Fundamental groups in non-archimedean geometry

PIOTR ACHINGER

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ABSTRACT

One of the main reasons non-archimedean geometry is useful to an algebraic geometer is the appearance of covering spaces of infinite degree. Key examples include Tate's uniformization of elliptic curves, its generalizations due to Mumford and Raynaud, and the theory of p-adic period mappings. At the same time, a comprehensive theory of the fundamental group of rigid spaces remains elusive. A major step in this direction has been taken by de Jong, who studied morphisms which locally on the base (in the sense of Berkovich analytic geometry) are disjoint unions of finite etale coverings. I will describe a vast generalization of this, based on the idea that good 'covering spaces' can be characterized by the property of 'unique arc lifting'. This is joint work with Marcin Lara and Alex Youcis.

Real forms on rational surfaces

Anna Bot

University of Basel, Switzerland

ABSTRACT

A real form of a complex algebraic variety X is a real algebraic variety whose complexification is isomorphic to X. For many families of complex algebraic varieties, it is known that the number of isomorphism classes over \mathbb{R} of real forms is finite. However, only recently Lesieutre [Les18] exhibited a complex projective variety with infinitely many nonisomorphic real forms; this work was then extended by Dinh, Oguiso and Yu. Yet, the case of a smooth complex projective rational surface still remains elusive. In this talk, we will present the content of [Bot21], in which for any positive integer r, we construct a smooth complex projective rational surface which has at least r real forms not isomorphic over \mathbb{R} , thus giving an unboundedness statement.

Primitive homogeneous varieties

MICHEL BRION

Université Grenoble Alpes. France

ABSTRACT

The starting point of the talk is the classification of birational actions of an algebraic group G in terms of pairs (K, X), where K is a function field and X a homogeneous variety under the group G_K obtained by base change. To analyze such varieties, one may start with the "primitive" ones, i.e., those which admit no nontrivial fibration with connected fibers. The talk will discuss the structure of primitive homogeneous varieties over a field K of characteristic zero, and their finiteness properties over the algebraic closure of K.

Equations for the K3-Lehmer map

SIMON BRANDHORST

Universität des Saarlandes, Germany

ABSTRACT

The dynamical complexity of an automorphism of a complex surface is measured by its topological entropy. The entropy is the logarithm of a Salem number, that is, a real algebraic integer $\lambda > 1$ which is conjugate to $1/\lambda$ and all whose other conjugates lie on the unit circle. Conjecturally the smallest Salem number is Lehmer's number λ_{10} . Lehmer's conjecture is true for entropies: $\log(\lambda_{10})$ is the minimum among (non-zero) entropies of automorphisms of complex surfaces. In a series of papers McMullen proved the existence of such an automorphism on a complex *projective* K3 surface. His strategy combines ideas from integer programming with the theory of lattices, number fields and reflection groups. The final step of the proof relies on the Torelli-type Theorem for K3 surfaces which is non constructive.

In this talk I present equations for this automorphism. To find it we used Kneser's neighbor method, elliptic fibrations and their linear systems, finite non-symplectic automorphisms and p-adic lifting.

This is joint work with Noam D. Elkies.

Folding pentagons and dynamics on K3 surfaces

SERGE CANTAT

Université de Rennes I, France

ABSTRACT

I will describe a simple geometric process, namely a way to fold euclidean pentagons, which is related to dynamics on K3 surfaces. Here, the dynamics is given by a large group of automorphisms. Then, I will state a few results concerning the stochastic properties of this action, which I recently obtained with Romain Dujardin.

On the minimal model for algebraically integrable foliations

PAOLO CASCINI

Imperial College, UK

ABSTRACT

Every fibration, or more in general every dominant rational map, between normal varieties define a natural foliation whose canonical sheaf behaves, in many aspects, as the canonical sheaf of a normal variety. I will describe some results and applications in this direction.

Classification and factorization of birational maps in 3-fold MMP

JUNGKAI CHEN

National Taiwan University, Taiwan

ABSTRACT

Divisorial contraction, flips and flops are considered to be the elementary maps in MMP. In this talk, we are going to gives various explicit examples of such maps. We will also demonstrate how these examples lead to factorization and classification of birational maps.

Variation of canonical height in families of maps on P^1

LAURA DEMARCO

Harvard University, USA

ABSTRACT

There are many parallels between the (arithmetic) dynamics of morphisms on algebraic varieties and the (arithmetic) geometry of abelian varieties. One example is the variation of canonical height, along a section of a family of elliptic curves or abelian varieties over a base curve, defined over a number field. Key results were proved by Tate, Silverman, Lang, Call, and Green in the 1980s. It is natural to wonder if the same properties carry over to the more general setting of families of maps, equipped with the Call-Silverman canonical height. I will talk about this problem and how it has led to interesting dynamical questions (and some answers), even in the simplest case of maps on P^1 . This is joint work with Myrto Mavraki.

Uniformity for the number of rational points on curves

ZIYANG GAO

CNRS and Sorbonne Université, France

ABSTRACT

Mazur conjectured, after Faltings's proof of the Mordell conjecture, that the number of rational points on a curve of genus g at least 2 defined over a number field of degree d is bounded solely in terms of g, d and the Mordell-Weil rank. In particular the height of the curve is not involved. In this talk I will explain how to prove this conjecture. This is joint work with Vesselin Dimitrov and Philipp Habegger.

If time permits, I will discuss how to generalize this approach to prove the full Uniform Mordell-Lang Conjecture for arbitrary subvarieties of abelian varieties. This is joint work with Tangli Ge and Lars Kühne.

Unlikely Intersections for polynomial dynamical pairs

THOMAS GAUTHIER

École Polytechnique, France

ABSTRACT

Let $(P_t)_{t\in C}$ be an algebraic family of degree $d \geq 2$ polynomials parametrized by an affine curve C and let $a, b : C \to \mathbb{C}$ be two marked points. Assume there exists infinitely many parameters $t \in C$ for which a(t) and b(t) are simultaneously preperiodic under iteration of P_t . Baker and DeMarco conjectured that, under this assumption, there exists a persistent dynamical relation between the orbits $\{P_t^n(a(t))\}_n$ and $\{P_t^m(b(t))\}_m$. In the particular case when $P_t(z) = z^2 + t$ with $t \in \mathbb{C}$ and $a, b \in \mathbb{C}$ are constants, they proved this actually implies $a^2 = b^2$.

The aim of this talk is to present a joint work with Charles Favre, where we prove this conjecture in the case when the curve is defined over a number field. The proof follows the lines of the general strategy that Baker and DeMarco have used. Nevertheless, thee are big difficulties to overcome and every step of the proof requires a completely non-trivial input.

Rationality constructions over non-closed fields

BRENDAN HASSETT

Brown University, USA

ABSTRACT

Let X be a smooth projective variety over a field k that becomes rational over the algebraic closure. When is X rational over k? We survey recent developments and present new results for even-dimensional complete intersections of two quadrics over the real numbers. (joint with Kollár and Tschinkel)

A dynamical approach to Weil's Riemann hypothesis

Fei Hu

University of Oslo, Norway

ABSTRACT

Inspired by a result of Esnault and Srinivas on automorphisms of surfaces and recent advances in complex dynamics, Truong raised a question on the comparison of two dynamical degrees, which are defined using pullback actions of dynamical correspondences on numerical cycle class groups and cohomology groups, respectively. An affirmative answer to his question would surprisingly imply Weil's Riemann hypothesis. In this talk, we consider more comparison problems on the norms and spectral radii of the pullback actions of certain correspondences. I will talk about their connections with Truong's dynamical degree comparison and the standard conjectures. Under certain technical assumption, some partial results will be given. I will also discuss some applications to Abelian varieties and surfaces. This is based on joint work with Tuyen Truong.

A birational map with transcendental dynamical degree

MATTIAS JONSSON

University of Michigan, USA

ABSTRACT

I will report on joint work with Bell, Diller and Krieger, where we construct a birational selfmap of projective 3-space, whose (first) dynamical degree is a transcendental number.

The (uniform) Hrushovski-Lang-Weil estimates

Shuddhodan Kadattur

Purdue University, USA

ABSTRACT

In 1994 E. Hrushovski combining techniques from Model theory and Intersection theory proved a vast generalization of the Lang-Weil estimates. This has since found numerous applications in Algebraic geometry, dynamics, number theory and group theory. Attempts were then made to obtain a purely geometric proof of the estimates, and partial results were obtained by Varshavsky (in 2018) and by the speaker (in 2019). In this talk we shall present a geometric proof of the estimates obtained in collaboration with Y. Varshavsky.

j-invariant and Borcherds Phi-function

Shu Kawaguchi

Doshisha University, Japan

ABSTRACT

The j-invariant is the $SL_2(Z)$ -invariant holomorphic function on the complex upper half-plane, which is fundamental in many branches of mathematics. In this talk, we explain that the difference of j-invariants is closely related to the Borcherds Phi-function, an automorphic form on the period domain for Enriques surfaces characterizing the discriminant divisor. This is joint work with Shigeru Mukai and Ken-Ichi Yoshikawa.

Rationality over non-closed fields of Fano threefolds with higher geometric Picard rank

Alexander Kuznetsov

Steklov Mathematical Institute, Russia

ABSTRACT

Over an algebraically closed field of characteristic zero there are six deformation families of rational Fano threefolds of Picard number higher than 1 which admit a finite group action with invariant Picard rank 1. I will talk about our recent results with Yuri Prokhorov about rationality of forms of these threefolds over non-closed fields.

Dominant rational maps from a very general hypersurface with high degree in the projective space

YONGNAM LEE

Korea Advanced Institute of Science and Technology (KAIST), Korea

ABSTRACT

There has been recent interest in studying measures of irrationality for hypersurfaces X of high degree in \mathbb{P}^{n+1} . The degree of irrationality of X is defined as the minimal degree of dominant rational maps from X to \mathbb{P}^n . It is known that the degree of irrationality of X is d-1 if X is a very general hypersurface of degree $d \ge 2n+1$.

In this talk, from a different point of view we will discuss dominant rational maps of finite degree from a very general hypersurface X of degree $d \ge n+3$ in \mathbb{P}^{n+1} to any smooth projective variety Z. The finite theorem states that these form a finite set, up to birational equivalence of Z, if Z is a variety of general type. It is an interesting question to determine Z when Z is not birational to X. This talk combines the joint work with Gian Pietro Pirola and the joint work with De-Qi Zhang.

Pathologies of the volume function

JOHN LESIEUTRE

Pennsylvania State University, USA

ABSTRACT

Suppose that X is a smooth projective variety, D is a divisor on X, and A is an ample divisor. The growth rate of $h^0(X, mD + A)$ as m increases plays an important role in birational geometry, where it is related to invariants called "numerical dimension". I will explain some examples in which this growth is badly behaved, by exploiting the geometry of Calabi-Yau threefolds with large groups of birational automorphisms.

Quantum cohomology of flag varieties via wonderful compactifications

CONAN LEUNG

Chinese University of Hong Kong, Hong Kong

ABSTRACT

Peterson conjectured that quantum cohomology ring of G/T is isomorphic to the homology of the based loop space of G after localization. Lam and Shimozono proved the conjecture by combinatorial method. We studied the wrapped Floer theory of the complexification of G and used the geometry of its wonderful compactification to give a geometric proof of this result.

Dynamical filtrations and applications in slow dynamics

HSUEH-YUNG LIN

Kavli Institute for the Physics and Mathematics of the Universe (IPMU), Japan

ABSTRACT

(Joint with T.-C. Dinh, K. Oguiso, and D.-Q. Zhang) Let X be a compact Kähler manifold and let f be an automorphism of X of zero entropy. Given these data, we will introduce some filtrations on the space $H^{1,1}(X)$ of (1,1)-classes of X, which capture the trade-off between the positivity of Kähler classes and the negativity arising from (mixed) Hodge-Riemann relations. These filtrations lead to new bounds of various dynamical invariants, only in terms of the dimension of X. The invariants that we will be focusing on are the polynomial growth of the pullback actions on $H^{1,1}(X)$, and the polynomial log-volume growth of an zero entropy automorphism.

Growth of heights and sizes of coordinates in arithmetic dynamics

Yohsuke Matsuzawa

Rikkyo University, Japan

ABSTRACT

I will discuss problems on growth of local and global heights associated with subschemes along orbits of self-morphisms on projective varieties.

One of the main theorems is a higher dimensional generalization of Silverman's theorem, which says the ratio (local height)/(ample height) goes to zero along orbits on a projective line under suitable conditions. There could be several applications of theorems of this type, but I will focus on applications concerning the simultaneous growth of sizes of coordinates. I will also present that if we assume Vojta's conjecture, then we can prove much stronger theorems, which, for example, imply the so-called Dynamical Lang-Siegel conjecture.

Fano threefolds with non-isomorphic endomorphisms

Sheng Meng

Korea Institute for Advanced Study (KIAS), Korea

ABSTRACT

Recently, together with De-Qi Zhang and Guolei Zhong, we show that a smooth Fano threefold with a non-isomorphic surjective endomorphism is either toric or a product of a rational curve and a del Pezzo surface. In this talk, I'll explain it for the polarized case.

Hochschild entropy and categorial entropy

Genki Ouchi

iTHEMS, Japan

ABSTRACT

In this talk, I will talk about categorical entropy, which is introduced by Dimitrov-Haiden-Katzarkov-Kontsevich, via homological mirror symmetry of K3 surfaces. We show the existence of a symplectic Torelli mapping class of positive categorical entropy, which is a counterexample of Gromov-Yomdin type conjecture. Furthermore, I will introduce the notion of Hochschild entropy and related topics.

Refined unramified homology of schemes

STEFAN SCHREIEDER

Leibniz Universität Hannover, Germany

ABSTRACT

We introduce refined unramified cohomology of algebraic schemes and show that it interpolates between Borel–Moore homology and algebraic cycles. Over finitely generated fields, l-adic Chow groups of algebraic schemes are computed by refined unramified cohomology. Over the complex numbers, our approach simplifies and generalizes to cycles of arbitrary codimensions on possibly singular schemes, previous results of Bloch—Ogus, Colliot-Thélène—Voisin, Voisin, and Ma. Our approach has several applications. For instance, it allows to produce the first example of a smooth complex projective variety whose Griffiths group has infinite torsion.

The Kawaguchi-Silverman conjecture for endomorphisms on affine surfaces

TAKAHIRO SHIBATA

National University of Singapore, Singapore

ABSTRACT

The Kawaguchi-Silverman conjecture is a conjecture predicting the coincidence of the first dynamical degrees and arithmetic degrees. This conjecture is verified in certain cases, but open variety case is hardly verified. In this talk, I show that the conjecture holds for certain affine surfaces.

This talk is based on a joint work with J. Jia, J. Xie, and D.-Q. Zhang.

Finite presentation for the tame fundamental group in characteristic p

VASUDEVAN SRINIVAS

Tata institute of Fundamental Research, India

ABSTRACT

We discuss a proof of finite presentation of the tame fundamental group of a smooth variety over an algebraically closed field of characteristic p, assuming the variety has a smooth projective compactification with a simple normal crossing divisor as boundary. This includes the case when the variety itself is smooth and projective. We note that the result is for the (tame) fundamental group itself, and not just its "prime to p" quotient. The talk will be a report on joint work with Hélène Esnault and Mark Shusterman.

A 10-line proof of a generalisation of Weil's RH for Abelian varieties

TUYEN TRUNG TRUONG

University of Oslo, Norway

ABSTRACT

Previous work on eigenvalues of endomorphisms of Abelian varieties (Weil's classical proof of positive characteristic analog of Serre's famous result on polarised endomorphisms of compact Kahler manifolds, and Fei Hu's proof of my conjecture for the case of general endomorphisms) use special properties of Abelian varieties (such as Rosati's involution and Riemann - Roch theorem). This talk gives a simple and more conceptual proof of a more general result (for correspondences), which relies only on a quantitative version of Standard conjecture C. In characteristic zero, a stronger version was proven by Dinh. In any characteristic, if Standard conjectures hold, then this quantitive version, for graphs of polarised endomorphisms (enough to solve the positive characteristic analog of Serre's result), is a corollary. Thus this points to an alternative and easier way, avoiding the difficult Standard conjectures, to achieve the main objective of Grothendieck and Bombieri's program. The talk is based on joint work with Fei Hu (University of Oslo).

Abelian Varieties not Isogenous to Jacobians - in arbitrary characteristic

JACOB TSIMERMAN

University of Toronto, Canada

ABSTRACT

(Joint w/ Ananth Shankar) We prove that over an arbitrary global field, for every g > 3 there exists an abelian variety which is not isogenous to a Jacobian.

Progress in the geometric Bogomolov conjecture

Kazuhiko Yamaki

University of Tsukuba, Japan

ABSTRACT

The geometric Bogomolov conjecture predicts that a closed subvariety that contains a dense subset of points of small canonical height should be a special subvariety. This conjecture is a generalization of the Bogomolov conjecture for curves and is a function-field analog of Zhang's theorem over number fields. In the last several years, there were not a few results on the conjecture established. In this talk, we explain how the progresses in the conjecture have been made.

Local fundamental groups of log terminal singularities via stringy invariants

TAKEHIKO YASUDA

Osaka University, Japan

ABSTRACT

As proved by Xu, the local etale fundamental group of a log terminal singularity is finite in characteristic zero. But it is an open problem whether or not the same assertion holds in positive characteristics, despite recent works by a number of people in this direction. One of the difficulties seems to be the presence of wild ramification in positive characteristics. In this talk, I will explain our affirmative solution in dimension two to this problem by using stringy motives. This is a joint work with Javier Carvajal-Rojas.

Structure of varieties admitting an int-amplified endomorphism

Sho Yoshikawa

University of Tokyo, Japan

ABSTRACT

In this talk, I will introduce questions and results about the structure of varieties admitting a special endomorphism, called int- amplified endomorphism. Polarized endomorphisms are typical examples of int-amplified endomorphisms. It is believed that if a variety has an int-amplified endomorphism, then it is close to a fiber product of toric varieties and abelian varieties.

First, I introduce questions and known results. about the structure of varieties admitting an int-amplified endomorphism. Moreover, I explain the two theorems. The first one is a characterization of toric varieties via int-amplified endomorphisms, which is an analogue of Thomsen and Achinger's characterization via the Frobenius morphism. The second one is that if a smooth variety has an int-amplified endomorphism, then it is of Fano type over the Albanese variety up to equivariant étale cover, and in particular, it is rationally connected if and only if it is of Fano typ.

On lattice polarizable cubic fourfolds

Xun Yu

Tianjin University, China

ABSTRACT

Hassett divisors (i.e., the moduli spaces of special cubic fourfolds introduced by Hassett) have played fundamental roles in many studies of cubic fourfolds. In this talk, we extend the non-emptyness and irreducibility of Hassett divisors to the moduli spaces of M-polarizable cubic fourfolds for higher rank lattices M, and show that Fermat cubic fourfold is contained in every Hassett divisor. As applications, we obtain an algorithm to determine the irreducible components of the intersection of any two Hassett divisors and give new examples of rational cubic fourfolds. Moreover, we derive a numerical criterion for the algebraic cohomology of a cubic fourfold having an associated K3 surface. This is based on a joint work with Song Yang.

Algebraic groups acting birationally on surfaces

SUSANNA ZIMMERMANN

Université Angers, France

ABSTRACT

Which algebraic groups act on rational smooth projective surfaces? And which ones over an arbitrary perfect field? In this talk, I will explain how to attack this problem and present the known classifications - up to conjugation by a birational map - over different fields. Among these I will focus on infinite algebraic groups and present the classification up to inclusion. This is joint work with Julia Schneider.