

Dealing with Information and Uncertainty >>>



Eric Maskin: "Hurry, hyperbolic discounts!"

Information and uncertainty often appear to us as threads of contrasting and conflicting colors woven into the tapestry of human knowledge. One well-defined and the other nebulous, they have long been axiomatically and quantitatively studied in science and economic theory. For the latter, it is only during the last two to three decades that major advances have been made in these two topics, adopted as the underlying themes for an Institute program (9 May – 3 July 2005) which focused on three areas of microeconomics: game theory, information economics and finance.

Game theory is generally considered to have begun with the publication in 1944 of the book *The Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern, which popularized a mathematical article in German that von Neumann published in 1927. Although

very little of that thick volume is relevant to present day game theory, it provided the terminology that allowed conflict to be analyzed mathematically. It was the subsequent work by John Nash in 1950 and 1951 that provided the breakthrough for later developments, not least because cooperation could also be accommodated in the theory.

Until the mid-1970s, game theory remained largely an autonomous field with little relevance to mainstream economics. During the 1970s, many economic models were devised which put emphasis on individuals acting rationally in the face of limited information. Since then, game theory and information economics have become closely intertwined, if not virtually inseparable. Auctions serve as valuable illustrations, and one of the most prominent applications, of games of incomplete information, as bidders' private information is the main factor affecting strategic behavior. In particular, combinatorial auctions, in which agents bid for bundles of goods, have emerged as an area in which we study algorithms for optimal resource allocation for autonomous and self-interested agents.

The Institute program also dealt with the study of the interplay between equilibrium theory and asset pricing with an emphasis on incomplete markets. In complete markets, complex financial securities can be synthetically replicated by sophisticated trading strategies involving considerably simpler instruments. This is not usually possible in incomplete markets though in some cases, the pricing of complex securities can still be accomplished via equilibrium arguments.

The program activities started with a one-day workshop on 16 May 2005, in which Eric Maskin of the Institute for Advanced Study at Princeton gave a very stimulating

Continued on page 2

Contents

• Dealing with Information and Uncertainty	1	• Programs & Activities	3	• Forthcoming Publications	19
• From the Editor	2	• Mathematical Conversations – Interviews with:		• IMS Staff	20
• People in the News	3	Eric Maskin	10		
		Theodore Slaman and	14		
		Hugh Woodin			

From the Editor >>>

Continued from page 1

Sometimes one wonders what breakthroughs would have occurred in bioinformatics if Gauss were born in the late 20th century. With the computing power and abundance of data available at his fingertips, there would be no way to stop the secrets of life from being unearthed by his phenomenal capacity for computations and interdisciplinary problem solving. For a young von Neumann or Kolmogorov armed with current mathematical tools, the large scientific unknown that beckons would undoubtedly become fertile ground for their irrepressible curiosity and creativity. For lesser mortals, it is both a blessing and a curse to be born in this age of plenty. It is a blessing because there is literally intellectual gold to be mined out there. It is a curse because there is so much to learn and so little time.

Knowledge creation is becoming a collective enterprise often requiring one to venture beyond the boundary of one's own field of specialization. And one thing is inescapable: more and more common pathways are being established running through the major disciplines in the world of scientific knowledge. New disciplines created at the interface of established ones are not perceived to encroach on the traditional territories of the latter but grow upwards in a different dimension. Nowadays, all kinds of interdisciplinary combinations seem possible, almost in vogue – evolutionary psychology, neuroeconomics, computational game theory, mathematical genomics and so on.

Mathematicians may or may not be in an enviable position of being in demand by people swarmed with problems, which are essentially of a mathematical nature, in fields that speak an unfamiliar language. For whatever (altruistic or pragmatic) reasons mathematicians lend a helping hand, how do they begin? Picking up a paper in another field to work on or learning a new field through tutorials or lectures might not provide enough sustaining power needed for a long journey. Perhaps we should learn from the multidisciplinary masters – they have always stressed the role of interaction and communication. Besides the individual's temperament and the interpersonal "chemistry" of collaboration, there are other external factors which could be alleviated. Inevitably, helping others to solve their problems will only end up in multiple-author papers. There used to be a time (perhaps still so) when joint papers carry reduced weights compared to single-author papers. Unless one is at the top of the academic ladder, this kind of reduced recognition will figure in one's career calculations. Perhaps, to encourage multidisciplinary research, some kind of new system of bona fide recognition for joint papers would have to be worked out.

Y.K. Leong



Anne Villamil: How to be out of debts

lecture based on his joint paper *Uncertainty and hyperbolic discounting* with Partha Dasgupta of Cambridge University, where they show that if the typical problem involves payoffs whose realization times are uncertain, then optimal preferences give rise to relatively patient behavior when the time horizon is long but induce a switch to impatience when the time horizon grows short.

At the main program workshop from 6 to 10 June 2005, 21 invited lectures were given on various aspects of uncertainty and information, including asset pricing, behaviors under risk or uncertainty, various kinds of games, general equilibrium, mechanism design, social choice rules, and relevant mathematical methods.

As part of the program, the Institute was honored with the task of hosting the Tenth Conference on Theoretical Aspects of Rationality and Knowledge (TARK-X) from 10 to 12 June 2005. It began with a joint session that was organized with the main program workshop, covering much that was of common interest to their respective themes. The 4 invited talks and 18 papers given at the conference dealt with the interdisciplinary issues involving reasoning about rationality and knowledge, and demonstrated a broad and deep collaboration among researchers from a wide variety of fields such as mathematics, computer science, economics, linguistics, philosophy and psychology.

During 30 May – 3 June and 13 – 17 June, there was also a lot of new and exciting knowledge to learn in depth for graduate students and non-specialists in 30 hours of tutorial lectures presented by the following invited speakers: Robert Anderson (University of California at Berkeley), Felix Kubler (Universitaet Mannheim), Parkash Chander (National University of Singapore), David Parkes (Harvard University), Sudhir Shah (University of Delhi), Anne Villamil and Nicholas Yannelis (both from University

Continued on page 3

People in the News >>>

Chua Kian Peng, an administrative officer of the Institute, left the Institute on 10 June 2005. His duties have since been taken over by Emily Chan who joined the Institute on 30 June 2005.

William Chen, a technical support officer of the Institute, left the Institute on 30 June 2005. Jolyn Wong joined the Institute as the Lab Officer on 28 June 2005.

Kok Khoo Phua to Head New Institute at NTU

Kok Khoo Phua, a member of the IMS Management Board, is the founding director of the Institute of Advanced Studies at Nanyang Technological University. All of us at IMS congratulate KK on his appointment and look forward to fruitful cooperation between our institutes.

Continued from page 2

of Illinois at Urbana-Champaign). They covered topics in finance, game theory and information economics such as equilibrium foundations, equilibrium models, asset pricing, coalitions, computational mechanism design, auctions, financial contracts and theory of debt. A spin-off of these tutorial lectures was a special graduate course "Topics in Applied Mathematics" offered by the Department of Mathematics for 10 graduate students, who were also given 9 additional hours of introductory lectures as preparation for the tutorials.

Junior college students too were not forgotten by the organizers. In May, two lectures were given by Peter Loeb (University of Illinois at Urbana-Champaign) and Walter Trockel (Bielefeld University) respectively at Victoria Junior College and Raffles Junior College. For the first time, students saw the language of calculus in a new light and caught a glimpse of modern game theory – from the masters themselves. Hopefully, it will not be the last for some.



Ilya Segal: How to choose your rules



Joe Halpern: Decisions, decisions, ...

Program & Activities >>>

Past Programs in Brief

Semiparametric Methods for Survival and Longitudinal Data (26 February – 24 April 2005)

Website: <http://www.ims.nus.edu.sg/Programs/semiparametric/index.htm>

Co-chairs:

Zhiliang Ying, *Columbia University*

Yougan Wang, *National University of Singapore*

The program attracted 34 international participants from North America, Europe, India, China, Hong Kong and Taiwan. NUS participants came from the Department of Mathematics, Department of Statistics and Applied Probability, Department of Economics, Faculty of Medicine and Lee Kuan Yew School of Public Policy among others. It also drew a substantial participation from Singapore Management University.

The program covered survival analysis and longitudinal data analysis, two major areas in statistics that are also studied extensively in biostatistics and econometrics. Five tutorials on the following topics were conducted:

- "Bayesian survival analysis" by Ming-Hui Chen (University of Connecticut),
- "Computational intensive methods with LASSO-type penalty" by Wenjiang Fu (Texas A&M University),
- "Mixed effects models and longitudinal data analysis" by Jiming Jiang (University of California at Davis),
- "Semiparametric models in survival analysis" by Zhiliang Ying (Columbia University),
- "Non- and semiparametric methods in econometrics" by Joel Horowitz (Northwestern University).

A lecture entitled "From data to decisions and discoveries" was given by Xuming He (University of Illinois at Urbana-Champaign) at the NUS High School on 2 March 2005.

The main activities, held in March, featured three workshops. The first workshop, covering survival analysis, was held from 7 – 18 March. The second workshop on nonparametric and semiparametric methods and models in econometrics was held from 21 – 26 March. Renowned econometricians who participated included Nobel laureate Daniel McFadden (University of California at Berkeley). A lecture by Jerry Hausman (MIT) was also given at the Department of Economics. The third workshop on longitudinal data analysis was held from 21 March – 2 April and overlapped the econometrics workshop.

Some encouraging feedback:

"This has been a nice experience. It was a privilege to come and meet so many eminent authorities in the field of semiparametric inference."

Continued on page 4

Continued from page 3

"The talks were interesting and the informal atmosphere was most conducive to discussion. I learnt a lot and had some very helpful feedback about my own work."

"I think it is a great idea to have econometricians and statisticians get together in the IMS to discuss each other's research. I have really enjoyed this conference."



Daniel McFadden and Jerry Hausman: Maximum likelihood models



Cupful of semi-parameters: (From Left) Xiaohong Chen, Oliver Linton, Jerry Hausman, Yougan Wang, Roger Koenker



Ying and yang of applause: (From Left) Anthony Kuk, Chen-Hsin Chen, Zhiliang Ying, Probal Chaudhuri

Uncertainty and Information in Economics (9 May – 3 July 2005)

Website: <http://www.ims.nus.edu.sg/Programs/uie/index.htm>

Co-chairs:

Robert Anderson, (University of California at Berkeley)

Parkash Chander, (National University of Singapore)

Peter Hammond, (Stanford University)

Yeneng Sun, (National University of Singapore)

The program focused on three areas of microeconomics where uncertainty and information play a key role: game theory, information economics and finance. Over thirty leading researchers were invited and foreign participants came from various places like Australia, Canada, Germany, Hong Kong, India, Israel, Japan, Switzerland, Taiwan, United Kingdom, and United States.

The program activities consisted of two workshops, one conference, thirty hours of tutorials given by seven speakers, two school lectures and ten seminar talks. A colloquium lecture "Minimal rationality" by Isaac Levi (Columbia University) was jointly organized with the Department of Philosophy on 6 June 2005. The Department of Mathematics offered a graduate course, "MA6252 Topics in Applied Mathematics II", based on the tutorials of the program.

A more detailed report on the program is featured in the cover story.

Some encouraging feedback:

"I wish this type of workshop series continues in future – that may be a challenge financially but I really think it is vital for promotion of international research activity in Asia."

"The quality of the papers were quite good and the people were nice. This was a rewarding visit, interacting with conference participants and working with coauthors."

"I enjoyed the conference very much. The tutorials provided an excellent learning experience"



Understanding economists



Double vision or multiple choice?

Computational Prospects of Infinity (20 June – 15 August 2005)

Website: <http://www.ims.nus.edu.sg/Programs/infinity/>

Organizing Committee :

Chi Tat Chong, National University of Singapore

Qi Feng, Chinese Academy of Sciences, China and National University of Singapore

Theodore A. Slaman, University of California at Berkeley

W. Hugh Woodin, University of California at Berkeley

Yue Yang, National University of Singapore

This program consists of two parts:

(a) Workshop in Set Theory (20 June to 16 July)

(b) Workshop on Recursion Theory (18 July to 15 August)

Continued on page 5

Continued from page 4

The program is sponsored by the Association for Symbolic Logic and comprises a series of seminars, tutorials, workshops, a public lecture and a math camp.

Tutorials lectures for the Workshop in Set Theory were conducted by John Steel (University of California at Berkeley) and Hugh Woodin. 29 overseas visitors participated in the Workshop.

A math camp, aimed at encouraging and stimulating students in the study of mathematics, was conducted by Hugh Woodin and Qi Feng on 22 June 2005. It was held at the institute and attended by 35 students from Temasek JC, Raffles JC, Victoria JC and NUS High School.

Theodore A. Slaman and Rodney Downey (Victoria University of Wellington) conducted tutorial lectures in recursion theory. Slaman also conducted a public lecture entitled "Logic and Computation" on 1 August 2005. 79 people attended the lecture.

35 overseas visitors, including a number of promising young postdocs and graduate students from various parts of the world, are currently participating in the ongoing workshop.

Next Program

Mathematical Modeling of Infectious Diseases: Dynamics and Control (15 August – 9 October 2005)

Website: <http://www.ims.nus.edu.sg/Programs/infectiousdiseases/index.htm>

Chair:

Bryan T. Grenfell, *University of Cambridge*

Co-chairs:

Stefan Ma, *Ministry of Health, Singapore*

Yingcun Xia, *National University of Singapore*

Program sub-themes and schedule:

- (a) New development of the SEIR models for the transmission of infectious diseases (15 - 19 Aug 2005)
- (b) Influenza-like diseases (22 - 26 Aug 2005)
- (c) Break for interaction and discussion (29 Aug - 2 Sep 2005)
- (d) Immunity, vaccination and other control strategies (5 - 9 Sep 2005)
- (e) Molecular analysis of infectious diseases (12 - 16 Sep 2005)
- (f) Break for interaction and discussion (19 - 23 Sep 2005)
- (g) Clinical and public health applications of mathematical modeling (26 - 30 Sep 2005)
- (h) Break for interaction and discussion (3 - 7 Oct 2005)

A public lecture on "The epidemic clockwork: Exploring the population dynamics of infectious diseases" will be given by Bryan T. Grenfell on 23 August 2005.

Tutorial lectures will be conducted by Neils Gunther Becker (Australian National University), Carlos Castillo-Chavez (Arizona State University), John W Glasser (US CDC Atlanta), Herbert W Hethcote (University of Iowa) and Oliver George Pybus (University of Oxford).

To date, 29 overseas visitors have confirmed their participation in this program.

Programs & Activities in the Pipeline

Workshop on Computational Finance (29 - 30 Aug 2005)

Website: <http://www.ims.nus.edu.sg/activities/wkcf/>

Co-chairs:

Kian-Guan Lim, *Singapore Management University*

Yeneng Sun, *National University of Singapore*

Keynote Speaker:

A.N. Shiryaev, *Steklov Mathematical Institute, Moscow*

The 2-day workshop will consist of seminars by expert researchers from academia and industry in the areas of Monte Carlo methods in computational finance, theoretical pricing of American and exotic options, quantum theory in interest rate pricing, credit risk and derivatives pricing, volatility, and risk management topics such as VaR and model risk. It is open to researchers as well as to finance and banking professionals in the quantitative finance, structured finance, and risk management industry.

Workshop on Genomics (14 - 17 Nov 2005)

Website: <http://www.ims.nus.edu.sg/activities/wkgenomics/>

Chair:

David Siegmund, *Stanford University and National University of Singapore*

Co-chairs:

Louis HY Chen, *National University of Singapore*

Louxin Zhang, *National University of Singapore*

Organizing Committee :

Guillaume Bourque, *Genome Institute of Singapore*

Kwok Pui Choi, *National University of Singapore*

Marti Tammi, *National University of Singapore*

Benjamin Yakir, *Hebrew University of Jerusalem and National University of Singapore*

Continued from page 5

Comparison of genomes, within and between species, has the potential to provide knowledge about the role of DNA as the basis for life, evolutionary relationships of different organisms, and the role of genomes in health and disease. Computational biology has a key role to play in genomic research. The core of comparative genome analysis is the establishment of the correspondence between genes or other genomic features in different genomes. By analyzing a genomic sequence or comparing different genomic sequences, one can learn about functional elements and structural organization of genomes. This gives rise to the problems of predicting functional elements, such as genes and transcriptional binding sites, and analyzing structural organization of genomes. Other problems include theoretical support for designing new bioinformatics tools for sequence annotation, exemplified by BLAST and GenScan. The complexity of genome evolution poses many challenges to researchers, who are interested in mathematical modeling, statistical analysis and development of annotation tools. The workshop will cover gene mapping, sequence analysis, evolutionary genetics and functional genomics.

Semidefinite Programming and its Applications (15 Dec 2005 - 31 Jan 2006)

Website: <http://www.ims.nus.edu.sg/Programs/semidefinite/index.htm>

Chair:

Michael J. Todd, *Cornell University*

Co-chairs:

Kim-Chuan Toh, *National University of Singapore*

Jie Sun, *National University of Singapore*

Activities:

(a) Tutorial (9 - 10 January 2006)

(b) Workshop (11 - 13 January 2006)

To date, 23 overseas visitors have agreed to participate in the program.

International Conference on Harmonic Analysis, Group Representations, Automorphic Forms and Invariant Theory (9 - 11 Jan 2006)

On the occasion of Professor Roger Howe's 60th Birthday

Website: <http://www.ims.nus.edu.sg/activities/rogerhoweconff/index.htm>

Organizing Committee:

Jian-Shu Li, *Hong Kong University of Science and Technology*

Eng-Chye Tan, *National University of Singapore*

Nolan Wallach, *University of California, San Diego*

Chen-Bo Zhu, *National University of Singapore*

Professor Roger E. Howe, member of the National Academy of Sciences, USA and fellow of the American Academy of

Arts and Sciences, and Professor of Mathematics at Yale University, is a scholar of distinction.

His major research interest is in applications of symmetry, particularly harmonic analysis, group representations, automorphic forms and invariant theory. "His pathbreaking contributions to the representation theory of p-adic groups and of dual reductive pairs establish him as a principal architect of a theory of central and growing importance. His originality and depth have far-reaching consequences." (From membership citation when Professor Howe was elected into the National Academy of Sciences, USA.)

He has had extensive contact with and significant influence within the mathematical community in the Asia-Pacific region. He has on many occasions visited universities in Australia, Israel, Japan, Singapore, Hong Kong and China. In particular, Professor Howe was a fellow of the Institute for Advanced Studies at Hebrew University in Jerusalem in 1988, a fellow of the Japan Society for the Advancement of Science in 1993 and is currently chair of the Scientific Advisory Board of the Institute for Mathematical Sciences at the National University of Singapore.

As Professor Howe celebrates his 60th birthday, we are organizing a conference to honor his achievements both as a scholar and as a teacher.

The conference is partially supported by IMS and academic research grants of NUS.

To date, 10 overseas visitors have confirmed their participation in the program.

Random Matrix Theory and Its Applications to Statistics and Wireless Communications (1 Feb to 31 Mar 2006)

Website: <http://www.ims.nus.edu.sg/Programs/randommatrix/>

Co-chairs

Yang Chen, *Centre for Combinatorics, Nankai University and Imperial College*

Zhi-Dong Bai, *National University of Singapore*

Ying-Chang Liang, *Institute for Infocomm Research, Singapore*

The notion of an ensemble of random matrices was put forward by Wigner to explain the observed structures of the energy levels of heavy nuclei. Through the early pioneering work of Wigner, Dyson and Mehta, random matrix theory has now permeated mathematics, from representation theory to integrable systems to combinatorics. On the applied side, random matrix theory has seen a great deal of activities in statistics and more recently in wireless communications. It has been found that the statistical efficiency of classical limit theorems decays rapidly with the increase in the number of parameters. Therefore there is an urgent need to develop a

Continued from page 6

new statistical theory. The same can also be said in wireless communications. Two scenarios can be well modeled as random matrix channels: direct-sequence code division multiple access (CDMA) with random spreading codes and multiple input multiple output (MIMO) antenna systems.

The main theme of the program is concerned with the applications of random matrix theory in mathematical statistics and wireless communications. Workers in probability, mathematical statistics, mathematical physics and wireless communications are invited to participate in a cross-fertilization of ideas. There will be tutorials to introduce the "classical" aspect of random matrix theory to graduate students and the more specialized topics in statistics and wireless communications.

Random Graphs and Large-Scale Real Life Networks (May to Jun 2006)

Website: <http://www.ims.nus.edu.sg/Programs/randomgraphs/>

Chair

Bela Bollobas, *University of Memphis and Cambridge University*

Co-chairs

Khee Meng Koh, *National University of Singapore*

Oliver Riordan, *Cambridge University*

Chung-Piaw Teo, *National University of Singapore*

Vikram Srinivasan, *National University of Singapore*

In the last four years, many hundreds of papers have been written studying networks in the real-world and attempting to understand their properties using random graphs as models. There are many complex networks (for example, communication networks, the world-wide-web, social networks, and biological interaction networks) where the exact structure cannot be predicted mathematically (because it depends on human behavior, for example), but which are large enough that the global effects of this unpredictability can be modeled using randomness. For this reason, the mathematical theory of random graphs is the natural starting point for understanding such "large-scale complex networks"; often, the mathematics explains real-world phenomena (such as the "small-world phenomenon") very well.

Complex networks tend to look very different from classical random graphs. This has led to the introduction of many new (often "scale-free") mathematical models. In designing such models, two of our main aims work against each other. We would like to have a model that faithfully represents the network, but the model should be simple enough to analyze. So far, most activity in this area is experimental, observing the properties of complex networks in the real-world, and heuristic, predicting the properties of the proposed

mathematical models in a mathematically non-rigorous way. Methods of statistical physics play an important role here; although they are not entirely rigorous mathematically, such arguments frequently lead to the discovery of intricate and surprising phenomena. Providing rigorous proofs (and sometime disproofs) of these phenomena is a great challenge to mathematics.

For this reason, the development and mathematical study of new models for complex networks is currently a very important area that is still in its infancy. This program will bring together mathematicians, particularly those with experience of classical random graphs, with others working on complex networks, to encourage the development of this new area.

Algorithmic Biology: Algorithmic Techniques in Computational Biology (1 Jun to 31 Jul 2006)

Website: <http://www.ims.nus.edu.sg/Programs/algorithmicbiology/>

Co-chairs

Hon Wai Leong, *National University of Singapore*

Pavel Pevzner, *University of California, San Diego*

Franco Preparata, *Brown University*

Ken W. K. Sung, *National University of Singapore*

Louxin Zhang, *National University of Singapore*

In the half century since the unraveling of the structure of DNA, molecular biology has experienced tremendous advances. The trend has accelerated since the completion of the Human Genome Project. Huge volumes of data have been generated. The rate of growth of the volume of data has outpaced our ability to process and understand them. Entirely new research problems that are multi-disciplinary in nature have emerged and there is an urgent need for collaboration between researchers in the life sciences and other disciplines, particularly those in the mathematical and computer sciences.

The theme of this program is algorithmic biology: algorithmic techniques in computational biology. The program will bring together researchers in algorithmic biology from a wide spectrum of application areas including, but not limited to, sequence comparison and analysis, microarray design and analysis, whole genome alignment, motif finding, recognition of genes and regulatory elements, gene network, phylogeny reconstruction, phylogenetic networks, molecular evolution, computational proteomics, and systems biology.

Two workshops are planned: (a) RECOMB Workshop on Regulatory Genomics (June 2006), and (b) Workshop on Bioalgorithmics (17-21 July 2006). There will also be several tutorials aimed at introducing beginners, especially graduate students, to algorithmic approaches in several problem

Continued from page 7

domains in computational biology as well as to highlight recent advances.

Braids (14 May - 13 Jul 2007)

Website: <http://www.ims.nus.edu.sg/Programs/braids/index.htm>

Co-chairs

Jon Berrick, *National University of Singapore*
Fred R. Cohen, *University of Rochester*

The main theme of the program is the mathematical structure of the braid group, together with applications arising from this structure both within mathematics, and outside of mathematics such as (a) magnetohydrodynamics, (b) robotics and (c) stereochemistry.

The interests of the organizers lie mostly in topology. Therefore it is likely that most long-term visitors will be from that area. Reflecting the theme of the program, it is intended to have tutorials that would:

- introduce outsiders (e.g. graduate students) to the mathematics of braid theory
- facilitate communication between those working in the mathematical theory of braids and those who apply braids elsewhere, specifically in magnetohydrodynamics, robotics and stereochemistry.

Activities:

Tutorials:

Week 1 (4 - 8 Jun 2007)

- Braids - definitions and braid groups:
Joan Birman (4 hrs)
- Simplicial objects, homotopy groups (Part 1):
Jie Wu (2 hrs)

Week 2 (11 - 15 Jun 2007)

- Simplicial objects, homotopy groups (Part 2):
Jie Wu (2 hrs)
- Stereochemistry: Kurt Mislow (2 hrs)
- Configuration spaces: Fred Cohen (2 hrs)

Week 3 (18 - 22 Jun 2007)

- Magnetohydrodynamics: Mitch Berger (4 hours)
- Configuration spaces and robotics:
Robert Ghrist (2 hours)

Conference: 25 - 29 Jun 2007

Public Lecture:

Braids and robotics by Robert Ghrist (University of Illinois, Urbana-Champaign):

Highlights of Other Activities

Workshop on Nonlinear Partial Differential Equations: Analysis, Computation and Applications (3 - 6 May 2005)
Jointly organized with the Institute of Mathematical Sciences at the Chinese University of Hong Kong.

Website: <http://www.ims.nus.edu.sg/activities/npde/index.htm>

Organizers:

Weizhu Bao, *National University of Singapore*
Ping Lin, *National University of Singapore*
Jian-Guo Liu, *University of Maryland and National University of Singapore*
Zhouping Xin, *Chinese University of Hong Kong*

Nonlinear partial differential equations (PDEs) are widely used to model many problems in materials science, engineering, physics, chemistry, biology, medicine, etc. Theoretical analysis and numerical simulation are two fundamental and important mathematical tools for studying these equations which arise in the applied sciences such as fluid mechanics and electromagnetics.

The 4-day workshop brought together scientists and mathematicians in this region working on nonlinear PDEs and their applications, and provided a platform for them to interact, exchange ideas and collaborate in research and to review recent developments in the analysis, computation and applications of nonlinear PDEs. Stimulating discussions were generated and research collaborations initiated. It also provided opportunities for local graduate students and junior researcher to learn state-of-the-art knowledge in PDEs and their applications. 23 active researchers, of whom 15 came from overseas and 8 from NUS, NTU and A*STAR institutes, were invited to present their work in the workshop. The workshop was attended by 34 participants.

A sequence of workshops in this direction was initiated; the next one will take place in Xian, China in 2006 and will be followed by one in Seoul, Korea in 2007.

Workshop on Data Analysis and Data Mining in Proteomics (9 - 12 May 2005)

Website: <http://www.ims.nus.edu.sg/activities/proteomics/index.htm>

Co-chairs:

Maxey C.M. Chung, *National University of Singapore*
Newman S.K. Sze, *Genome Institute of Singapore*

2 D gel electrophoresis and liquid chromatography coupled with mass spectrometry are the current work-horses for proteomics. These technologies generate huge amount of data, especially raw mass spectra and tandem mass spectra. The quality and reliability of these data and their mode of analysis and data mining methods by bioinformatics

Continued from page 8

are extremely important in the correct identification and subsequent characterization of proteins.

The objective of the workshop was to review and discuss recent developments and advancements in data analysis and data mining in proteomics by a panel of experts, as well as to foster the exchange of ideas and collaboration between local and overseas participants. The workshop attracted 224 local and overseas participants. The scientific program consisted of 8 sessions involving 24 overseas and local speakers. Almost every aspect of data analysis and data mining in proteomics, including the application of proteomics in life science were covered in the lectures. Many students from local institutes also participated actively in the workshop program and were given an idea of the key challenges in current proteomics research. A one-day satellite MASCOT workshop was also held on the following day and attracted 84 participants.

From David Creasy on behalf of invited speakers:

This workshop far exceeded all possible expectations of the invited speakers. The organization of the workshop was magnificent – everything was catered for. The welcome and friendliness was amazing. The choice and variation of the topics covered by the speakers seemed to work perfectly because it encouraged new thoughts and discussions from people outside their own area of expertise. I certainly learnt a great deal – perhaps equally from invited speakers and local speakers.

Second IMS Math Camp (22 June 2005)

Conducted by W. Hugh Woodin (University of California at Berkeley) and Qi Feng (Chinese Academy of Sciences, China and National University of Singapore)

The math camp was held from 9.00 am to 3.00 pm at the Institute and attracted 35 students from Temasek Junior College, Raffles Junior College, Victoria Junior College and NUS High School. Talks were given by two visitors of the Institute's program on logic. The theme of the talks and ensuing discussions revolved around some fundamental questions on the foundations of mathematics in logic and set theory, and opened up a whole new world of mathematical thinking.

Asian Mathematical Conference 2005 (20 – 23 July 2005)

Website: <http://www1.math.nus.edu.sg/AMC/index.htm>

Chair (International Scientific Committee):
Kenji Ueno, *Kyoto University*

Chair (Steering Committee and Organizing Committee):
Eng Chye Tan, *National University of Singapore*

Jointly organized with Southeast Asian Mathematical Society, Singapore Mathematical Society, Department of Mathematics and Department of Statistics and Applied Probability.

The Asian Mathematical Conference (AMC) series was initiated by the South East Asian Mathematical Society (SEAMS) as a platform to showcase talents from Asian countries and to encourage academic exchange within the region. The earlier conferences took place in Hong Kong (1990), Thailand (1995) and Philippines (2000).

The eight plenary speakers came from China, Hong Kong, Israel, Japan, Singapore and United States. More than 60 invited speakers delivered lectures covering 16 major areas in the mathematical sciences. Sessions of contributed talks enabled mathematicians from within and without Asia to share their work with each other and to establish mathematical connections.



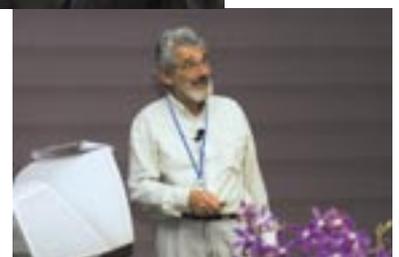
Protein power



Maxey Chung: Your data or mine?



Pro-tea-mix



Peter Roepstorff: Hitchhiker's guide to proteomics

Mathematical Conversations

Eric Maskin: Game Theory Master >>>



Eric Maskin

Interview of Eric Maskin by Y.K. Leong

Eric Maskin has made fundamental and groundbreaking contributions to numerous areas of game theory and economic theory, such as implementation theory, auction theory, the economics of incentives, and social choice theory. Among the wide range of topics he is currently studying are the design of auctions, comparison of different electoral rules, the pros and cons of political accountability, and the advantages and drawbacks of protecting intellectual property. His work, sometimes in collaboration with coauthors including Partha Dasgupta, John Riley, Jean Tirole, Jean-Jacques Laffont, Peter Diamond, and Drew Fudenberg, has been widely applied to industrial organization, finance, development, and other fields within economics and political science.

He has published over one hundred articles and book chapters, and has served on the editorial boards of leading economics journals such as the *Quarterly Journal of Economics*, *Economics Letters*, *Social Choice and Welfare*, *Games and Economic Behavior*, the *Review of Economic Design*, the *Review of Economic Studies*, and the *Journal of Economic Perspectives*.

He has been frequently invited to give named lectures, in particular, the Arrow Lectures, the McKenzie Lecture, the Zeuthen Lectures, the Schwartz Lecture, the Marshall Lectures, and the Pareto Lecture. He is a Fellow of the American Academy of Arts and Sciences and of the Econometric Society, a Corresponding Fellow of the British Academy, and an Honorary Fellow of St. John's College, Cambridge. He was President of the Econometric Society in 2003.

He was a Research Fellow at Cambridge University (1976 – 77) and taught at MIT (1977 – 84) and Harvard University (1985 – 2000), where he was Louis Berkman Professor of Economics (1997 – 2000). He then moved to the Institute for Advanced Study at Princeton and is currently Albert O. Hirschman Professor of Social Science.

When he was at the Institute as an invited speaker of the program on uncertainty and information in economics, the Editor (Y.K. Leong) of *Imprints* interviewed him on 16 May 2005. The following is an edited account of the interview in which Maskin talked passionately about the revolutionary game-theoretic ideas that are changing economic theory and influencing the social and political sciences in practice.

Imprints: Your doctorate at Harvard was in applied mathematics. What kind of applied mathematics was it? Was it related to economics?

Eric Maskin: In those days, applied math at Harvard at the graduate level was a fairly free-form program. Each student designed his own program of study, and the only common requirement was that the dissertation had to have significant mathematical content. My own program included a fair amount of economics, and, in fact, my advisor was the economist Kenneth Arrow.

I: You were in the Mathematics Department?

M: No, I was in Applied Math. It wasn't a "department" *per se*, but an interdisciplinary committee, including some people from the math department, a few from the economics department (in particular, Arrow), and assorted others from statistics, engineering, and so on. I did quite a bit of mathematical economics. I also did some mathematics not related to economics simply because I was interested in it. My thesis was in game theory and social choice theory.

I: You went to Cambridge University immediately after your doctorate. Was there any specific reason for that?

M: While I was studying at Harvard, the Cambridge economist Frank Hahn – a close friend and collaborator of Arrow – visited for several months. Hahn encouraged me to spend some time in Cambridge in a postdoctoral position. The idea appealed to me: Hahn was a leading mathematical economist and so it made sense educationally; I also thought it would also be interesting culturally.

I: Did you do any joint work with Hahn?

M: I never wrote a joint paper with him, but talked to him a great deal about my own work—he was extremely generous about making time for that. I've remained personally very close to him since those days. In fact, I'm going to England at

Continued from page 10

the end of this week to help celebrate his 80th birthday.

I: You taught at MIT for a short period after returning to the United States from England. Is it true that one of your most important works was done during this period? Could you tell us something about it?

M: Yes. Actually, I wasn't at MIT for such a short a period; it was 7 years. I worked on many things there, but probably the thing that I'm best known for now was a series of papers on implementation theory. In implementation theory, the idea is to construct a game or mechanism for attaining the goals that you as the mechanism designer wish to achieve. Suppose, for example, that there are some economic resources to be distributed among the agents in the economy and that you, the designer, have a particular set of criteria in mind for evaluating different possible distributions. Imagine however, that determining the best distribution according to these criteria calls for information that you don't have. Assume that the agents in the economy have this information, but you don't. Then, what you can try to do is design a game for the agents to play so that, when equilibrium is reached, the outcome is the same as the one you would have imposed had you had that information in the first place. In effect, the game itself "compiles" the agents' information, enabling the right outcome to emerge in spite of your own ignorance.

I: It sounds a bit psychic.

M: Well, it's what economic systems are supposed to do. An important reason why market economies have historically worked better than planned economies is that typically economic planners don't have enough information to allocate resources appropriately, even if their intentions are benevolent. Markets, by contrast, provide a remarkably good way of aggregating or gathering information. Implementation theory can be thought of as a generalization of the sort of things markets do.

I: Is implementation theory part of game theory?

M: Yes, it is part of game theory. Implementation theory actually intersects a number of areas of economics, but because it presumes that people act strategically, it is certainly game-theoretic. Still, it differs from much other game theory in an important respect. Usually in game theory, we start with the game to be analyzed and then try to predict what will happen, i.e., what the outcome will be. In implementation theory, the process is reversed. We start with the outcome that we want and then consider how we might construct a game that achieves that outcome. So implementation theory can be thought of as the "reverse engineering" part of game theory.

I: You went back to teach at Harvard after MIT. Was it some

kind of loyalty that made you return to Harvard?

M: It was partly loyalty. Also I was intellectually close to a number of people at Harvard, which had been my first academic home. Still, MIT was a terrific place to be. I enjoyed and profited from my time there enormously.

I: You once mentioned that game theory has revolutionized many fields, especially economics. Could you give us some examples of this in economics?

M: Yes. Before game theory, most of economics dealt with models of large markets—so-called "perfectly competitive" models, where there are lots of economic agents: lots of buyers, lots of sellers. Paradoxical as it may seem, the large numbers actually made the analysis easier, because the agents in those models didn't have to act strategically. If you are only one seller among many, you are not going to have much influence on the other sellers' outcomes, and so won't affect their behavior much. This means that you don't have to take into account how they are going to respond when you decide what price to set or how much to sell; your decision is relatively straightforward. Game theory provided economists with the tools for analyzing the small numbers case, which is the relevant case for many industries. In the U.S. automobile industry, for instance, each of General Motors, Ford, and Chrysler is big enough so that, whenever it acts, it has to calculate how that action is going to affect its competitors and how those competitors are going to react. For economists interested in the automobile industry, calculating what is going to happen is therefore harder than predicting what would happen in an industry where there are lots and lots of sellers. Game theory helps us to make those calculations. It's been developed precisely to deal with the case where each player has a significant influence on the payoffs of the other players.

I: Someone once said that every economist should know auction theory. How much do you agree with it?

M: These days auctions are highly visible economic institutions. They are used in settings ranging from online sales of everyday items to the privatization of major public assets. So, one reason economists should know some auction theory is that auctions constitute an ubiquitous practical way of allocating resources. They also provide us with an explicit mechanism of price determination. How prices actually get set is an issue that is left out of many economic models. Auction theory gives economists the foundation for a theory of price formation.

I: Does auction theory apply to any number of participants?

M: Oh yes, auction theory is all-inclusive in that sense.

Continued on page 12

Continued from page 11

But the theory is probably most interesting when there are relatively few participants because that is when the game-theoretic aspects come to the fore.

I: Is auction theory a species of game theory?

M: Auction theory can definitely be thought of as part of game theory; an auction is a game in which the bidders are the competing players. If I'm a bidder in an auction and considering what bid to make, I have to take into account how the other bidders might behave. Similarly, they will be thinking about what I'm going to do. So there is a strategic interaction that calls for game-theoretic analysis.

I: Have you personally organized any auctions?

M: Well, I tend not to get too involved in practical consulting, but I have made a few exceptions. One was a couple of years ago. The British government was interested in creating an auction to help reduce carbon dioxide emissions by British companies. The problem raised some interesting theoretical questions, and so I agreed to help design the auction.

I: Did they raise more money than they expected to?

M: They weren't actually trying to raise money in this case; they were trying to spend it effectively. The government budgeted about 300 million pounds (or about 500 million U.S. dollars) that they were prepared to spend to induce British firms to decrease their CO₂ output. The question they put to us auction designers was how to construct an auction in which companies would be awarded money in exchange for pledging reductions so that this budget would stretch as far as possible, that is, the biggest reduction in greenhouse-gas pollution would result.

I: I understand that you have been recently applying game theory to the study of electoral procedures within a democratic system. Are there any clear-cut answers?

M: In fact, that work, which is joint with Partha Dasgupta, is not entirely game-theoretic. The issue we examine is whether there exist voting procedures that satisfy certain basic desirable properties. A famous theorem obtained by my thesis advisor Kenneth Arrow over 50 years ago establishes that there is no voting procedure that satisfies all these properties all the time. Thus, the natural question to ask is: Which voting procedures satisfy the properties in the largest class of cases? It turns out that there is indeed a sharp answer.

Specifically, the procedure called "simple majority rule" (or "Condorcet's method," after the 18th-century scholar, the Marquis de Condorcet) is the voting rule that satisfies these properties more often than any other. Of course, I have to

tell you what these basic properties are. One, called the "Pareto (or consensus) principle" says that if everybody in society prefers candidate X to candidate Y, then candidate Y should not be elected. Another property requires that all voters should count equally. It's sometimes called the "anonymity" or "one-person, one-vote" principle. A third property, "neutrality," has two components. The first is symmetry, which means that the electoral rules should not favor one candidate over another. The second requires that the voters' choice between candidates X and Y should not depend on their views about some third candidate Z. The final property, called "transitivity," demands that if candidate X is chosen over Y, and Y is chosen over Z, then X should be chosen over Z. From Arrow's theorem, there is no voting procedure that satisfies these four principles all the time. But simple majority rule satisfies them more often than any other rule.

Simple majority rule compares candidates pairwise. If there are three candidates X, Y, and Z running, we should elect candidate X provided that X would defeat Y in a head-to-head contest and also beat Z in such contest. Of course, in most elections, you can't vote for more than one candidate at a time, which means that it may be impossible to tell from the returns alone whether X would beat both Y and Z in this way. Therefore, Dasgupta and I recommend that voters should be allowed to express their rankings of all three candidates. Rather than simply voting for candidate X, you might, for example, write X first, then Z, then Y. When all voters supply these rankings, one can make the pairwise comparisons that I was just talking about.

I: Is it possible to get a situation where there is no clear-cut winner, that is, no candidate who will beat the other two?

M: That is possible. In fact, that possibility was discovered by Condorcet himself. In such a case, one would have to use some tie-breaking rule to determine the winner. Still, even though simple majority rule doesn't always work as it is supposed to, it works, as I was suggesting, more often than any other voting rule. The cases in which there is no clear-cut winner turn out to be less numerous than the instances in which other voting rules run into trouble. Thus, there is a sense in which the problem you point out is less serious than those that beset other voting rules.

I: Has this method been tried in out in practice?

M: I don't believe it has been used in a large-scale election, say on the national level. But it has certainly been used in smaller elections, e.g., those for committee officers. Now that it's so easy to vote and count votes electronically, I think the time has come to try the method out on a bigger scale. The current way of determining winners in Congressional and Parliamentary elections in the United States and the

Continued on page 13

Continued from page 12

United Kingdom—the winner is the candidate with the most votes even if that total is short of a majority—is, in my view, highly objectionable. With more than two candidates, it's quite possible for the winner to be a “minority” candidate in the sense that a majority of voters prefer some other candidate. I would like to see simple majority rule used instead in these elections.

I: You have also done some work on social choice theory. Could you tell us briefly what this is?

M: In a sense, we have already been talking about social choice theory: it includes implementation theory and voting theory. Social choice theory examines the question: How do we go from individual preferences to social preferences? Of course, we do that when we vote, but the theory goes well beyond voting. Almost any public policy question involves passing from individual to social preferences. When we decide how much public education to provide, how much to spend on national defense, or whether to redistribute income from rich to poor, we have to answer that question. Thus, all these issues belong to social choice theory.

I: These are very practical issues.

M: They certainly are. The theory itself is generally worked out at a fairly abstract level, but it often has strong implications for practical issues.

I: Is it easy to convince the politicians?

M: I never try [laughs]. I wouldn't know how to begin to convince politicians.

I: In some of your papers, you drew some analogies between economic behavior and animal behavior with respect to evolutionary biology. Could we interpret this to mean that animals are also “economic” creatures?

M: If by “economic” creatures, you mean creatures that “optimize”, I think they definitely are. In order to survive, an animal needs (i) food, (ii) shelter from the elements, and (iii) some way of dealing with other animals that might harm it. How an animal pursues these objectives is its “strategy”. Evolution serves to select animals with successful strategies over those with strategies that don't work so well. The animals with the successful, “optimizing” strategies will survive. They will have the opportunity to reproduce, and their offspring will carry on the optimizing legacy. So, evolution forces animals to be optimizers in the same sense that consumers and producers in standard economic models are optimizers.

I: Have you tried to apply your ideas to evolutionary biology systems?

M: Absolutely. The paper I gave today at the conference—another project with my old friend Partha Dasgupta—is on this exact topic. We are trying to understand a certain kind of perplexing behavior that has been observed in certain bird species and also perhaps in humans. Humans are considerably more difficult to study because their behavior is so complicated; it's usually harder to do good laboratory experiments with human subjects than with pigeons. Anyway, the paper constructs an evolutionary model that attempts to account for the documented behavior.

I: A related question: does this mean that economic behavior may have genetic origins?

M: Undoubtedly, many aspects of our daily behavior do have genetic origins. The fact that we are impelled to eat when our stomachs are empty is programmed into our genes. But more complicated optimizing behavior (such as the way we invest in the stock market) is often so complex that usually one can't say how much of it, if any, is genetic in origin. That's why evolutionary psychology is such a controversial subject; in the case of humans, it is very difficult to disentangle what is the result of biology, what is the result of rational calculation, and what is the result of past experience.

I: It seems that economics is now so intertwined with so many other fields.

M: That's certainly true. In particular, the boundary with psychology is where some of the most exciting work in economics is being done today. But the boundaries with other social sciences—especially political science—are also very lively.

I: Is the urge to take risk genetically driven?

M: I don't know the answer to that. There are those who suggest that there may be a genetic disposition toward taking risks. It has been conjectured that males may be more disposed to taking certain kinds of risks than females. That's not a question I have looked into myself, but it's fascinating.

I: Economics is so different from what it was 20 years ago.

M: Yes, it's a subject that has evolved quite rapidly. That's one reason why it's stimulating to be an economist these days – the subject changes so quickly.

I: Do you do simulation in your type of work?

M: I don't personally use simulations often, not because I'm opposed to them but because I prefer, as a matter of

Theodore Slaman and Hugh Woodin: Logic and Mathematics >>>



From right to left: Theodore Slaman and Hugh Woodin

Interview of Theodore Slaman and Hugh Woodin by Y.K. Leong

Theodore Slaman and Hugh Woodin have recently made important contributions to logic, especially to recursion theory and set theory respectively.

Slaman did a bachelor's degree in physics at Pennsylvania State University before going to Cambridge, Massachusetts to do his doctorate in mathematics at Harvard University. He taught at University of Chicago from 1983 to 1996 and subsequently at University of California at Berkeley, where he is a professor of mathematics and is currently chairman of the mathematics department. He has received the Presidential Young Investigator Award and the Alexander von Humboldt Research Award. He has

been invited to give lectures at major mathematical meetings such as the International Congress of Mathematicians, meetings of the American Mathematical Society, British

Continued on page 15

Continued from page 13

taste, doing things the old-fashioned way by calculation. Simulation can be a useful first step for formulating and ruling out conjectures but, in my view, it should rarely be a last step. After you have done enough simulations to have a reasonably good intuition about what is true, then I think you should sit down and try to prove things analytically.

I: In auction theory, for example, there should be some scope for simulation.

M: Sure. In fact, often before a new sort of auction is used in practice, the designers will try to simulate bidding to make sure something won't happen that they didn't think of. Simulation can be a powerful short-cut. I just don't think it should take the place of theorem-proving.

I: Was there any specific reason why you moved from Harvard to the Institute for Advanced Study at Princeton?

M: Yes, there were a couple of reasons. Harvard was a fantastic place to be, and it has as good an economics department as one could hope for. But one advantage I have at the Institute is unparalleled freedom – freedom that a professorship in an economics department could never bring me. At the Institute I have few formal duties and the time to do pretty much whatever I want. The other thing I get there is interdisciplinary contact. In the School of Social Science (where I am), I bump up against anthropologists, political scientists, historians, etc. all the time. I find this

interdisciplinary mix stimulating and interesting. I think one of the ironies about modern universities like Harvard is that one has an enormous variety of interesting people nearby, but little opportunity to talk to them. If you are an economist, you see the economists, of course, but you don't run across people in other fields nearly as often. One nice feature of the Institute is that you talk to non-economists every day, and so and it's not an extraordinary event to have a truly interdisciplinary conversation.

I: Are there other economists in the School of Social Science?

M: I'm the only economist on the faculty, but we always have economists visiting. At the moment there are 5 economists at the Institute for the year, and so I certainly don't feel lonely.

I: Do you have any students at the doctoral level?

M: I do, I have always had Ph.D. students. I think of that as an essential part of my professional life. It's not only fun supervising these students, but useful for my research. Advanced students, particularly those working in areas close to mine, often have stimulating new ideas, which cause me to rethink what I'm doing. In some cases, these students turn out to be collaborators. Some of my favorite coauthors are former students.

Continued from page 14

Mathematics Colloquium and the Association of Symbolic Logic's Logic Colloquia. He has given the Gödel Lecture and has been invited to give lectures and engage in research collaboration in many parts of the world, in particular Japan, United Kingdom, Germany, France, Canada, China, and Singapore. Slaman is best known for his contributions to Recursion Theory.

Woodin obtained his bachelor's degree in mathematics from California Institute of Technology and doctorate from University of California at Berkeley. He taught at CalTech and then at University of California, Berkeley, where he is professor of mathematics. He has received many awards and grants for his research work, among them the NSF Principal Investigator, SERC Senior Visiting Fellowship Research Grant, Presidential Young Investigator Award, Sloan Research Fellow, Carol Karp Prize, Humboldt Research Award, Miller Research Professorship. He is a member of the American Academy of Arts and Sciences. He has been invited to give lectures at the International Congress of Mathematicians and in many parts of the world, in particular, UK, France, Netherlands, Italy, Spain, Canada, New Zealand, Germany and China. He is well-known for fundamental contributions to modern set theory and large cardinals.

The Editor (Y.K. Leong) interviewed Slaman and Woodin at the Department of Mathematics on 12 June 2004 when they were invited to participate in the Institute's program on the Computational Prospects of Infinity. The following is an enhanced version of the edited transcript of the interview in which they share their passion for and fascination with the highest form of abstraction in one of the possibly most remote area of knowledge at the boundaries of mathematics, logic and philosophy.

Imprints: Why did you choose logic and set theory in your graduate studies?

Slaman: When I was an undergraduate, I majored in physics and I took courses in physics and pure mathematics. But nothing fit my way of thinking as well as mathematical logic. I cannot say how logic felt about me, but I was attracted to it from the beginning. It was an intuitive rather than rational decision.

Woodin: When I was an undergraduate, I began in analysis and the problems in analysis that I was interested in led naturally to set theory. So I studied set theory.

I: Mathematical logic and set theory seem to be linked together like inseparable Siamese twins. Why is that so?

W: Methods from logic form an essential part of the study of set theory because the study of definable sets is such an important part of the subject. In that sense, logic is very

much a part of set theory.

S: There are many natural questions in set theory that cannot be settled within a naive mathematical setting. The best example is probably the continuum hypothesis. Gödel and Cohen made a big breakthrough by showing that the continuum hypothesis is neither provable nor refutable within the axioms of Zermelo-Frankel set theory. It is a mathematical question in set theory but you have to use logic to understand it.

I: Logic seems to be a closed book to many people, even to many mathematicians. What do you think is the reason for this?

S: In fact, much of mathematical logic percolated into the general mathematical consciousness without the general mathematician's being aware of it. For example, logic is instrumental in the analysis of solvability problems for Diophantine equations. Given a polynomial with integer coefficients, can you tell whether it has integer solutions? A lot of people contributed and then Matiyasevich provided the final step in the 1970's to show that the answer is "no": there is no such algorithm. Of course, before Matiyasevich's theorem can be proven, one has to analyze what it means to have an algorithm. That preliminary work was done by Gödel, Church, Turing, and others in the 1930's. The very recent result that primality can be decided in polynomial time is something any contemporary mathematician should be able to understand. Again, one is building upon: what it means to be computable, what it means to have an algorithm, the model of computation using Turing machines. All of that comes from logic. These days, there's a concrete feet-on-the-ground understanding of logical issues.

I: Probably for the new generation of mathematicians, but for the older generation of mathematicians, they are quite happy that they never had a course in logic.

S: I recall taking a variety of courses in analysis, topology, algebra, and geometry, and I am very happy about that. Yes, there are mathematicians who are happy that they don't know some branch of mathematics. But with a different attitude, they would have a richer mathematical experience. People should be happy for all the courses that they have taken.

W: I think the Gödel incompleteness theorem is one of the greatest theorems of the 20th Century. How could one be happy about being ignorant of it?

I: They may know it but only at the superficial level. Even now, a first course in logic is not part of the essential mathematics curriculum, even in the second year.

Continued from page 15

S: It's true that in most universities a course in logic is not required but usually it is available.

I: Do you think that logic itself could be subject to immutable logical laws? How do we know that the logical processes that we use in mathematics follow the genuine laws and not some approximation of them?

W: I think that if one comes up with a proof that is a formal proof, there is no question that you have proved what you have set out to prove. In that sense it is a sufficient condition. Of course, no one exhibits a formal proof. On the other hand, I do happen to believe that there are truths of number theory or set theory which are not formal consequences of the axioms. But it's not that we are missing a logic to discover those truths. We discover those truths by understanding formal consequences of the axioms and by following our intuition. But, I don't think there is a super logic that transcends classical logic waiting to be discovered and things you can prove in super logic from the axioms give you more truth than the formal truth.

I: If I remember correctly, when a person defines "implication", it is not a "real" definition, it's just a symbol and somehow it assumes that the person already knows what it means. It is one of those things that you don't define. It seems to me that the logical process is taken for granted right from the beginning.

S: Sure, a logical process is taken for granted at the beginning. It's similar to taking the integers for granted when introducing the axioms for a ring. The point is that we analyze logic with the same mathematical precision and the same success that we achieve with the integers.

W: The issue is if you want to show that a problem is unsolvable, to make that precise you have to formalize reason. Otherwise how do you get mathematical content into the statement like "The continuum hypothesis cannot be solved from the axioms of set theory"? You have to set up some formal system of logic and there are many different ways to do it.

I: Is there a unique system of logic?

W: No, but I think whatever approach you take, you are going to end up with the same collection of unsolvable propositions.

I: Has logic or set theory been able to solve any long-standing problems in areas like analysis or algebra that are not essentially of a logical nature?

S: One way to give a logician's solution to a problem would be to show, to say, that the problem does not have a solution

within the axioms of set theory - that the basic principles used by people in the field who originally formulated the problem are insufficient to settle the problem - in the same way that the axioms of a group are insufficient to settle whether the operation commutes. There are lots of examples like that.

Another way that logic could be used to settle a problem would be to actually give an answer, in the usual sense, which rests upon perspectives or techniques which are intrinsically logical, having something to do with language, definability, and so on. In the past few years, Hrushovski has brilliantly applied model theory to algebra and number theory. Of course, ideas flow in both directions; Sela recently solved a long-standing problem in logic by applying ideas from topology.

W: I give you another example. When I was an undergraduate, I became interested in set theory and was given a summer research project on a well-known problem of Kaplansky in Banach algebras to think about. I did an analysis which led to the theorem of Solovay that it was unsolvable. It was a non-logical problem and the answer is impossible within set theory.

I: What exactly is that problem?

W: The simplest formulation is this question. Consider the set of continuous functions on the unit interval. This is naturally a Banach algebra with the norm being the sup norm. Suppose you have an algebra homomorphism of that Banach algebra into another Banach algebra. Must it be continuous? You can recast the problem as the following. $C[0,1]$ is a Banach algebra under the sup norm. Suppose you put another norm on it that makes it a normed Banach algebra. Must the norm topology be the same? In other words, are all algebraic norms on $C[0,1]$ topologically equivalent? This problem is independent. If the continuum hypothesis is true, then there exist mutually inequivalent algebraic norms on $C[0,1]$. However it is possible to build a universe of sets so that there is only one norm up to topological equivalence.

In fact there is an area where methods of logic and set theory have led to a solution of a classical problem in analysis and not just by showing that the problem is unsolvable. Here there are various ways to cast the problem. A nice way of doing it is to use projective sets which are the sets of real numbers which can be generated from the Borel sets by closing under continuous images and complements. These were studied extensively in the beginning of 1900. By 1925, I would say, two kinds of questions had emerged. One is a measure-theoretic question. Are the projective sets Lebesgue measurable? The second was a structural question called the uniformization question: given a projective subset of a

Continued from page 16

plane; is there a function whose graph is contained in that projective set and whose domain is the projection of that projective set? These are the two classical questions about projective sets. It turns out that both questions are unsolvable in set theory. We can say now that we have solved the questions. They are unsolvable in set theory because the axioms of set theory are insufficient.

The study of the projective sets is really second-order number theory. Beyond the integers, the next structure you might want to look at is the set of integers together with all sets of integers. Many fundamental questions you can ask there are unsolvable, even in set theory. The measure problem for the projective sets, level by level, is a question of second-order number theory. So the question is really what are the axioms for second-order number theory? We now know what they are, and it took many years to come to that understanding. In some sense, we have found the axioms for second-order number theory that correspond to the Peano axioms for number theory. So here we have an example where classical problems have been solved by other than purely formal means.

I: It would be dramatic if one could use set theory to settle the Riemann Hypothesis. Any chance of doing that?

W: Well, the Riemann Hypothesis is formally equivalent to the consistency of a theory. So it could be equivalent to the consistency of the axioms of set theory. If that is the case, probably set theory will be used. But there is no evidence that the problem cannot be solved just on the basis of the axioms for number theory.

There are examples now of algebraic questions that have been solved using set theory. There is a very nice example dealing with free left-distributive algebras with one operation. The algebra generated by one element is left-distributive: $a*(b*c) = (a*b)*(a*c)$. There are some natural questions you can ask about this. It turns out that the natural model for this comes from set theory and that model was used to gain insights into that algebra. In particular, the first solution to the word problem came from this natural model. Subsequently it was done without it. But there are still questions about this algebra that have to do with whether the free algebra is an inverse limit of a canonical sequence of finite approximations. The only proof known still uses the natural model from set theory, but there is no evidence that it really needs it.

This is an example where a truth was discovered first by invoking very powerful axioms. And there are other examples, such as those coming from the determinacy of infinite games.

S: Determinacy talks about two players and a given set

of reals. You use the set to organize a game between the players. The players alternate playing integers to form an infinite sequence. If the sequence is in the set, the first player wins. If not, the second player wins.

W: Suppose the given set is a subset of the unit interval and we are going to play integers between zero and nine. We create the decimal expansion of a real from the integers played. Player 1 wins if that real is in the set, otherwise Player 2 wins.

S: Once we have defined the games we can ask whether they are determined, i.e. if one of the two players has a winning strategy. For example, if we start with the set of all reals, then the first player is going to win. Any strategy will work. The theorem that any Borel set is determined has a remarkable history and is due to Tony Martin. The first proof used measurable cardinals (so, very strong principles from set theory). Later on, Martin proved the determinacy of all Borel games using just the axioms of set theory.

I: What do you think is the greatest contribution of logic to computer science?

S: My answer to that would be the definition of a computable function. It grew purely from logical considerations. Turing presented his model of a computing machine as part of his argument that if a function could be computed at all, then it could be computed by a machine of his sort, namely a Turing machine.

I: This was introduced before the first computers ...

S: Yes, the work on the foundations of computability took place in the 1930s and predates actual computers considerably. There are other examples. Logic has to do with the analysis of language and definability, the resources of definability and algorithms. If you look at logic as having a scope that wide, then computer science is looking at a certain section of logic. So you cannot in any way think of computer science without logic. Theoretical computer scientists are analyzing different levels of complexity of computation, and that's all logic.

I: We know from Gödel's incompleteness theorem that there are true results about numbers that cannot be proved within arithmetic. Is it possible to produce concrete, non-metamathematical statements of this nature?

W: A famous example concerns Goodstein sequences. Goodstein published a paper in the forties presenting a number-theoretic fact which he thought was a good candidate for statements which cannot be proved from the axioms of number theory. In the seventies, that was shown to be the case. You start with a seed number N and generate

Continued from page 17

sequences of integers, called Goodstein sequences. The theorem is that the sequence is eventually 1.

You start with your seed number N and expand it to base 2 as a sum of powers of 2. Then you take every exponent and expand it as a sum of powers of 2 – exponents of exponents and so on. Now change all the 2s to 3s this defines a new number expanded in base 3. Subtract 1 and re-expand to base 3. Now replace the 3s by 4s. Subtract 1 and re-expand to base 4, and so on. The theorem is that you will eventually reach 1. It may take a long time but you will always reach 1. If you use your calculator and start with 33, you will overflow your calculator.

S: Whether the sequences converge is not a metamathematical question. In theory, you can sit down and write them out. Remarkably, the proof that the Goodstein sequences converge involves the transfinite ordinals.

I: But practically it is not possible to compute those numbers.

W: Well, you can do it with small numbers, like 5. Very soon it gets very complicated. This is predicted by the metamathematics.

I: Are there other examples?

W: Harvey Friedman has other examples which are quite interesting. In fact, there are lots of examples of number-theoretic statements which are not solvable within number theory, which are true and which are purely combinatorial.

I: What about the Goldbach conjecture?

W: Well, it could be equivalent to the consistency of ZF or very strong theory but there is absolutely no evidence for that.

S: Or it could be false.

I: Probability has been successfully axiomatized by measure theory, which is essentially about set theory. Yet it is not clear that this gives a true understanding of randomness. Has there been any progress on shedding some light on the intrinsic concept of randomness from the logical point of view?

S: I think there has been a lot of progress. You can trace it back to Kolmogorov. He had this nice idea. He looked at infinite sequences. A random sequence should be as “complicated” as possible. It should be “unpredictable”, it has no historical pattern. It’s just noise. But, what does it mean to have no pattern? What is a pattern? That way of speaking has to be made mathematically precise. To have no

pattern means that there is no algorithm that will describe the sequence. It has a pattern if it behaves according to some algorithmic law, like the sequence of primes – you can write a program that will give you more and more of the primes. So the sequence of primes is not random. It does not have an obvious pattern such as “every other digit is 1,” but it does have a pattern – a computational pattern. Now, if you want to say that a sequence has no pattern at all, then you can say there is no way to compute a pattern. Kolmogorov brought a logical perspective to the concept of randomness.

I: That has nothing to do with probability?

S: Probability and randomness seem closely related to me, but logic has had more to say about randomness. It is not hard to describe. A finite sequence is said to be “simple” if there is a program that will compute the sequence digit by digit and the length (the number of symbols) of the program is less than the length of the sequence. A random infinite sequence has the property that after some finite clustering, none of its initial segments are simple. That’s the notion of Kolmogorov complexity; a sequence can be descriptively random. That definition of “random”, with some technical adjustments, is equivalent to one asserting that the sequence does not belong to easily described sets of measure 0.

On the one hand, you have the measure-theoretic definition of “random”. On the other hand, you have the property about an individual sequence that says that it has this bit-by-bit indescribability. The equivalence of the two is very pretty.

I: What are the greatest advances in logic and set theory in the last century?

S: First, a caveat. The caveat is that the further in the past you look, the easier it is to tell the contributions that had the most impact, changed the way people think about mathematics. Those would be the greatest. I would say the identification of what it means to be provable and the Gödel completeness and incompleteness theorems are great in that way. Gödel’s incompleteness theorem states that the method of proof which he showed captures logical implication is insufficient to axiomatize the basic facts about number theory. That’s a real advance and it changed the way people thought about the mathematical enterprise. A 19th Century mathematician might think, “We know what the proofs are and we should be able to find the correct set of axioms on which to base all of mathematics.” This mindset was completely gone by the middle, certainly by the end, of the 20th Century.

Giving a clear definition of “computable function” was another great achievement. With that available, we can prove theorems about algorithmic solvability and

Continued on page 19

Forthcoming Publications >>>

Continued from page 18

unsolvability within a mathematical setting.

W: In set theory, I think Gödel's isolation or identification of the constructible universe is certainly one of the greatest achievements in set theory in the 20th Century. Cohen's discovery of forcing was also one of the great achievements in set theory of the last century. I think the validation and understanding of projective determinacy is also an important advance, but even among set theorists it is debated whether the axiom should be regarded as true.

S: Professor Woodin would have to be modest about this since he was involved in that work. I do think logicians looking back 50 years from now would see it as something great.

I: There were many scientists and mathematicians who started off as logicians, like Norbert Wiener, John von Neumann.

S: And Saunders MacLane.

I: Any advice for graduate students in mathematics?

S: There are two things you have to do when you are a graduate student. First, read papers (a lot of them), work out the details, learn the body of mathematics, and learn as much as possible of applied mathematics, physics and biology. Then, identify a field of interest and learn it in depth. It's very important to choose the questions to study. You should have the big questions in mind. The smaller questions you work on should have a bearing on the big ones. Secondly, and maybe this should have been primarily, you have to choose the right advisor.

I: What are your hobbies when you are not doing mathematics?

S: The richest part of my life is the company of my family. Mathematics is second to that. There is no candidate for third place. Third place is always going up for rent: running marathons, fixing my house, programming my computer, doing administrative work at the university.

W: Sleeping.

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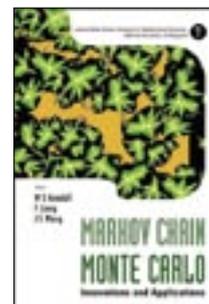
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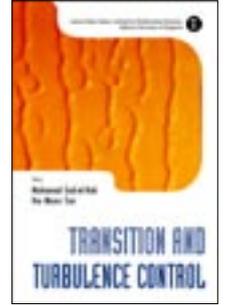
Continued on page 20

Continued from page 19

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