If $(E, \nabla^{E''})$ is a holomorphic vector bundle, and if g^E is a Hermitian metric on E , we get an associated Chern connection on E , and corresponding Chern character forms.

I will show how to extend this construction to arbitrary coherent sheaves, using the antiholomorphic superconnections of Block. An antiholomorphic

superconnection is a differential operator of degree 1, with the same principal symbol as $\bar{\partial}$, whose square vanishes.

Using generalized metrics, we obtain a Chern character with values in Bott-Chern cohomology.

For complex varieties and arbitrary coherent sheaves, we establish a Riemann-Roch-Grothendieck theorem with values in Bott-Chern cohomology. One can interpret part of this theorem as a refined version of the Atiyah-Singer families index theorem.

This is joint work with Shu SHEN and Zhaoting WEI.
<https://arxiv.org/abs/2102.08129>.

3 Xianzhe Dai

UC Santa Barbara, USA

[Analytic torsion for Witten deformation on noncompact manifolds](#)

Abstract

The Calabi-Yau/Landau-Ginzburg correspondence in mirror symmetry has highlighted the importance of studying Witten deformation on noncompact manifolds. I would like to report on joint work with Junrong Yan in which we develop the notion of analytic torsion for Witten deformation on noncompact manifolds following our earlier work on the cohomologies and local index theorems and the pioneering work of Bismut-Zhang for Witten deformation on compact manifolds. In particular we establish the Cheeger-Muller/Bismut-Zhang theorem for the analytic torsion in this setting.

4 Hao Guo

Texas A&M University, USA

[Higher A-hat genera for proper actions and positive scalar curvature](#)

Abstract

I will report on recent joint work with V. Mathai, in which we formulate new topological obstructions to invariant metrics of positive scalar curvature that take into account the cohomology of the classifying space for proper actions. This leads to a natural generalisation of Gromov-Lawson's notion of higher

A-hat genera to the setting of proper actions by groups with torsion, and it is conjectured that these invariants are obstructions to invariant positive scalar curvature. We verify this when the cohomology class is generated by elements of degree at most 2, under suitable assumptions, via index theory.

5 Pedram Hekmati

The University of Auckland, New Zealand
[Twisted torsion for finite group actions](#)

Abstract

Analytic torsion is an invariant of flat vector bundles on a closed odd dimensional manifold. It was generalized by Mathai and Wu to 2-periodic complexes associated to flat superconnections, where the prototype is the twisted de Rham complex. In this talk we extend the twisted analytic torsion to the equivariant setting for finite group actions. We also consider a corresponding equivariant combinatorial torsion and use an analytic deformation argument to establish a twisted equivariant Cheeger-Muller theorem. This is joint work with Olivia Sorto.

6 Nigel Higson

The Pennsylvania State University, USA
[On Perrot's index cocycles](#)

Abstract

Denis Perrot has pioneered a very original approach to the construction of index cocycles in cyclic cohomology (Adv. Math., 2013). The approach has found significant applications. For instance Perrot and Rodsphon have used it to answer long-standing questions in the transverse index theory of Connes and Moscovici. But despite its originality and utility, Perrot's work hasn't yet been widely embraced by the index theory community. In an effort to address this, I shall try to give an accessible introduction to the method, focusing on the basic case of the Atiyah-Singer index theorem, and the even more basic case (from the point of view of the method) of the vanishing of the noncommutative residue in cyclic cohomology. This is joint work with Jonathan Block.

7 Peter Hochs

Radboud University, Netherlands

[Index theory on generalised hyperbolic cusps](#)

Abstract

We study manifolds that are compact apart from a finite number of “generalised hyperbolic cusps”: infinitely long ends of a type that includes both hyperbolic cusps and cylindrical ends. We give conditions for Dirac operators on such manifolds to be Fredholm, and give an index formula for such operators. As one might expect, this index formula consist of a contribution from the interior of the manifold, and a contribution from the cusps, or “from infinity”. The contribution from a cusp is given by an expression that reduces to the eta-invariant in the case of a cylindrical end. In general, this expression depends on the spectrum of a Dirac operator on a compact hypersurface, and involves eigenfunctions of a Sturm–Liouville operator on the half-line that depends on the specific cusp shape. These cusp contributions vanish if the Dirac operator on the hypersurface has symmetric spectrum around zero. So as in the work of Atiyah, Patodi and Singer on manifolds with boundary, spectral asymmetry plays an important role. This is joint work with Hang Wang.

8 Bo Liu

East China Normal University, China

[Eta invariant and localization](#)

Abstract

In 1968, Atiyah and Segal established a localization formula for the equivariant index which computes the equivariant index via the contribution near the fixed point sets of a group action. It is natural to ask whether such localization property holds for more complex spectral invariants, e.g. eta invariant. In this talk, we will prove a version of localization formula for equivariant eta invariant by using differential K-theory. This is a joint work with Xiaonan Ma.

9 Richard Melrose

Massachusetts Institute of Technology, USA

[The Dirac-Ramond operator](#)

Abstract

In this talk I plan to spend some time explaining the background to the definition of the Dirac-Ramond operator on the loop space of a string manifold. Since there is still quite a gap in the (or at least my) understanding of the analytic properties of this operator I will try to indicate what I believe remains to be done.

10 Martin Puchol

Université Paris-Saclay, France

[A comparison between the Bismut-Lott torsion and the Igusa-Klein torsion](#)

Abstract

We consider a smooth fibration with compact fiber together with a flat complex vector bundle over the total space. The associated analytic torsion form is an even differential form on the base manifold, which arises from a transgression of Bismut and Lott's Riemann-Roch-Grothendieck formula. Igusa and Klein constructed the topological counterpart of the analytic torsion form, known as the higher topological torsion. The relation between the analytic torsion form and the higher topological torsion is a natural and important problem in the theory of higher torsion invariants, and a higher version of the Cheeger-Müller/Bismut-Zhang theorem is expected. As a step towards this problem, it was conjectured in 2003 that the analytic torsion form should satisfy a gluing formula. In this talk, we confirm this conjecture, and use it to obtain a comparison formula between the Bismut-Lott torsion and the Igusa-Klein torsion. This talk is based on a joint work with Yeping Zhang and Jialin Zhu.

11 Shu Shen

Sorbonne Université, France

[The fried conjecture for admissible twists](#)

Abstract

The relation between the spectrum of the Laplacian and the closed geodesics on a closed Riemannian manifold is one of the central themes in differential geometry. Fried conjectured that the analytic torsion, which is an alternating product of regularized determinants of the Laplacians, equals the zero value of the dynamical zeta function. In this talk, I will show the Fried conjecture on locally symmetric spaces twisted by an acyclic flat vector bundle obtained by the restriction of a representation of the underlying Lie group. This generalises the results of myself for unitarily twists, and the results of Brocker, Muller, and Wotzker on closed hyperbolic manifolds.

12 Yanli Song

Washington University in St. Louis, USA

[Higher orbital integrals and APS index theorem](#)

Abstract

In this talk, we will present an index theorem for proper cocompact Lie group actions on manifolds with boundary, which generalizes the Atiyah-Patodi-Singer index theorem for compact manifolds and the Atiyah-Bott fixed point index theorem for compact Lie group actions on closed manifolds. In addition, We give sufficient conditions ensuring the well-definedness of the delocalized eta invariant associated to a Dirac operator on a cocompact manifold without boundary. The talk is based on the joint work with Paolo Piazza, Hessel Posthuma and Xiang Tang.

13 Xiang Tang

Washington University in St. Louis, USA

[Higher orbital integrals and cyclic cohomology](#)

Abstract

In this talk, we will report our exploration of cyclic cohomology of the Harish-Chandra Schwartz algebra of a connected real reductive Lie group. We will introduce explicit cyclic cocycles generalizing the classical orbital integrals.

As an application, we will present an index theorem for proper cocompact Lie group actions on closed manifolds. This talk is based on the joint work with Pierre Clare, Nigel Higson, Peter Hochs, Markus Pflaum, Hessel Posthuma, and Yanli Song.

14 Boris Vertman

University of Bonn, Germany

[Low energy resolvent on manifolds with fibered boundary](#)

Abstract

We present our recent results on the low energy resolvent on manifolds with fibered boundary, equipped with complete ϕ -metrics in the open interior. These manifolds generalize scattering manifolds. Their natural examples are non-abelian magnetic monopoles and gravitational instantons. Applications of our results relate to Riesz transform as well as to analytic torsion.

15 Hang Wang

East China Normal University, China

[A delocalized Riemann-Roch theorem for proper actions](#)

Abstract

For a discrete group, we use deformation of Lie groupoids to define a pairing between the left-hand side of the Baum-Connes assembly map and the cyclic cohomology of the group ring. As an application, we obtain a delocalized Riemann-Roch theorem for proper actions. This is joint work with Paulo Carrillo Rouse and Bai-Ling Wang.

16 Siye Wu

National Tsinghua University, Taiwan

[Anomaly and symmetry breaking in quantisation](#)

Abstract

In quantum physics, a symmetry is an automorphism of the quantum operator algebra. A classical symmetry is anomalous if it can not be realised as a quantum symmetry consistently. The quantum Hilbert space is an irreducible representation of the operator algebra. A symmetry is spontaneously broken if it is an outer automorphism mapping one irreducible representation to another. In this talk, we explore some of these conceptual issues in the framework of deformation and geometric quantisation.

17 Zhizhang Xie

Texas A&M University, USA

[Index theory and Gromov's conjectures on positive scalar curvature](#)

Abstract

I will talk about my recent work on solutions to some Gromov's conjectures and open questions on positive scalar curvature. These include solutions to Gromov's cube inequality on positive scalar curvature, and Gromov's open question on λ -Lipschitz rigidity of positive scalar curvature metrics on hemispheres or more generally spheres with certain subsets removed. If time permits, I will also talk about my recent joint work with Jinmin Wang and Guoliang Yu on a proof for Gromov's (strict) cube inequality on positive scalar curvature with the optimal constant.

18 Ken-Ichi Yoshikawa

Kyoto University, Japan

[Degeneration of Riemann surfaces and small eigenvalues of Laplacian](#)

Abstract

Let $f : X \dashrightarrow S$ be a one parameter family of compact Riemann surfaces over a curve. Assume that X is a compact Kaehler surface. Then the fibers of f are endowed with the metric induced from the Kaehler metric on X . If the singular fiber of f is not irreducible, then some eigenvalues of the Laplacian of the regular fiber converge to zero as the regular fiber approaches the singular fiber. We call such eigenvalues small eigenvalues. Here the Laplacian means the one acting on the functions of each regular fiber. In this talk, when the

singular fiber is reduced, I explain the asymptotic behavior of the “product” of all small eigenvalues of Laplacian.

19 Guoliang Yu

Texas A&M University, USA

[Quantitative K-theory and decay of scalar curvature](#)

Abstract

Gromov proved a quadratic decay inequality of scalar curvature for a class of manifolds. In this talk, I will explain how to use quantitative K-theory to prove that if a uniformly contractible manifold has finite asymptotic dimension, then the scalar curvature decays to zero at a rate depending only on the contractibility radius and the asymptotic dimension. We also construct examples of uniformly contractible manifolds with finite asymptotic dimension whose scalar curvature decays to zero arbitrarily slowly. This is joint work with Jingmin Wang and Zhizhang Xie.

20 Yeping Zhang

Korea Institute for Advanced Study, Korea

[Bbirational invariance of the BCOV invariant](#)

Abstract

Bershadsky, Cecotti, Ooguri and Vafa constructed a real valued invariant for Calabi-Yau manifolds, which is now called the BCOV invariant. In this talk, we extend the BCOV invariant to such pairs (X, D) , where X is a compact Kaehler manifold and D is a canonical divisor on X with simple normal crossing support. Based on this extension of BCOV invariant, we prove the birational invariance of the BCOV invariant. This result is a joint work with Lie Fu.