Contents

Abstracts

Conference on Complex Analysis and Geometry

16–20 August 2021

1   Fabrizio Bianchi

University of Lille, France
Higher bifurcations for polynomial skew products

Abstract

Given a holomorphic family of rational maps on the Riemann Sphere, one can decompose the parameter space into a stability locus and a bifurcation locus. The latter corresponds to maps whose global dynamics are very sensitive to a perturbation of the parameter and is characterized as the support of the so-called bifurcation current.

The changes in the global dynamics are dictated by changes in the dynamics of the critical set. When several critical points are present, it makes sense to define a stratification of the bifurcation locus, depending on how many critical points bifurcate independently. The good framework to do this is by means of the self-intersections of the bifurcation current, and one can prove that the resulting stratification is strict.

We consider in this talk dynamical systems on C^2 of the form f(z,w)= (p(z),q(z,w)), for suitable polynomials p,q. The stability/bifurcation dichotomy in higher dimensions was developed in previous joint work with Berteloot and Dupont. We prove here that, in contrast with the one-dimensional case, all the self-intersections of the bifurcation current have the same support.

Joint work with Matthieu Astorg, Orleans.

3

2 Judith Brinkschulte

University of Leipzig, Germany
Dynamical aspects of holomorphic foliations with ample normal bundle

Abstract

I will report on a joint paper with M. Adachi, where we proved a conjecture by Brunella: Let X be a compact complex manifold of dimension ≥ 3. Let $\mathcal{F}$ be a codimension one holomorphic foliaton with ample normal bundle. Then every leaf of $\mathcal{F}$ accumulates to the singular set of $\mathcal{F}$. I will also discuss related results concerning the nonexistence problem for Levi-flat real hypersurfaces.

3 Hoang-Son Do

Vietnam Academy of Science and Technology, Vietnam
Viscosity solutions to parabolic complex Monge-Ampère equations

Abstract

In this talk, we present our recent results about the Cauchy-Dirichlet problem for the Parabolic complex Monge-Ampère equation on a strictly pseudoconvex domain. Our results extend the earlier works by Eyssidieux-Guedj-Zeriahi (2015) on the existence of viscosity solution and the convergence at infinity. We also analyze the solvability of the Cauchy-Dirichlet problem by giving several characterizations for the admissible condition.

4 Peter Ebenfelt

University of California San Diego, USA
Obstruction flat rigidity of the standard CR 3-sphere

Abstract

Obstruction flat rigidity of the standard CR 3-sphere On a bounded strictly pseudoconvex domain in $\mathbb{C}^n$, $n > 1$, the smoothness of the Cheng-Yau solution to Fefferman's complex Monge-Ampère equation up to the boundary is obstructed by a local curvature invariant of the boundary, the CR obstruction

density $\mathcal{O}$. While local examples of obstruction flat CR manifolds are plentiful, the only known compact examples are the spherical CR manifolds. In this talk, we shall consider the problem of characterizing obstruction flatness for small deformations of the standard CR 3-sphere. In the case of embeddable CR structures, it was previously known that obstruction flat rigidity holds for the standard CR sphere (in all dimensions), where this rigidity also holds at the infinitesimal level. In the 3-dimensional case, however, a CR structure need not be embeddable. Unlike in the embeddable case, it turns out that in the non-embeddable case there is an infinite dimensional space of solutions to the linearized obstruction flatness equation on the standard CR 3-sphere and this space defines a natural complement to the tangent space of the embeddable deformations. In spite of this, as we shall explain in this talk, the CR 3-sphere does not admit nontrivial obstruction flat deformations, embeddable or nonembeddable. This is a report on recent joint work with Sean Curry.

5   Teng Fei

Rutgers University - Newark, USA
A geometric flow for Type IIA superstrings

Abstract

The equations of flux compactifications of Type IIA superstrings were written down by Tomasiello and Tseng-Yau. To study these equations, we introduce a natural geometric flow on symplectic Calabi-Yau 6-manifolds. We prove the well-posedness of this flow and establish the basic estimates. As an application, we make use of our flow to find optimal almost complex structures on certain homogeneous symplectic half-flat manifolds. This is based on joint work with Phong, Picard and Zhang.

6   Hendrik Herrmann

Bergische Universität Wuppertal, Germany
G-equivariant embedding theorems for CR manifolds of high codimension

Abstract

A famous result due to Boutet de Monvel shows that any compact strongly pseudoconvex codimension one CR manifold of dimension greater than or equal to five is CR embeddable into the complex Euclidean space. Based on the approach used in his work we present an embedding result for the high codimension case under the hypothesis of a CR Lie group action. As an application we obtain a Boutet de Monvel style embedding result for CR orbifolds.

This is a joint work with Kevin Fritsch and Chin-Yu Hsiao.

7 Chin-Yu Hsiao

Institute of Mathematics, Academia Sinica, Taiwan
Geometric quantization on CR manifolds

Abstract

Let X be a compact connected orientable CR manifold with the action of a connected compact Lie group G. Under natural pseudoconvexity assumptions we show that the CR Guillemin-Sternberg map is Fredholm at the level of Sobolev spaces of CR functions. As application we study this map for holomorphic line bundles which are positive near the inverse image of 0 by the momentum map. We also show that "quantization commutes with reduction" for Sasakian manifolds. This is a joint work with Xiaonan Ma and George Marinescu.

8 Xiaojun Huang

Rutgers University, USA
Bergman-Einstein metric over a normal Stein space with a compact strongly pseudo-convex boundary

Abstract

In this talk, I give a discussion when a normal Stein space with a strongly pseudo-convex boundary is holomorphically equivalent to the ball when its Bergman metric is Einstein. The materials are taken from a joint paper with Ming Xiao and a recent preprint with X. Li.

9   Jun-Muk Hwang

Institute for Basic Science, Korea
Minimal rational curves and 1-flat irreducible G-structures

Abstract

1-flat irreducible G-structures, equivalently, irreducible G-structures admitting torsion-free affine connections, have been studied extensively in differential geometry, especially in connection with the theory of affine holonomy groups. In a joint work with Qifeng Li, we study them in a setting in algebraic geometry, where they arise from varieties of minimal rational tangents (VMRT) associated to families of minimal rational curves on uniruled projective manifolds. We prove that such a structure is locally symmetric when the dimension of the uniruled projective manifold is at least 5. By the classification result of Merkulov and Schwachhoefer on irreducible affine holonomy, the problem is reduced to the case when the VMRT at a general point of the uniruled projective manifold is isomorphic to a subadjoint variety. In the latter situation, we prove a stronger result that, without the assumption of 1-flatness, the structure arising from VMRT is always locally flat. The proof employs the method of Cartan connections. An interesting feature is that Cartan connections are considered not for the G-structures themselves, but for certain geometric structures on the spaces of minimal rational curves.

10   Eiji Inoue

RIKEN, Japan
Canonical metrics and Perelman's entropy in Kahler geometry

Abstract

Extremal metric and Kahler-Ricci soliton are both generalizations of Kahler-Einstein metric from different perspectives. I will talk about a natural generalization of both of these metrics, which I call mu-cscK metric. On one hand, the notion is deeply connected to Perelman's entropy firstly appeared in the study of Ricci flow. On the other hand, it is also connected to algebraic geometry, K-stability. The goal of this talk is to explain an algebro-geometric bound on Perelman's mu-entropy for Kahler metrics.

11 Lucas Kaufmann

Institute for Basic Science, Korea
Random walks on $SL_2(C)$

Abstract

Given a sequence of independent and identically distributed random 2 by 2 complex matrices, it is a classical problem to study the statistical properties of their products.
In this talk I will show how methods from complex analysis can be used to obtain several new limit theorems for these random processes, often in their optimal version. I will also discuss new results on the Fourier coefficients of the Furstenberg measure. This is based on joint works with T.-C. Dinh and H. Wu.

12 Sung Yeon Kim

Institute for Basic Science, Korea
Effective Kohn algorithm for holomorphic subelliptic multipliers

Abstract

In 1979, J.J. Kohn proposed an algorithm to produce subelliptic multipliers for the $\bar{\partial}$-Neumann problem. Kohn's original procedure gives no effective bound on the order of subellipticity in subelliptic estimates. In 2010, Y.-T. Siu proposed an effective procedure to terminate Kohn's algorithm for holomorphic subelliptic multipliers. In this talk, we present Siu's effective algorithm as well as Kohn's full radical algorithm and their difference. Then we introduce a triangular system of multipliers. Finally, we give applications of triangular system for domains with holomorphic subelliptic multipliers including Kohn's special domains. This is a joint work with D. Zaitsev.

13 Qifeng Li

Institute for Basic Science, Korea
Unbendable rational curves of Goursat type and Cartan type

Abstract

In this talk we discuss the relation between the geometry of unbendable rational curves on a complex manifold and natural differential systems on the corresponding Douady space parameterizing these curves. We give a correspondence between the germs of unbendable rational curves of Goursat type with ordinary differential equations of order three, and give a correspondence between the germs of unbendable rational curves of Cartan type with germs of contact manifolds. This is a joint work with Jun-Muk Hwang.

14   Kefeng Liu

University of California, Los Angeles, USA
Solving equations with Hodge theory

Abstract

I will discuss several applications of Hodge theory in solving equations related to deformations of complex structures.

15   George Marinescu

University of Cologne, Germany
On the singularities of the Bergman projections for lower energy forms on complex manifolds with boundary

Abstract

We determine the boundary behavior of the spectral kernel of the d-bar-Neumann Laplacian of a domain with smooth boundary near points where the Levi form is non-degenerate. As a consequence we show that the Bergman projection admits an asymptotic expansion under a certain closed range condition for d-bar in L2. This is a joint work with Chin-Yu Hsiao.

16   Nordine Mir

Texas A&M University, Qatar
Finite jet determination of CR maps of positive codimension

Abstract

We will present recent results about finite jet determination of CR maps of positive codimension from real-analytic CR manifolds into Nash manifolds in complex spaces of possibly different dimension. This is joint work with B. Lamel.

17   Ngoc-Cuong Nguyen

Korea Advanced Institute of Science and Technology (KAIST), Korea
The complex Sobolev space and Hölder continuous solutions to Monge-Ampère equations

Abstract

This is a joint work with Tien-Cuong Dinh and Sławomir Kołodziej. Let X be a compact Kähler manifold of dimension n and ω a Kähler form on X. We consider the complex Monge-Ampère equation $(dd^c u + \omega)^n = \mu$, where μ is a given positive measure on X of suitable mass and u is an ω-plurisubharmonic function. We show that the equation admits a Hölder continuous solution *if and only if* the measure μ, seen as a functional on a complex Sobolev space $W^*(X)$, is Hölder continuous. A similar result is also obtained for the complex Monge-Ampère equations on domains of $\mathbb{C}^n$.

18   Viet-Anh Nguyen

University of Lille, France
Positive plurisubharmonic currents: Generalized Lelong numbers and Tangent theorems

Abstract

Dinh–Sibony theory of tangent and density currents is a recent but powerful tool to study positive closed currents. Over twenty years ago, Alessandrini–Bassanelli initiated the theory of the Lelong number of a positive plurisubharmonic current in C^k along a linear subspace. Although the latter theory is intriguing, it has not yet been explored in-depth since then. Introducing the concept of the generalized Lelong numbers and studying these new numerical values, we extend both theories to a more general class of positive

plurisubharmonic currents and in a more general context of ambient manifolds.

More specifically, in the first part of our talk, we consider a positive plurisubharmonic current T of bidegree (p, p) on a complex manifold X of dimension k, and let $V \subset X$ be a Kähler submanifold of dimension l and B a relatively compact piecewise $\mathcal{C}^2$-smooth open subset of V. We impose a mild reasonable condition on T and B, namely, T is weakly approximable by $T_n^+ - T_n^-$ on a neighborhood U of $\overline{B}$ in X, where $(T_n^\pm)_{n=1}^\infty$ are some positive plurisubharmonic $\mathcal{C}^3$-smooth forms of bidegree (p, p) defined on U such that the masses $\|T_n^\pm\|$ on U are uniformly bounded and that the $\mathcal{C}^3$-norms of $T_n^\pm$ are uniformly bounded near ∂B if $\partial B \neq \varnothing$. Note that if X is Kähler and T is of class $\mathcal{C}^3$ near ∂B (for example, this $\mathcal{C}^3$-smoothness near ∂B is automatically fulfilled if either $\partial B = \varnothing$ or $V \cap \mathrm{supp}(T) \subset B$), then the above mild condition is satisfied. Then, we define the notion of the j-th Lelong number of T along B for every j with $\max(0, l - p) \leq j \leq \min(l, k - p)$ and prove their existence as well as their basic properties. We also show that T admits tangent currents and that all tangent currents are not only positive plurisubharmonic, but also partially V-conic and partially pluriharmonic. When the current T is moreover pluriharmonic (resp. closed), then every tangent current is also V-conic pluriharmonic (resp. V-conic closed). We also prove that the generalized Lelong numbers are intrinsic. In fact, if we are only interested in the *top degree* Lelong number of T along B (that is, the j-th Lelong number for the maximal value $j = \min(l, k - p)$), then under a suitable holomorphic context, the above condition on the uniform regularity of $T_n^\pm$ near ∂B can be removed. Our method relies on some Lelong-Jensen formulas for the normal bundle to V in X, which are of independent interest.

The second part of our talk is devoted to geometric characterizations of the generalized Lelong numbers. As a consequence of this study, we show that the top degree Lelong number of T along B is strongly intrinsic. This is a generalization of the fundamental result of Siu (for positive closed currents) and of Alessandrini–Bassanelli (for positive plurisubharmonic currents) on the independence of Lelong numbers at a single point on the choice of coordinates.

19 Yûsuke Okuyama

Kyoto Institute of Technology, Japan
Parabolic bifurcation loci in the dynamical moduli spaces of rational functions

Abstract

The dynamical moduli space M_d of rational functions of a given degree $d > 1$ is the orbit space of the conjugation action of PSL_2 over the space of rational functions of degree d on the projective line. In this talk, we would give a precise description of the parabolic bifurcation locus in M_d, that is, the collision locus of periodic orbits of rational functions of degree d in terms of the multipliers of periodic orbits of rational functions of degree d.

20 Keiji Oguiso

The University of Tokyo, Japan
Some aspects of real form problem of a smooth complex projective variety

Abstract

My talk is a report of current joint works with Professors Tien-Cuong Dinh and Xun Yu.

After Lesieutre, there made several works related to a long standing open question since Kharlamov: "Is there a smooth complex projective rational surface with infinitely many real forms up to isomorphisms, i.e. , with infinitely many ways to describe by a system of equations with real coefficients up to isomorphisms over the real number field?"

After a brief introduction on the real form problem with currently known results and one way to reduce the problem to a problem of concrete complex geometry, I would like to explain our affirmative answer to the question above, with an outline of proof using a special Kummer K3 surface and its rich geometry.

I would like to close this talk by discussing relevant results in higher dimensions and/or non-negative Kodaira dimensions, together with related open problems.

21 Luca Rizzi

The University of Tokyo, Japan
Fujita decomposition and Massey product for fibered varieties

Abstract

Let $f\colon X \to B$ be a semistable fibration where X is a smooth complex variety of dimension n and B is a smooth complex curve. By a famous result by Fujita the direct image $f_*\omega_{X/B}$ is the sum $f_*\omega_{X/B} = \oplus$ of a unitary flat vector bundle and an ample vector bundle . Unitary flat bundles are in one-to-one correspondence with local systems and monodromy representations on the base curve B; in this talk we present a way to study this flat bundle using the theory of Massey products. We give an interpretation of the Fujita decomposition which allows to show the existence, up to base change, of higher irrational pencils and the finiteness of the associated monodromy representations under natural Castelnuovo-type hypothesis on local subsystems. This will give interesting results on the semi-ampleness of .

Based on a joint work with Prof. Francesco Zucconi.

22 Min Ru

University of Houston, USA
Nevanlinna hyperbolicity for complex manifolds

Abstract

Similar to the notion of the algebraic hyperbolicity introduced by J.P. Demailly and Xi Chen, we introduce the notion of Nevanlinna hyperbolicity for a pair (X, D), where X is a complex projective variety and D is an Cartier divisor on X which may be empty. This notion links and unifies the Nevanlinna theory, the complex hyperbolicity (Brody and Kobayashi hyperbolicity), the big Picard type extension property (more generally the Borel hyperbolicity). It also implies the algebraic hyperbolicity. This is a joint work with Yan He.

23 Nikhil Savale

University of Cologne, Germany
Bergman-Szegő kernel asymptotics in weakly pseudoconvex finite type cases

Abstract

We construct a pointwise Boutet de Monvel-Sjöstrand parametrix for the Szegő kernel of a weakly pseudoconvex three dimensional CR manifold of finite type assuming the range of its tangential CR operator to be closed; thereby extending the earlier analysis of Christ. This particularly extends Fefferman's boundary asymptotics of the Bergman kernel to weakly pseudoconvex domains in C2, in agreement with D'Angelo's example. Finally our results generalize a three dimensional CR embedding theorem of Lempert.

24 Laurent Stolovitch

Université Côte d'Azur, France
Geometry of hyperbolic Cauchy-Riemann singularities and KAM-like theory for holomorphic involutions

Abstract

This joint work with Zhiyan Zhao (Nice) is concerned with the geometry of germs of real analytic surfaces in $(\mathbb{C}^2, 0)$ having an isolated Cauchy-Riemann (CR) singularity at the origin. These are perturbations of *Bishop quadrics*. There are two kinds of CR singularities stable under perturbation : *elliptic* and *hyperbolic*. Elliptic case was studied by Moser-Webster who showed that such a surface is locally, near the CR singularity, holomorphically equivalent to *normal form* from which lots of geometric features can be read off.

In this talk, we focus on perturbations of *hyperbolic* quadrics. As was shown by Moser-Webster, such a surface can be transformed to a formal *normal form* by a formal change of coordinates that may not be holomorphic in any neighborhood of the origin.

Given a *non-degenerate* real analytic surface M in $(\mathbb{C}^2, 0)$ having a *hyperbolic* CR singularity at the origin, we prove the existence of a Whitney smooth

family of holomorphic curves intersecting M along holomorphic hyperbolas. This is the very first result concerning hyperbolic CR singularity not equivalent to quadrics.

This is a consequence of a non-standard KAM-like theorem for pair of germs of holomorphic involutions $\{\tau_1, \tau_2\}$ at the origin, a common fixed point. We show that such a pair has large amount of invariant analytic sets biholomorphic to $\{z_1 z_2 = const\}$ (which is not a torus) in a neighborhood of the origin, and that they are conjugate to restrictions of linear maps on such invariant sets.

25 Song-Yan Xie

Academy of Mathematics and Systems Science, China
On Ahlfors currents

Abstract

We answer a basic question in Nevanlinna theory that Ahlfors currents associated to the same entire curve may be nonunique. Indeed, we will construct one exotic entire curve which produces infinitely many cohomologically different Ahlfors currents. Moreover, concerning Siu's decomposition, for an arbitrary positive integer k or k=infinity, some of the obtained Ahlfors currents have singular parts supported on k irreducible curves. In addition, they can have nonzero diffuse parts as well, which answers a question of Brunella. This is joint work with Dinh Tuan Huynh (JMPA 2021, arxiv.org/abs/2101.11973).

26 Ben Weinkove

Northwestern University, USA
The Chern-Ricci flow

Abstract

I will give a survey on the Chern-Ricci flow, a parabolic flow of Hermitian metrics on complex manifolds. I will emphasize open problems and new directions.

27 Xiangyu Zhou

Academy of Mathematics and Systems Science, China
Siu's lemma: generalizations and applications

Abstract

Siu's lemma related to multiplier ideal sheaves plays an important role in Siu's work on some open problems in complex algebraic geometry. In this talk, we'll present some generalizations which are related to optimal L^2 extension and strong openness of multiplier ideal sheaves, and their applications in some problems such as optimal L^2 extension satisfying the property of Siu's lemma, comparison between singular metrics on the twisted relative pluri-canonical bundles, subadditivity of Kodaira-Iitaka's dimensions with multiplier ideal sheaves, the extension of cohomology classes. This is based on joint work with Langfeng Zhu.