Topics at the Interface of Low Dimensional Group Actions and Geometric Structures (4 - 15 Jan 2021)

> Workshop (11 - 15 Jan 2021)

Institute for Mathematical Sciences

# ORGANIZING COMMITTEE

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# Topics at the Interface of Low Dimensional Group Actions and Geometric Structures

Workshop: 11 - 15 Jan 2021

Warning: the day is indicated with respect to GMT, please adjust for your time zone ZOOM ID: <u>864 3240 1572</u> Tea & Informal Discussion: <u>Gathertown</u>

Monday, 11 Jan 2021			
GMT Time	Title	Speaker	
<b>Q&amp;A Session</b> for Asia and Europe			
1115 - 1145	Welcome Tour of <u>Gathertown</u>	Organizers	
1145 - 1200	On the space of properly convex projective structures ( <u>Video</u> )	Stephan Tillmann The University of Sydney, Australia	
1200 - 1215	Complex Coxeter groups ( <u>Video</u> )	Martin Deraux Université Grenoble Alpes, France	
1215 - 1230	Braids, triangles and Lissajous curves ( <u>Video</u> )	Eiko Kin Osaka University, Japan	
1230 - 1300	Informal discussions in the recitation rooms		
<b>Q&amp;A Session</b> for Europe and America			
1915 - 1945	Welcome Tour of <u>Gathertown</u>	Organizers	
1945- 2000	Actions of big mapping class groups on the arc graph (Video)	Carolyn Abbott Columbia University, USA	
2000 - 2015	Complex Coxeter groups ( <u>Video</u> )	Martin Deraux Université Grenoble Alpes, France	
2015 - 2045	Informal discussions in the <u>recitation rooms</u>		
Tuesday, 12 Jan 2021			
GMT Time	Title	Speaker	
<b>Q&amp;A Session</b> for Asia and America			
0315 - 0345	Welcome Tour of <u>Gathertown</u>	Organizers	
0345 - 0400	On the space of properly convex projective structures ( <u>Video</u> )	Stephan Tillmann The University of Sydney, Australia	
0400 - 0415	Actions of big mapping class groups on the arc graph (Video)	Carolyn Abbott Columbia University, USA	

Tuesday, 12 Jan 2021		
GMT Time	Title	Speaker
0415 - 0430	Braids, triangles and Lissajous curves ( <u>Video</u> )	Eiko Kin Osaka University, Japan
0430 - 0500	Informal discussions in the <u>recitation rooms</u>	
	<b><u>Q&amp;A Sessions</u></b> for Asia and Eu	ırope
1115 - 1145	Tea	
1145 - 1200	Realisation of measured laminations on boundaries on convex cores ( <u>Video</u> )	Ken'ichi Oshika Gakushuin University, Japan
1200 - 1215	The model theory of the curve graph (Video)	Valentina Disarlo Heidelberg University, Germany
1215 - 1230	Monodromy of projective structures on punctured surfaces ( <u>Video</u> )	Subhojoy Gupta Indian Institute of Science, India
1230 - 1400	Informal discussions in the recitation rooms	
1400 - 1500	Social event: Game night & address by Director of the IMS Location: <u>Gathertown</u> Library	
	<b>Q&amp;A Session</b> for Europe and A	merica
1915 - 1945	Tea	
1945 - 2000	Topological restrictions on Anosov representations ( <u>Video</u> )	Richard Canary University of Michigan, USA
2000 - 2015	The model theory of the curve graph (Video)	Valentina Disarlo Heidelberg University, Germany
2015 - 2045	Informal discussions in the <u>recitation rooms</u>	
Wednesday, 13 Jan 2021		
GMT Time	Title	Speaker
<b>Q&amp;A Session</b> for Asia and America		
0315 - 0345	Теа	
0345 - 0400	Realisation of measured laminations on boundaries on convex cores ( <u>Video</u> )	Ken'ichi Oshika Gakushuin University, Japan
0400 - 0415	Topological restrictions on Anosov representations ( <u>Video</u> )	Richard Canary University of Michigan, USA

Wednesday, 13 Jan 2021		
GMT Time	Title	Speaker
0415 - 0430	Monodromy of projective structures on punctured surfaces ( <u>Video</u> )	Subhojoy Gupta Indian Institute of Science, India
0430 - 0500	Informal discussions in the <u>recitation rooms</u>	
<b>Q&amp;A Sessions</b> for Asia and Europe		
1115 - 1145	Tea	
1145 - 1200	Dynamics of actions of automorphisms of locally compact groups <i>G</i> on Sub <i>G</i> ( <u>Video</u> )	Riddhi Shah Jawaharlal Nehru University, India
1200 - 1215	Spaces of enumerated orderable groups ( <u>Video</u> )	Yash Lodha EPFL, Switzerland
1215 - 1230	Generalized Dehn twists on surfaces and homology cylinders ( <u>Video</u> )	Yusuke Kuno Tsuda University, Japan
1230 - 1300	Informal discussions in the recitation rooms	
	<b>Q&amp;A Session</b> for Europe and A	merica
1915 - 1945	Tea	
1945 - 2000	Exotic real projective Dehn surgery space ( <u>Video</u> )	Jeffrey Danciger University of Texas at Austin, USA
2000 - 2015	Spaces of enumerated orderable groups ( <u>Video</u> )	Yash Lodha EPFL, Switzerland
2015 - 2045	Informal discussions in the <u>recitation rooms</u>	
Thursday, 14 Jan 2021		
GMT Time	Title	Speaker
Q&A Session for Asia and America		
0315 - 0345	Tea	
0345 - 0400	Dynamics of actions of automorphisms of locally compact groups <i>G</i> on Sub <i>G</i> ( <u>Video</u> )	Riddhi Shah Jawaharlal Nehru University, India
0400 - 0415	Exotic real projective Dehn surgery space ( <u>Video</u> )	Jeffrey Danciger University of Texas at Austin, USA
0415 - 0430	Generalized Dehn twists on surfaces and homology cylinders ( <u>Video</u> )	Yusuke Kuno Tsuda University, Japan
0430 - 0500	Informal discussions in the <u>recitation rooms</u>	

Thursday, 14 Jan 2021			
GMT Time	Title	Speaker	
<u>Q&amp;A Sessions</u> for Asia and Europe			
1115 - 1145	Tea		
1145 - 1200	Hyperbolic jigsaw groups: arithmeticity, pseudomodularity and algorithms ( <u>Video</u> )	Ser Peow Tan National University of Singapore, Singapore	
1200 - 1215	Punctual Hilbert schemes and higher complex structures ( <u>Video1</u> ) ( <u>Video2</u> )	Alexander Thomas IRMA, Université de Strasbourg, France	
1215 - 1230	Convex real projective Dehn filling ( <u>Video</u> )	Gye-Seon Lee Sungkyunkwan University, Korea	
1230 - 1400	Informal discussions in the <u>recitation rooms</u>		
1400 - 1500	Trivia Night	Zoom Lecture Hall	
<b>Q&amp;A Session</b> for Europe and America			
1915 - 1945	<u>Tea</u>		
1945 - 2000	Right-angled Coxeter groups commensurable to right-angled Artin groups ( <u>Video</u> )	Pallavi Dani Louisiana State University, USA	
2000 - 2015	Punctual Hilbert schemes and higher complex structures ( <u>Video1</u> ) ( <u>Video2</u> )	Alexander Thomas IRMA, Université de Strasbourg, France	
2015 - 2045	Informal discussions in the recitation rooms		
Friday, 15 Jan 2021			
GMT Time	Title	Speaker	
<b>Q&amp;A Session</b> for Asia and America			
0315 - 0345	Tea		
0345 - 0400	Hyperbolic jigsaw groups: arithmeticity, pseudomodularity and algorithms ( <u>Video</u> )	Ser Peow Tan National University of Singapore, Singapore	
0400 - 0415	Right-angled Coxeter groups commensurable to right-angled Artin groups ( <u>Video</u> )	Pallavi Dani Louisiana State University, USA	
0415 - 0430	Convex real projective Dehn filling ( <u>Video</u> )	Gye-Seon Lee Sungkyunkwan University, Korea	
0430 - 0500	Informal discussions in the <u>recitation rooms</u>		

Friday, 15 Jan 2021		
GMT Time	Title	Speaker
<b>Q&amp;A Sessions</b> for Asia and Europe		
1115 - 1145	Tea	
1145 - 1200	Hyperbolic lattices in diffeomorphisms of the circle ( <u>Video</u> )	Nicolas Tholozan CNRS, École normale supérieure de Lyon, France
1200 - 1215	Percolation on hyberbolic groups ( <u>Video</u> )	Mahan Mj Tata Institute of Fundamental Research, India
1215 - 1230	Inhomogeneous Diophantine approximation and homogeneous dynamics ( <u>Video</u> )	Seonhee Lim Seoul National University, Korea
1230 - 1300	Informal discussions in the recitation rooms	
	<b>Q&amp;A Session</b> for Europe and A	merica
1915 - 1945	Tea	
1945 - 2000	Convex co-compact representations of 3-manifold groups ( <u>Video</u> )	Andrew Zimmer University of Wisconsin-Madison, USA
2000 - 2015	Hyperbolic lattices in diffeomorphisms of the circle ( <u>Video</u> )	Nicolas Tholozan CNRS, École normale supérieure de Lyon, France
2015 - 2045	Informal discussions in the <u>recitation rooms</u>	
Saturday, 16 Jan 2021		

GMT Time	Title	Speaker
<b>Q&amp;A Session</b> for Asia and America		
0315 - 0345	<u>Tea</u>	
0345 - 0400	Convex co-compact representations of 3-manifold groups ( <u>Video</u> )	Andrew Zimmer University of Wisconsin-Madison, USA
0400 - 0415	Percolation on hyberbolic groups ( <u>Video</u> )	Mahan Mj Tata Institute of Fundamental Research, India
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0430 - 0500	Informal discussions in the <u>recitation rooms</u>	

# Actions of big mapping class groups on the arc graph

### CAROLYN ABBOTT

Columbia University, USA

#### ABSTRACT

Given a finite-type surface (i.e. one with finitely generated fundamental group), there are two important objects naturally associated to it: a group, called the mapping class group, and an infinite-diameter hyperbolic graph, called the curve graph. The mapping class group acts by isometries on the curve graph, and this action has been extremely useful in understanding the algebraic and geometric properties of mapping class groups. One particularly important class of elements of the mapping class group are those which act as loxodromic isometries of the curve graph; these are called "pseudo-Anosov" elements. Given an infinite-type surface with an isolated puncture, one can associate two analogous objects: the so-called big mapping class group, and the (relative) arc graph. In this talk, we will consider the action of big mapping class groups on the arc graph, and, in particular, we will construct an infinite family of "infinite-type" elements that act as loxodromic isometries of the arc graph, where (roughly) an infinite-type element is one which is not supported on any finite-type subsurface. This is joint work with Nick Miller and Priyam Patel.

# Topological restrictions on Anosov representations

### RICHARD CANARY

University of Michigan, USA

### ABSTRACT

Anosov representations are the higher rank analogues of convex cocompact representations into rank one Lie groups. However, very little is known about the class of groups which admit Anosov representations. In this talk, we discuss characterizations of groups which admit Anosov representations into SL(3,R), projective Anosov representations into SL(4,R) and Borel Anosov representations into SL(4,R). In general, we provide restrictions on the cohomological dimension of groups admitting Anosov representations into SL(d,R). Joint work with Konstantinos Tsouvalas.

# Exotic real projective Dehn surgery space

JEFFREY DANCIGER

University of Texas at Austin, USA

#### ABSTRACT

We study properly convex real projective structures on closed 3-manifolds. A hyperbolic structure is one special example, and in some cases the hyperbolic structure may be deformed non-trivially as a convex projective structure. However, such deformations seem to be exceedingly rare. By contrast, we show that many closed hyperbolic manifolds admit a second convex projective structure not obtained through deformation. We find these examples through a theory of properly convex projective Dehn filling, generalizing Thurston's picture of hyperbolic Dehn surgery space. Joint work with Sam Ballas, Gye-Seon Lee, and Ludovic Marquis.

# Right-angled Coxeter groups commensurable to right-angled Artin groups

# PALLAVI DANI

Louisiana State University, USA

#### ABSTRACT

A well-known result of Davis–Janukiewicz states that every right-angled Artin group (RAAG) is commensurable to some right-angled Coxeter group (RACG). I will talk about joint work with Ivan Levcovitz, in which we explore the converse statement. We establish criteria for constructing finite-index RAAG subgroups of RACGs. As an application, we prove that a 2-dimensional, one-ended RACG with planar defining graph is quasi-isometric to a RAAG if and only if it is commensurable to a RAAG.

# **Complex Coxeter groups**

### MARTIN DERAUX

Université Grenoble Alpes, France

#### ABSTRACT

I will survey some results about complex reflection groups in complex space forms (complex projective, euclidean or hyperbolic spaces). In the hyperbolic case, I will present joint work with Parker and Paupert, which can be thought of as an important step in the classification of well-generated complex reflection groups in the complex hyperbolic plane.

# The model theory of the curve graph

### VALENTINA DISARLO

Heidelberg University, Germany

#### ABSTRACT

The curve graph of a surface is a graph encoding the combinatorics of simple closed curves on a surface. It was introduced by William Harvey in analogy with Tits buildings, and it is a fundamental tool in Teichmueller theory and the study of the mapping class groups. Nikolai Ivanov proved that the automorphism group of the curve graph is isomorphic to the (extended) mapping class group of the surface. Many other complexes encoding the combinatorics of topological objects on the surface also has the extended mapping class group as automorphism group, and their proofs rely on Ivanov's theorem. Ivanov posed a metaconjecture that all "natural" complexes should have automorphism group isomorphic to the extended mapping class group, and that the proofs should factor through his theorem. In this talk, we will propose a rigorous interpretation of Ivanov's metaconjecture from a model theoretic point of view. We will see that the curve graph and many of its analogues are omega-stable. The talk will be self-contained, and no background in mathematical logic will be assumed.

This is joint work with T. Koberda (UVirginia) and J. de la Nuez Gonzalez (UPV Bilbao).

# Monodromy of projective structures on punctured surfaces

### SUBHOJOY GUPTA

Indian Institute of Science, India

### ABSTRACT

Given a surface S, one can ask: which representations of the surface-group to PSL(2,C) arise as the monodromy of some complex projective structure on S? Such a structure is determined by a choice of a complex structure on S and a holomorphic quadratic differential, via the Schwarzian differential equation. When S is a closed surface, this question was answered by the work of Gallo-Kapovich-Marden. I shall talk about the case when S has punctures, and the quadratic differential has poles of order at most two at the punctures. I shall discuss two new results, including one that provides a complete answer when S has at least two punctures. This is partly joint work with Gianluca Faraco.

# Braids, triangles and Lissajous curve

### Eiko Kin

Osaka University, Japan

#### ABSTRACT

The purpose of this talk is to introduce Lissajous 3-braids. Suppose we have a closed curve on the plane, and we consider the periodic motion of n points along the closed curve. If the motion is collision-free, then we get a braid obtained from the trajectory of the set of n points in question. In this talk, we consider 3-braids coming from the periodic motion of 3 points on Lissajous curves. We classify Lissajous 3-braids and present a parametrization in terms of natural numbers together with slopes. We also discuss some properties of pseudo-Anosov stretch factors for Lissajous 3-braids. Interestingly, there is a nice relation between Farey sequences and stretch factors of Lissajous 3-braids. Our main tool is the shape sphere — the configuration space of the oriented similarity classes of triangles. This is a joint work with Hiroaki Nakamura and Hiroyuki Ogawa.

# Generalized Dehn twists on surfaces and homology cylinders

## YUSUKE KUNO

Tsuda University, Japan

#### ABSTRACT

This is a joint work with Gwénaël Massuyeau (University of Burgundy). Lickorish's trick describes Dehn twists along simple closed curves on an oriented surface in terms of surgery of 3-manifolds. I discuss one possible generalization of this description to the situation where the curve under consideration may have self-intersections.

I also mention one application of our result in the context of the theory of the Johnson homomorphisms.

# Convex real projective Dehn filling

## Gye-Seon Lee

Sungkyunkwan University, Korea

#### ABSTRACT

Hyperbolic Dehn filling theorem proven by Thurston is a fundamental theorem of hyperbolic 3-manifold theory, but it is not true anymore in dimension > 3. Since hyperbolic geometry is a sub-geometry of convex real projective geometry, it is natural to ask whether Thurston's Dehn filling theory for hyperbolic 3-manifolds can generalize to convex real projective manifolds in any dimension. In this talk, I will give evidence towards a positive answer to the question.

Joint work with Subyoung Choi and Ludovic Marquis.

# Inhomogeneous Diophantine approximation and homogeneous dynamics

## SEONHEE LIM

Seoul National University, Korea

### ABSTRACT

We show that the Hausdorff dimension of epsilon-badly approximable vectors is less than the full dimension. We will start from relating the problem to homogeneous dynamics and discuss further about the effective upper bound of the dimension. (The talk is based on a joint work with Taehyung Kim and Wooyeon Kim.)

# Spaces of enumerated orderable groups

### Yash Lodha

SEPFL, Switzerland

#### ABSTRACT

An enumerated group is a group structure on the natural numbers. Given one among various notions of orderability of countable groups, we endow the class of orderable enumerated groups with a Polish topology. In this setting, we establish a plethora of genericity results using elementary tools from Baire category theory and the Grigorchuk space of marked groups. In this talk I will describe these spaces and some of their striking features. This is ongoing joint work with Srivatsav Kunnawalkam Elayavalli.

# Percolation on hyperbolic groups

Mahan Mj

Tata Institute of Fundamental Research, India

#### ABSTRACT

We study first passage percolation (FPP) in a Gromov-hyperbolic group G with boundary equipped with the Patterson-Sullivan measure. We associate an i.i.d. collection of random passage times to each edge of a Cayley graph of G, and investigate classical questions about asymptotics of first passage time as well as the geometry of geodesics in the FPP metric. Under suitable conditions on the passage time distribution, we show that the 'velocity' exists in almost every direction, and is almost surely constant by ergodicity of the G-action on the boundary. For every point on the boundary, we also show almost sure coalescence of any two geodesic rays directed towards the point. Finally, we show that the variance of the first passage time grows linearly with word distance along word geodesic rays in every fixed boundary direction.

This is joint work with Riddhipratim Basu.

# Realisation of measured laminations on boundaries on convex cores

### Ken'ichi Ohshika

Gakushuin University, Japan

### ABSTRACT

I shall present a generalisation of the theorem by Bonahon-Otal concerning realisation of measured laminations as bending laminations, to the case of Kleinian surface groups which might be geometrically infinite. Our proof is based on analysis of geometric limits, and is independent of the technique of hyperbolic cone-manifolds employed by Bonahon-Otal.

This is a part of joint work with Shinpei Baba.

# Dynamics of actions of automorphisms of locally compact groups G on $Sub_G$

### Riddhi Shah

Jawaharlal Nehru University, India

#### ABSTRACT

For a locally compact group G,  $\operatorname{Sub}_G$  is the compact space of closed subgroups of G endowed with the Chabauty topology. There is a natural action of  $\operatorname{Aut}(G)$  or  $\operatorname{Sub}_G$ . In this talk we discuss the dynamics of this action. A homeomorphism Tof a Hausdorff topological space is said to be *distal* if every T-orbit of (x, y) does not intersect the diagonal unless x = y. Distal maps were introduced by David Hilbert on compact spaces to study non-ergodic maps. We characterise the distal actions of automorphisms of a connected Lie group G on  $\operatorname{Sub}_G$  under a certain condition. We also characterise the actions of automorphisms of  $\Gamma$  on  $\operatorname{Sub}_{\Gamma}$  for a lattice  $\Gamma$  in a connected Lie group. We briefly discuss expansive actions and show that neither a connected Lie group G nor a lattice  $\Gamma$  in it admit automorphisms which act expansively on  $\operatorname{Sub}_G$  or  $\operatorname{Sub}_{\Gamma}$  respectively, unless G is trivial or  $\Gamma$  is finite. (These results are from joint works with Rajdip Palit, Manoj B. Prajapati and Alok K. Yadav).

# References

- Riddhi Shah and Alok K. Yadav. Distal actions of automorphisms of Lie Groups G on Sub<sub>G</sub>. Preprint 2019. arXiv:1909.04397[math.GR]
- [2] Rajdip Palit and Riddhi Shah. Distal actions of automorphisms of nilpotent groups G on Sub<sub>G</sub> and applications to lattices in Lie groups. Glasgow Math. J. 1-20. doi:10.1017/S0017089520000221
- [3] Rajdip Palit, Manoj B. Prajapati and Riddhi Shah. Dynamics of actions of automorphisms of discrete groups G on  $Sub_G$  and applications to lattices in Lie groups. Preprint 2020. arXiv:2010.12958 [math.GR]
- [4] Manoj B. Prajapati and Riddhi Shah. Expansive actions of automorphisms of locally compact groups G on  $Sub_G$ . Monatsh. Math. 193 (2020), 129–142.

# Hyperbolic jigsaw groups: arithmeticity, pseudomodularity and algorithms

### SER PEOW TAN

National University of Singapore, Singapore

#### ABSTRACT

We will explain a hyperbolic jigsaw construction which produces non-uniform lattices in PSL(2,R) generated by involutions about certain marked points on the sides of the jigsaws. We explore conditions under which the resulting lattice is arithmetic/non-arithmetic, pseudomodular (non-arithmetic but having the same cusp set as the modular group); and certain pseudo-euclidean algorithms and recursions associated to these constructions. In particular, we answer a couple of questions posed by Long and Reid in their seminal paper "Pseudomodular surfaces" (Crelle, 2002). This is joint work with Beicheng Lou and Anh Duc Vo.

#### References

D.D. Long and A.W. Reid, Pseudomodular surfaces, J. Reine Angew. Math. 552, (2002), 77–100.

B. Lou, S.P. Tan and A.D. Vo, Hyperbolic jigsaws and families of pseudomodular groups I, Geometry & Topology, 22, 2339—2366.

B. Lou, S.P. Tan and A.D. Vo, Hyperbolic jigsaws and families of pseudomodular groups II, Preprint, arXiv:2010.10725.

# Hyperbolic lattices in diffeomorphisms of the circle

## NICOLAS THOLOZAN

CNRS, École normale supérieure de Lyon, France

### ABSTRACT

The fundamental group of a closed hyperbolic 3-manifold fibering over the circle acts naturally by homeomorphisms on the circle at infinity of a surface group. Here I explain why this action can be conjugated to an action of class C1.

# Punctual Hilbert schemes and higher complex structures

### ALEXANDER THOMAS

IRMA, Université de Strasbourg, France

### ABSTRACT

Part 1: Introduction to the punctual Hilbert scheme of the plane

In the first talk, I give an introduction to the punctual Hilbert scheme of the plane, the main tool to construct the higher complex structure. We give three approaches to the Hilbert scheme and explore their relations. Finally we discuss some symplectic aspects.

Part 2: Higher complex structures

The goal of the second talk is to present a geometric approach to higher Teichmüller theory, in particular Hitchin components. For this, I present the construction of the higher complex structure, a new geometric structure on surfaces generalizing the complex structure. We analyze its local and global properties and outline a program to prove that its moduli space is isomorphic to Hitchin's component. Finally, we discuss a GL(2,R) - action in our setting. Joint work with Vladimir Fock.

# On the space of properly convex projective structures

## STEPHAN TILLMANN

The University of Sydney, Australia

### ABSTRACT

I will outline joint work with Daryl Cooper concerning the space of holonomies of properly convex real projective structures on manifolds whose fundamental group satisfies a few natural properties. This generalises previous work by Benoist for closed manifolds. Key examples illustrating our results will be given.

# Convex co-compact representations of 3-manifold groups

### ANDREW ZIMMER

University of Wisconsin-Madison, USA

### ABSTRACT

A representation of a finitely generated group into the projective general linear group is called convex co-compact if it has finite kernel and its image acts convex co-compactly on a properly convex domain in real projective space. We prove that the fundamental group of a closed irreducible orientable 3-manifold can admit such a representation only when the manifold is geometric (with Euclidean, Hyperbolic, or Euclidean cross Hyperbolic geometry) or when every component in the geometric decomposition is hyperbolic. In each case, we describe the structure of such examples. This is joint work with Mitul Islam.