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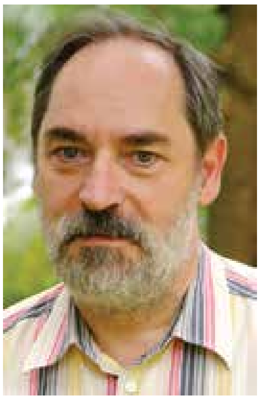
Mathematical Horizons for Quantum Physics >>>


Berge Englert


Hui Khoon Ng

[Editor's note: From August to October 2013, the Institute, in collaboration with Centre for Quantum Technologies, National University of Singapore, hosted the program "Mathematical Horizons for Quantum Physics 2". Two of the program organizers, Berge Englert and Hui Khoon Ng, contributed this invited article to Imprints *as a follow-up to the program.]*

Quantum theory is one of the most important intellectual developments in the early twentieth century. Since then, there has been much interplay between theoretical physics and mathematics, both pure and applied. Arguably, the mathematical foundations of quantum theory – equally at home in physics and in mathematics – emerged from John von Neumann's seminal work on the spectral theory of linear operators in Hilbert space, triggered by the birth of quantum theory in the mid 1920s. Yet, this is just one example of how mathematical insights and tools developed in the course of answering challenging physical problems have contributed to the advance of both mathematics and physics.

It was with this hope of intellectual cross-fertilization that we brought together mathematicians, whose work has a bearing on quantum physics, with researchers in mathematical and theoretical physics, whose work will benefit from mathematical progress, under the second IMS Program on Mathematical Horizons for Quantum Physics (MHQP 2013) (12 August - 11 October 2013). The first such program was held, also at IMS, from 28 July to 21 September 2008, under the overarching theme of operator theory and operator algebra theory. In this second installment, we focused on the rich areas of entanglement and operator spaces in quantum information theory (Session 1), information-theoretic approaches to thermodynamics (Session 2), many-particle systems (Session 3), and open quantum systems (Session 4). Each session spanned three weeks, with a week of overlap between consecutive sessions to allow for potential cross-interaction between the different themes.

For the benefit of Ph.D. students at NUS, a graduate module was offered as part of MHQP 2013. It consisted of the lectures given at the various sessions in conjunction with individual assignments and presentations by the enrolled students.

Session 1: Quantum Information Theory. The fruits of close interaction between mathematicians and physicists are particularly abundant and visible today in the active field of quantum information – the research focus of the Centre for Quantum Technologies (CQT), an NRF Research Centre of Excellence, housed in NUS. Elucidating the mathematical structure and quantifying the amount of multipartite entanglement represent some of the major challenges facing researchers today in trying to advance the understanding

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of ground states of many-particle systems. Answers to the question of separability of multipartite states and violation of Bell inequalities rely heavily on notions from operator spaces. The associated positive but not completely positive maps give some of the most powerful and general entanglement witnesses, and the study of cones of positive maps remain a topic of intense interest in the study of the geometry of separable states as well as the distillability of entangled states. All these topics formed the content of Session 1.

Session 2: Information-Theoretic Approaches to Thermodynamics. Beginning with the Landauer principle – that the work required to erase the information in a system is proportional to the initial thermodynamic entropy of the system – one can establish quantitative links between information and thermodynamics. Carried further, this opens the door to the study of thermodynamics of physical systems through the use of typicality analyses and other information-theoretic approaches standard in information theory. This topic, sitting squarely between physics and mathematics/computer science, was the focus of Session 2 where, in addition to mathematicians and physicists, computer scientists played a major role. Information theory perspectives can offer general statements about issues like the approach to equilibrium and thermalization, as well as work extractable from a quantum system, in the presence of quantum entanglement.

Session 3: Many-Particle Systems. This session examined the mathematical aspects of many-particle quantum systems. The phenomenon of Anderson localization in many-particle disordered systems continue to be of interest in various areas of physics. For instance, the question of the effects of disorder on Bose-Einstein condensation and superfluidity formed the subject of many conversations between participants of Session 3. Another topic of discussion was the question of Lieb-Robinson bounds – mathematical limits on the speed of propagation of information – for fermionic many-body systems in continuous space, with the hope that such bounds would be similarly illuminating as they are for lattice systems.

Session 4: Open Quantum Systems. Despite much work, the classic question of a mathematically precise and physically reasonable delineation between quantum Markovian and non-Markovian dynamics has not been completely answered. Measures of non-Markovianity have been proposed and certain properties like CP-divisibility seem to be commonly agreed upon as characterizing Markovian dynamics. Yet, there remain missing links between such measures and properties and the varied approaches to Markovian dynamics. Another interesting question in open quantum systems that arose from the Session 4 discussions was whether and how much one can learn about the properties of the environment by observing the non-Markovian dynamics of the system immersed in that environment.

At the heart of open quantum systems is also the notion of quantum machines – quantum versions of classical heat engines that do work and are coupled to reservoirs, and the question of relaxation and equilibration of a quantum system in a heat bath. At the quantum level, there is even a debate – conducted at MHQP 2013 as well – on how to exactly distinguish between work and heat. The topic of quantum machines has close links between Sessions 2 and 4, the former dealing with formulation of thermodynamics of quantum systems from an information-theoretic perspective, the latter dealing with quantum heat engines, the classic tool to studying and understanding thermodynamics.

MHQP 2013 brought together researchers in theoretical physics, mathematical physics, and computer science, from Singapore and overseas. All sides benefitted from the intense discussions during the workshop, with each community learning from the other. Many new collaborations emerged from the interactions among the participants, and our Singaporean participants benefitted from the increased visibility among the international researchers. We believe that this unique collaboration between these scientists of different background, different expertise, and different scientific culture has inspired and will continue to trigger new ideas and progress in the mathematics and physics of quantum theory.

Berge Englert
Centre for Quantum Technologies and Department of Physics, National University of Singapore

Hui Khoon Ng
Yale-NUS College, and Centre for Quantum Technologies, National University of Singapore

New Management Board Members >>>

The Institute is pleased to welcome two new members to its Management Board: Professor Kee Chaing CHUA (National University of Singapore (NUS)) and Professor Chengbo ZHU (NUS).

Kee Chaing CHUA

Professor Kee Chaing Chua is Dean of the Faculty of Engineering at NUS (starting July 2014) and Professor in its Department of Electrical and Computer Engineering. His research interests encompass various areas of communication networks, and in particular, wireless and optical networks. He joined the NUS in 1990, and has served as Vice Dean (Research) of the NUS Faculty of Engineering twice (2003-2006 and 2008-2009) and Head of its Department of Electrical and Computer Engineering (2009-2014). He was seconded to the Center for Wireless Communications as its Deputy Director (1995-2000), and he was seconded to National Research Foundation as a Director (2006-2008). From 2001 to 2003, he was on leave of absence from NUS to work at Siemens Singapore where he was the Founding Head of the Mobile Core R&D Department funded by Siemens' ICM Group. He has served on several advisory committees, research/technical review panels and conference organization committees. He is a recipient of an IEEE Third Millennium Medal (2000).

Chengbo ZHU

Professor Chengbo Zhu is currently Professor and Head of the Department of Mathematics at NUS. He was educated as an undergraduate in Zhejiang University, China from 1980-1984 and received his PhD from Yale University in 1990. Professor Zhu joined NUS in 1991, and has served as the President of the Singapore Mathematical Society (2009-2012), and Vice President of the Southeast Asian Mathematical Society (2012-2013). Professor Zhu's research interests are in representation theory of Lie groups, especially classical groups. He was an organizer of several IMS programs – most recently the Chair of the Organizing Committee for the IMS program "Branching Laws" (March 2012).

The Institute would like to express its thanks to the outgoing members of the Management Board: Professor Eng Soon CHAN and Professor Andrew WEE. Professor Chan joined the Board in 2010, while Professor Wee had been a Board member since 2007. Both members contributed substantially in overseeing the Institute's operations and activities.

The Institute looks forward to continuing its smooth running and making further strides under the mentorship of the new and incumbent members of the Management Board.

Wing-Keung TO
National University of Singapore

People in the News >>>

IMS Associates elected as SNAS Fellows

Five associates and friends of IMS, Professors LIM Hock, LING San, PHUA Kok Khoo, SUN Yeneng and ZHU Chengbo have recently been elected as Fellows of the Singapore National Academy of Science. They were among a group of nine scientists in Singapore who received the honor in a ceremony held on 9 May 2014.

Professor Lim Hock (National University of Singapore (NUS)) is a former member of the IMS Management Board. Professor Ling San (Nanyang Technological University (NTU)) served as a member of the organizing committees for the IMS programs “Coding, Cryptology and Combinatorial Designs” (May – June 2011) and “Coding Theory and Data Integrity” (July – December 2001). Professors Phua Kok Khoo (NTU, NUS and World Scientific Publishing Company) and Zhu Chengbo (NUS) are serving members of the IMS Management Board. Professor Sun Yeneng (NUS) is a former Deputy Director of IMS.

Agnes’ baby girl
Annika Roellin

Agnes’ baby girl

Agnes Wu, the Institute’s secretary, became the proud mother of a baby girl, Annika Roellin on 13 March 2014. Congratulations to Agnes and her family!

Personnel movements at IMS

Maisarah Binte Abu Bakar joined IMS as management assistant officer on 10 February 2014. She will cover the secretarial duties of Agnes, who is currently on leave till March 2015.

Nurleen Binte Mohamed, former management assistant officer, left IMS in March 2014. She had been the Institute’s housing officer since April 2009. The Institute takes the opportunity to thank Nurleen for her service during the past five years and wish her success in her future endeavors.

Past Programs in Brief >>>

Inverse Moment Problems: the Crossroads of Analysis, Algebra, Discrete Geometry and Combinatorics (18 November 2013 - 25 January 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014inverse/index.php

Co-chairs

Dmitrii Pasechnik, *Nanyang Technological University*
Sinai Robins, *Nanyang Technological University*

The program covered the following topics: classical moment problems, real algebraic geometry, complex variables, numerical methods, quantum information theory and discrete geometry. These topics were well represented by senior and well established experts though their informative lectures.

There were in total four workshops, four tutorials and 73 invited talks in this program. It started with a workshop “Optimization, Moment Problems and Geometry I” which consisted of 18 invited talks. The second “Quantum Computing Workshop on Inverse Moment Problem” continued with a two-hour tutorial by Jop Briet (New York University) and eight invited talks. The program then continued with a third workshop “Optimization, Moment Problems, and Geometry II” which consisted of 13 invited talks. A graduate winter school was held amidst this workshop and had three four-hour tutorials by Anton Leykin (Georgia Institute of Technology), Josephine Yu (Georgia Institute of Technology) and Jordan Stoyanov (Newcastle University). The last workshop for this program on “Polyhedra, Lattices, Algebra, and Moments” consisted of 34 invited talks.

A lot of active discussions took place between the experts and the junior researchers, postdocs, graduate students, and even the undergraduates that took part in the program.

Björn Gustavsson: Exponential transforms, resultants and moments

Jean Bernard Lasserre: Recovering an homogeneous polynomial

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Raul Curto: Cubic column relations in truncated moment problems

Sharing thoughts amidst coffee (From left: A. Benedict Balbuena, Peter Wittek and Sabine Burgdorf)

A memorable moment for analysts, algebraists, geometers and combinatorialists

Some participants even gave additional informal lectures, even after all the workshops were complete. There were 30 research collaborations/papers that the participants began, worked on, or completed during the program. A total of 98 participants attended the program, and among them were 12 graduate students.

Workshop on Living Analytics: Analyzing High-Dimensional Behavioral and Other Data from Dynamic Network Environments 1 (26 - 28 February 2014)

.... Jointly organized with Living Analytics Research Centre, Singapore Management University

Website: http://www2.ims.nus.edu.sg/Programs/014wliv/index.php

Organizing Committee:

Stephen E. Fienberg, *Carnegie Mellon University*
Ee-Peng Lim, *Singapore Management University*
Wei-Liem Loh, *National University of Singapore*

The joint two-day workshop sought to introduce the current living analytics research activities to the mathematical science, machine learning, and statistical researchers in Singapore. One of the workshop goals was to broaden statistical research underpinnings of models and computational algorithms for living analytics research and other research activities involving the analysis of large high-dimensional databases. The workshop provided a forum for scientists to interact and develop methodology for "big data" problems using living analytics as a focal point, as well as to chart new directions of research and explore possible collaborations.

Day One of the workshop was held at the Ngee Ann Kongsi Auditorium at Singapore Management University and it commenced with a joint talk by the Co-directors of LARC - Professor Fienberg and Professor Lim Ee-Peng. Day One featured a total of nine speakers and a poster session. Among the demos showcased include live tracking and analysis of user mobility data in SMU campus, realtime analytics of social media data, dynamic itinerary planning, and randomized online experimentation. These projects demonstrate synergies with the workshop's theme.

Day Two of the workshop was held at the NUS University Hall Auditorium. An expert in the development of large scale machine learning theories and systems, Eric Xing (Carnegie Mellon University) began the day by introducing Petuum – a general purpose framework for distributed machine learning. The day featured a total of six speakers, including Richard De Veaux (Williams College) who concluded the event with a 2.5-hour tutorial on "Successful data mining in practice".

A total of 179 participants attended the workshop, comprising a good mix of academic and industry participants. Following up the success of the workshop, a one-week program on the same theme will be organized in February 2015.

Stephen Fienberg: LARC-LiveLabs Overview

Richard De Veaux: Successful data mining in practice

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Analyzing big data in a small discussion (From left: Wei-Liem LOH, Ee-Peng LIM, and Eric XING)

An engaging poster session

A network of living analytics scientists

Workshop on IDAQP and their Applications (3 - 7 March 2014)

… Dedicated to Professor Takeyuki Hida

… Co-sponsored by Research Institute for Science & Technology (RIST), Quantum Bio-informatics Research Division, Tokyo University of Science, and Aichi Prefectural University

Website: http://www2.ims.nus.edu.sg/Programs/014widaqp/index.php

Chair:
Masanori Ohya, *Tokyo University of Science*

In the past few years the fields of infinite dimensional analysis and quantum probability (IDAQP) have undergone increasingly significant developments and have found many new applications, in particular, to classical probability and to different branches of physics. The fields of infinite dimensional analysis and quantum probability are rather wide and strongly related in interdisciplinary nature. This workshop aimed at building bridges among these interdisciplinary fields, and mainly focused on quantum information theory and white noise analysis in line with IDAQP

Through the research discussions, the participants of this workshop found clues on how to build a bridge among these interdisciplinary fields. They explored new research directions by interacting with researchers in other fields. This five-day workshop consisted of a total of 23 invited talks, and was attended by 28 participants and one NUS PhD student.

Takeyuki Hida:
Some future directions of white noise theory

Igor Volovich:
Quantum holography and classical random fields

Takashi Matsuoka: Quantum correlation of quantum composite system

Yuh-Jia LEE listening attentively to Ludwig Streit

A union of IDAQP theorists

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School and Workshop on Classification and Regression Trees (10 - 26 March 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014swclass/index.php:

Organizing committee:
Wei-Yin Loh, *University of Wisconsin-Madison*
Probal Chaudhuri, *Indian Statistical Institute, Kolkata*
Ben Haaland, *Duke-NUS Medical School*
Tao Yu, *National University of Singapore*

Classification and regression trees are an integral part of the toolbox of data mining, machine learning, and statistics. The school aimed to introduce the subject to other researchers and practitioners, while the workshop brought together current experts in the field to discuss recent developments and generate ideas for future research. Wei-Yin Loh (University of Wisconsin-Madison) gave a tutorial on "Classification and regression trees" spread over four days, after which the workshop continued and had a total of 17 invited talks.

Despite fifty years of research in the topic of classification and regression trees, the workshop was the first international conference on the subject. The school and workshop attracted a total of 62 participants from academia and industry, and among them were 23 graduate students.


Achim Zeileis: Model-based recursive partitioning


Wei-Yin LOH: Tutorial on classification and regression Trees


Jeffrey Simonoff: Regression trees for longitudinal and clustered data


Sharing light moments (From left: Yu-Shan SHIH and Hongshik AHN)


Climbers of classification and regression trees

Self-normalized Asymptotic Theory in Probability, Statistics and Econometrics (1 – 30 May 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014self/index.php

Co-chairs:
Ngai Hang Chan, *The Chinese University of Hong Kong*
Xiaohong Chen, *Yale University*
Qi-Man Shao, *The Chinese University of Hong Kong*

This program provided the probabilists, statisticians and econometricians a unique platform to discuss interesting fundamental problems and results and explore possible solutions related to asymptotic theory in their fields. It was also intended to bring young researchers to the frontier of this fascinating area.

In total, there were three tutorial lectures and 23 invited talks presented during the program. The tutorial lectures on "Introduction to Self-normalized Limit Theory" was delivered by Qi-man Shao (The Chinese University of Hong Kong), while the tutorial lectures on "Applications in econometrics" were delivered by Kengo Kato (University of Tokyo) and Xiaohong Chen (Yale University).


Andrew Chesher: Instrumental variable methods with set-valued residuals


Qi-Man SHAO: Introducing self-normalized limit theory

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There were a total of 49 participants and among them were 15 graduate students.

Thomas Mikosch: The self-normalized sample extremogram and ex-periodogram

Big smiles among tea breaks (From left: Tze Leung LAI and Ching-Kang ING)

An assembly of probabilists, statisticians and econometricians

Public lecture:

Probal Chaudhuri: Shape of the Earth, Motion of the Planets and the Method of Least Squares

Professor Probal Chaudhuri of the Indian Statistical Institute delivered a public lecture titled "Shape of the Earth, Motion of the Planets and the Method of Least Squares" at NUS on 20 March 2014. In the lecture, Professor Chaudhuri first described a statistical problem in the 18th century - known as "the problem of combining inconsistent equations" in those days - which arose when scientists dealt with astronomical and geodesic measurements. Then he talked about the solutions to the problem as contributed by various mathematicians including Boscovich, Euler, Gauss, Laplace and Legendre, and how these developments led to the invention of the method of least squares. A total of 41 people attended the lecture.

Current Program

Algorithmic Randomness (2 – 30 June 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014algo/index.php

Chair:
Frank Stephan, *National University of Singapore*

Randomness is a mathematical concept that spans over a broad class of mathematical objects from finite words to transfinite cardinals. While classical probability theory does not formulate or even allow for a definition of an individual random object, there is an enduring appeal to the intuitive notion of a real number or of an infinite binary sequence chosen at random. This intuitive notion can be made precise when it is interpreted within an effective framework. There, an object is random if it passes all effective tests for randomness. This means that no algorithmically devisable test could detect features of an object that would contradict its randomness. This descriptive, or algorithmic, perspective is both generally applicable and conceptually useful. With it, one can calibrate degrees of randomness, characterize applications of randomness, and prove preservation of randomness across type, such as between real numbers on the line and sample paths in Brownian motion. These are among the topics that will be studied during the 2014 IMS Programme on Algorithmic Randomness.

The conference series "Computability, Complexity and Randomness" is centered on developments in Algorithmic Randomness, and the conference CCR 2014 will be part of the IMS program.

Activities

- Informal collaboration and talks before conference: 2 - 8 June 2014
- Ninth International Conference on Computability, Complexity and Randomness (CCR 2014): 9-13 June 2014
- Informal collaboration and talks after conference: 14 - 30 June 2014

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Next Program

IMS Graduate Summer School in Logic (23 June – 4 July 2014)

... Jointly organized with Department of Mathematics, NUS

Website: http://www2.ims.nus.edu.sg/Programs/014logicss/index.php

The Summer School bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject.

Activities

- Week 1: Lectures in Recursion Theory by Liang Yu, Nanjing University
- Week 2: Lectures in Set Theory by Qi Feng, Chinese Academy of Sciences and Hugh Woodin, Harvard University

Programs & Activities in the Pipeline

The Geometry, Topology and Physics of Moduli Spaces of Higgs Bundles (7 July - 29 August 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014geometry/index.php

Co-chairs:
Richard A. Wentworth, *University of Maryland*
Graeme Wilkin, *National University of Singapore*

The subject of this program is the moduli space of Higgs bundles and its connections with different areas of mathematics and physics. This program aims to bring together experts who study the geometry, topology and physics of Higgs bundles; invite leading researchers to give talks on recent results and the latest developments in the field; have experts give mini-courses explaining the background to their fields; encourage collaborative work and introduce graduate students and young researchers to the latest research and open problems in the field.

The activities at the IMS – consisting of a summer school, a workshop and a conference – will be a continuation of the program "The geometry and physics of moduli spaces" at Instituto de Ciencias Matemáticas (ICMAT) in Madrid from 14 April - 11 July 2014. The conference will consist of research talks on the latest developments relating to the moduli space of Higgs bundles. The conference will be accepted as a satellite conference to the International Congress of Mathematicians in Seoul, South Korea, which begins on 13 August 2014.

Activities

- Summer School: 7 - 18 July 2014
- Workshop on the Themes of the Summer School: 21 - 25 July 2014
- Conference: 4 - 8 August 2014

IMS-JSPS Joint Workshop in Mathematical Logic and Foundations of Mathematics (1 - 5 September 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014wlogic/index.php

Organizing Committee:
Chi Tat Chong, *National University of Singapore*
Frank Stephan, *National University of Singapore*
Kazuyuki Tanaka, *Tohoku University*
Yue Yang, *National University of Singapore*

This workshop is jointly sponsored by the Japan Society for the Promotion of Science and the National University of Singapore. This workshop is intended to provide a venue for continued interaction and to serve as an opportunity to explore new research collaborations in three broad areas of common interest: reverse mathematics (involving both standard and nonstandard models of arithmetic), algorithmic randomness (in both classical and higher setting), and set theory (particularly cardinal characters of the continuum).

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Scalar Curvature in Manifold Topology and Conformal Geometry (1 November - 31 December 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014scalar/index.php

Organizing Committee:
Fei Han, *National University of Singapore*
Wei-ming Ni, *University of Minnesota*
Xingwang Xu, *National University of Singapore*
Weiping Zhang, *Nankai University*

Activities

- Workshop on Positive Curvature and Index Theory: 17 - 21 November 2014
- Workshop on Partial Differential Equation and its Applications: 8 - 12 December 2014
- Winter School on Scalar Curvature and Related Problems: 16 - 19 December 2014
- Public Lecture

High Performance and Parallel Computing for Materials Defects and Multiphase Flows (1 January - 31 March 2015)

... Jointly organized with Department of Mathematics, NUS

Website: http://www2.ims.nus.edu.sg/Programs/015hiper/index.php

Co-chairs:
Weizhu Bao, *National University of Singapore*
Weiqing Ren, *National University of Singapore and Institute of High Performance Computing, A*STAR*
Ulrich Rude, *Universität Erlangen-Nürnberg*

Activities

- Collaborative Research: 1 January - 31 March 2015
- Embedded meeting: 7 - 11 January 2015
- Workshop I (Recent Advances in Parallel and High Performance Computing Techniques and Applications): 12 - 16 January 2015
- Workshop II (High Performance and Parallel Computing Methods and Algorithms for Materials Defects): 9 - 13 February 2015
- Workshop III (High Performance and Parallel Computing Methods and Algorithms for Multiphase/Complex Fluids): 2 - 6 March 2015
- Tutorial and Public Lectures

Joint international workshop of the National University of Singapore Institute for Mathematical Sciences and Yong Siew Toh Conservatory of Music Mathemusical Conversations: Mathematics and Computation in Music Performance and Composition (13-15 February 2015)

... Jointly organized with Centre for Digital Music, Queen Mary University of London, UK, and Science and Technology of Music and Sound Lab, IRCAM, CNRS, UPMC, France

Website: http://www2.ims.nus.edu.sg/Programs/015wmusic/index.php

Program Chairs:
Gérard Assayag, *Institut de Recherche et Coordination Acoustique/Musique*
Elaine Chew, *Queen Mary University of London*

Workshop on Living Analytics: Analyzing High-Dimensional Behavioral and Other Data from Dynamic Network Environments 2 (23 - 27 February 2015)

... Jointly organized with Living Analytics Research Centre, Singapore Management University

Website: http://www2.ims.nus.edu.sg/Programs/015wliv/index.php

Organizing committee:
Stephen E. Fienberg, *Carnegie Mellon University*
Ee-Peng Lim, *Singapore Management University*
Wei-Liem Loh, *National University of Singapore*

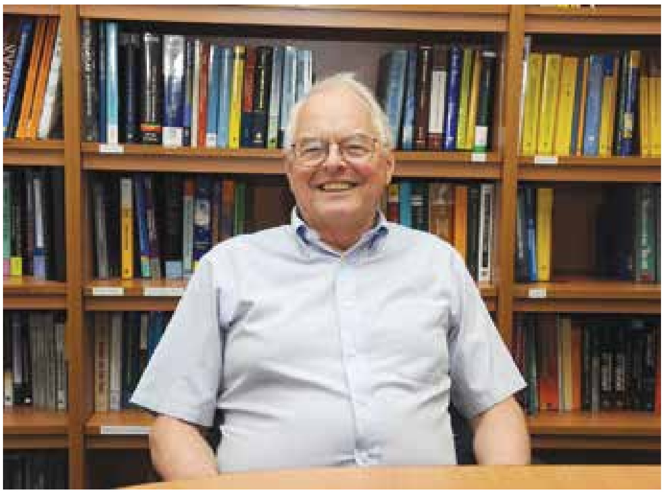
Sets and Computations (30 March - 30 April 2015)

Workshop on Stochastic Processes in Random Media (4 - 15 May 2015)

Workshop on New Directions in Stein's method (18 - 29 May 2015)

Mathematical Conversations

Ralph T. Rockafellar: Convexity, Optimization, Risk >>>



Ralph T. Rockafellar

Interview of Ralph Tyrrell Rockafellar by Y.K. Leong

Ralph Tyrrell Rockafellar made pioneering and significant contributions to convex analysis, variational analysis, risk theory and optimization, both deterministic and stochastic.

Rockafellar had his undergraduate education and PhD from Harvard University with a one-year Fulbright scholarship break at University of Bonn. Except for an initial short stint at University of Texas at Austin, he has taught at the University of Washington at Seattle since 1966. He became professor emeritus there in 2003 and was concurrently appointed as an adjunct research professor at the University of Florida at Gainesville. During his distinguished career, he has been invited to numerous scientific meetings in various parts of Europe and has held positions as visiting professor and researcher in Denmark, France and Austria. During the last decade, however, he has spread his wings of mathematical research and scientific collaboration also to South America, especially Chile and Brazil, and to emerging centers of scientific activities in Japan, China and Taiwan. He is fluent in German and knowledgeable in French and Russian.

He has served on the editorial boards of numerous international journals on applied mathematics, optimization and mathematical finance and continues to do so for at least four major journals. He has contributed organizational services to the Mathematical Programming Society, the International Institute for Applied Systems Analysis, Institut des Sciences Mathématiques (Montreal) and the FONDAP Program in Applied Mathematics in Chile.

His total research output in the form of research papers, scholastic articles and books exceeds 230 in number; his collaborative research is prodigious. His most famous book *Convex Analysis*, published in 1970, is the first book to systematically develop that area in its own right and as a framework for formulating and solving optimization problems in economics and engineering. It is one of the most highly cited books in all of mathematics. (Werner Fenchel (1905-1988) was generously acknowledged as an "honorary co-author" for his pioneering influence on the subject.) In addition, Rockafellar has written five other books, two of them with research partners. His books have been influential in the development of variational analysis, optimal control, mathematical programming and stochastic optimization. In fact, he ranks highly in the Institute of Scientific Information (ISI) list of citation indices.

Rockafellar is the first recipient (together with Michael J.D. Powell) of the Dantzig Prize awarded by the Society for Industrial and Applied Mathematics (SIAM) and the Mathematical Programming Society (MPS) in 1982. He received the John von Neumann Citation from SIAM and MPS in 1992. His scientific contributions have been further recognized by honorary doctorates from universities in the Netherlands, France, Spain and Chile.

In 1965 Rockafellar began a long period of collaboration with Roger J-B Wets. That led eventually to the unified development of a new field which they termed "variational analysis". It extends the concepts and methodology of classical calculus and convex analysis to cover, among other things, broader problems of optimization that require set-valued convergence and generalized differentiation. The resulting monograph *Variational Analysis*, which earned the 1997 Frederick W. Lanchester Prize from the Institute for Operations Research and the Management Sciences (INFORMS), systematically laid out that subject. This was followed in 1999 by INFORMS's award of the John von Neumann Theory Prize to the joint authors.

While he was attending the Third Sino-Japan Optimization Meeting (31 October – 2 November 2005), which was organized by the National University of Singapore and the first such meeting to be held outside China and Japan, that meeting celebrated his 70th birthday. In January 2011, he was invited to NUS's Risk Management Institute, its Department of Decision Sciences and its Institute for

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Mathematical Sciences. He was also an invited speaker at the Institute's Workshop on the Probabilistic Impulse behind Modern Economic Theory, held from 11 to 18 January 2011. On behalf of *Imprints*, Y.K. Leong took the opportunity to interview him on 18 January 2011. The following is an edited and enhanced transcript of a lively interview in which he traces his early years in the United States and Europe and imparts the passion of a trail blazer of a path less trodden. One also sees a less well-known side of him – a spirit of physical adventure that is closely intertwined with the spirit of mathematical exploration.

Imprints: You went to University of Bonn on a Fulbright Scholarship in 1957. What attracted you to Bonn then?

Ralph Tyrrell Rockafellar: That goes back to the early days of my career when I did not know anything except that I was a good student and did well. I had come from a limited background. No member of my family had a college education. In my undergraduate years at Harvard, I already took two of the main sequences in graduate studies of mathematics, in real analysis and algebra. But I didn't know I wanted to be a mathematician. What I wanted was to have a year abroad while making up my mind. The easiest way to do this was to get a Fulbright Scholarship. I had a good chance and did get one, but then had to choose the country to go to. (The university would then be assigned automatically.) I chose Germany because I knew a lot of German, which I had learned by myself as a teenager. (My home city of Milwaukee, Wisconsin, has a strong German background.) I was very interested in languages, perhaps even more than in mathematics. Nevertheless I worked very hard on mathematics that Fulbright year at the University of Bonn, although I hardly went to classes at the university. In those days there were no exams at German universities, only lectures until a final graduation period, but the effort was exceedingly important to me. I learnt in fact that I really wanted to be a professional mathematician.

I: You were in Bonn for a number of years, isn't it?

R: I was in Bonn for just one year. It seemed that right afterwards I would have to go to the army, which I dreaded. It would be two years of service, but three years if you volunteered so as to maybe get into military intelligence and work with languages. I concluded my time in Germany, being reconciled to going to the army in some way or other, instead of proceeding with education. However, Sputnik then went up and everything changed. I found out that I could be deferred from the military once more. It was already too late to apply to Harvard for graduate studies, so I spent a year in my hometown, Milwaukee, teaching at Marquette University. There I learnt from a statistician friend, Joseph Talacko, about optimization, which was a new field, and that was extremely formative for me. For the following year opportunities came up to go either to Harvard or to Princeton. I decided on Harvard because I knew the place well, had friends there, and enjoyed the cultural life in Boston. (Princeton is in a small town more than an hour south of New York City.)

I: You took your PhD at Harvard University. What was the topic of your PhD thesis and who was your thesis advisor?

R: The topic was in optimization, which as a subject was only about 8 years old at that time and totally unrepresented at Harvard. Before writing a thesis I had to take the standard graduate courses along with various electives and had to pass the required comprehensive exams for a PhD. Then, since there was nobody in the mathematics department who had any idea about optimization, I basically had to do my research on my own. My designated advisor was Garrett Birkhoff [(1911-1996)], who was a specialist in lattice theory and differential equations but knew nothing about the topics I was exploring. I anyway completed a thesis in optimization and got it approved with the help of kind words from [Albert W.] Tucker [(1905-1995)] at Princeton, who had heard about my interests from Talacko at Marquette.

I: Did you pick up the research problem yourself?

R: Exactly, exactly. In the year before I had gone on to graduate studies, but had heard about optimization at Marquette, I learned that it had a phenomenon called duality. Mathematicians are generally familiar with some aspects of this, such as dual vector spaces, but this was a new and intriguingly different kind of duality. I was told it was well understood for "linear" problems of optimization but nobody knew how to extend it to "nonlinear" problems of optimization. That got me fired up and put me on my own track. I worked on it by myself even during the first two years back at Harvard, devoted to courses and exams. But later I got some help from the outside, not in research but in support from Tucker at Princeton, who was one of the

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founders of optimization. It was he who had arranged that I could have pursued graduate studies at Princeton instead. He was able to tell my advisor sometimes: "He's okay. What this young man is doing is good. Don't worry." My advisor himself did not know how to deal with me except always to say "Work harder, work longer." That's how I was able to finish. And a couple of years after I got out, Tucker invited me to be a visiting professor at Princeton, which was very important to my career.

I: How did you know him [Tucker]?

R: The statistician I knew back in my hometown, Talacko, had contacted him at an early meeting in optimization in the year before I returned to Harvard [1959]. He told him about me and that got the relationship started. Tucker was shown some of the duality work I was doing, and he was impressed. It's very good, by the way, to see professors who encourage young people, especially young people who are trying to cross an academic ocean all by themselves. In those days I didn't even know I wanted to be a professor. To me a professor seemed just like one step above a high school teacher, maybe with more prestige. I knew very little about the life of a professor or the academic world.

I: If I understand it correctly, convex optimization or programming generalizes many different kinds of programming methods. Would that mean that in some sense convex optimization unifies a number of areas and would that also not mean that the most important or central problems in optimization are those in convex optimization?

R: This is a good question because it helps me explain a lot of conceptions that people may have about optimization and programming. The original idea of "programming" was synonymous with optimization. The only way "programming" turned out to be computer programming as we know it now is that "programming" was connected with running or managing a government program on a computer. Early examples, like food distribution programs, very much involved optimization in the sense of finding the best ways to do a job, and computers were essential for that. It was then called computer programming. The word "programming" as a synonym for optimization is going out of fashion, though. In fact, there is an organization called the Mathematical Programming Society which this year changed its name to Mathematical Optimization Society because they found itself more and more uncomfortable with some misunderstanding created increasingly by its original name. "Programming" as optimization referred to a new kind of mathematics which demanded fresh ways of looking at things. I got fascinated by the theory behind it – how to create the needed mathematics. I've always been more of a theory builder. I like the idea that mathematics can organize ideas across disciplines. People in some particular application area may have a very narrow view of what they are doing, but a mathematician can see similarities and analogies and can put together structures that will work for many different practical purposes and at the same time generate deeply interesting mathematical concepts and results.

I: Just to pursue it a bit further, does convex optimization sort of unify different kinds of things?

R: I would like to put it differently. I have to explain what optimization is about. First let me say that in mathematics most people make a distinction between linear and nonlinear things, but in optimization it's between convex and non-convex things. This means that the core entities for which the theory is nicest and computation is the easiest in optimization are those that have properties of convexity, just as in engineering and physics it's the linear things that mostly serve for approximations and computations. But this had to be discovered. Optimization was not a known subject in those days, and its essentials had to be found out. In some way the impetus for that started with computers. Anytime there is a decision and choices have to be made, you want to make a better decision, or in other words, optimize. Once computers came in, people were able to look at problems on an entirely different scale of magnitude. In huge problems of optimization, inequalities are very important. You are not modelling with equations. You have certain ranges in which you can do things – not too much, not too little. You have a large number of these one-sided constraints, but you don't know which ones will ultimately be active. Maybe some of them are superfluous in determining the solution, but there is no way to know that in advance of laborious computations. Optimization draws on many different things, including a new kind of geometry, but relatively little of classical mathematics. The new geometry centers on convexity but carries over then to the treatment of functions as well.

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I: You wrote a book on convexity. Was that the first book on convexity?

R: It was, at least in combining convex sets with convex functions and in that way with analysis. The title of this book was suggested by Tucker in the year when he invited me to Princeton. I had already done a lot of work on convex sets, convex functions and applications to optimization. He suggested that I give a course on this, and when that was nearly over, with a set of lecture notes, he said, "Write it up as a book." The title "Convex Analysis" was prescient because this marks a transition from geometry to analysis in which profound changes take place. The way we look at functions ordinarily in calculus is through their graphs. Differentiation corresponds to approximations that linearize these graphs. But in much of optimization you shouldn't look at a function this way; you should focus instead on the epigraph – the set of points on and above the graph. Convex functions are characterized by the fact that the epigraph is a convex set. The epigraph may not have a smooth boundary suitable for linearization, but nonetheless there can be convex tangential approximations leading to new forms of differentiation, and so forth. This creates a wholly different outlook on analysis. The convex analysis book is the one people best know me for. I wrote it at the beginning of my thirties, very early in my career, and it put the subject on the map.

I: What about a later edition? Did you revise it?

R: Never had a chance to really revise it. Sorry about that, but there also didn't seem to be much need. I have a friend at *Mathematical Reviews*. He had access to certain databases that they keep, for example a list of the 100 most cited books, not just the most recent, but of all time. Out of the first 100, he told me – that was a few years ago – it was number 6. In 1997 that book also came out in paperback, still going strong some 27 years after its first printing, and it is probably one of the most enduring books published by Princeton University Press. Why? Because, besides being a unique contribution, it came at the beginning of a subject which was growing enormously with many applications, so it became a key reference for everybody. Later on, my idea was to unite convex analysis with classical analysis in some bigger and grander scheme. I was able finally to do that, along with the help of others, and the larger subject is called "variational analysis".

I: It seems that this book is more of a theory book than a methods book.

R: That's right. Optimization has an unusual status in mathematics. It really has to stand on three legs. One leg is some kind of basic theory like convex analysis. Another leg is the understanding of the various ways of formulating problems, what are the important things, not important things – in other words, artful mathematical modelling. What are the tools for that? In a new subject, you are obliged to develop new tools. The third leg on which optimization stands is computation. All three interact deeply. Computation is often based on optimality conditions, which come from analysis and especially duality. On the other hand, the models you set up should be ones suitable for finding solutions effectively. The challenges of modeling and computation inspire advances in theory. The trouble with this three-legged field of optimization, however, is that it doesn't fit into a single branch of mathematics, pure, applied or numerical. So it doesn't really have a home *[Laughs]*.

I: Or it belongs everywhere …

R: Belongs everywhere. If you look at the academic world there are traditions. Chemistry belongs to the school of science, but where is optimization? You find it in engineering schools, mathematics departments, business schools. You find it in all sorts of different locations. It has undergone a sort of random social development in different universities and countries.

I: That's very interesting. This book came out before personal computers came in, didn't it?

R: You want to go back earlier. Computers came out in the forties with those enormous things like ENIAC (Electronic Numerical Integrator and Computer). People like [John] von Neumann [(1903-1957)] were very much involved in computers. He was a mathematical genius looking at many things – quantum mechanics, mathematical economics, computers and early aspects of optimization. He was at Princeton and basically that's how optimization got going there – his connections with Tucker. Some foundational work in optimization was promoted at Princeton. The thing about computers is that they got involved early on with algorithms for certain classes of optimization problems –

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linear programming – there is something called the "simplex method" invented by George Dantzig [(1914-2005)], which turns out to be extremely efficient for solving many practical problems. These are problems we could never solve by hand. That is basically the spark that made everything grow, in 1949, eight years before I started to be involved.

I: You have been Visiting Professor in quite a number of universities in Europe between 1964 and 1997. Does Europe hold a certain kind of attraction for you?

R: It does from a number of angles. One is that Europe is easy to understand beyond mathematics, even with my limited background growing up. I love to travel. I love languages, and as a graduate student in those days you needed to study European languages. I was fluent in German and I knew enough French to be a visiting professor in France and give lectures in French. I also learned a lot of Russian. I like that side of the world and jumped at the opportunity to spend time there. Another more important aspect of my connection with Europe is that in the United States there was hardly any activity [in convexity] except possibly at Princeton. It was different in Europe. In France the field of convex analysis quickly became very popular. In Russia, too, there was intense interest. Of course, because I was a founder of the subject, I got many invitations related to it to participate in meetings for which financing was available. You could say that this was in contrast to Asia at the time. There were not many things happening in mathematics in Asia in the `60s, certainly not the kind of things I've been connected with. Now I have many chances to go to both Asia and South America. I have connections with University of Chile in Santiago; I go there several times a year. I went to a conference in Brazil last month, and that wasn't the first time. I have strong research connections in that southerly direction and more recently also in the direction of Asia – China, Japan and Taiwan. Part of what we see in Asia is a wave of professors who studied in the United States, Canada and Europe and then came back to their home countries and made mathematics grow. If I may put a footnote somewhere, in the business school here, which I am visiting for two weeks, I have a former student [Jie Sun]. He was originally from mainland China, but had his PhD with me, worked in the United States for some time, and came to Singapore in 1992. He was then the only person here in optimization, but now there are probably 15 to 20 people, just in the business school at the National University of Singapore, who are connected in one way or another to optimization.

I: Are you also interested in economics?

R: From the very beginning I was interested in economics. One of the reasons is that a lot of mathematical economics involves convex sets and convex functions. Another fundamental reason is that a lot of economic ideas involve optimization, for example in maximizing utility or minimizing cost. That connection has continued to grow, and now I'm all the more engaged with economics. As you perhaps know, I was involved with an economics conference here just now. With me at the moment at NUS is a close collaborator [Alejandro Jofré] from Chile.

I: Until your retirement, you were at the University of Washington (Seattle) for 40 years. Is there any particular reason for this attachment?

R: We can start with why I went there in the first place. There were people on the faculty who had worked on convexity of a more geometric kind in a setting of functional analysis, for instance the study of the unit balls associated with norms. That enabled me to be invited there in the first place. Then I found a great working environment – not too many rules, the personal freedom to focus on research I liked, and the possibility of teaching courses on topics under development. Later I was able to set up a broad program of optimization-related courses in a department that supported me well, a department of mathematics where theory was welcome to thrive. I didn't want a job in a business school or an engineering school. I really felt myself to be a mathematician, so this was a wonderful thing. However, an underlying answer to your question about the 40 years is that I love the natural setting of Seattle. It has made me very much of an outdoor adventurer. I have spent many, many years climbing around in the wilderness, hiking in the mountains, fishing in high lakes with my family and friends, kayaking from one island to another, camping, catching crabs. This became an important part of me and the way I related to my family and friends, even students. After I became so tightly bound up with it, there was no substitute ever to be found in another location.

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I: Do you still do mountain climbing?

R: I still do mountain climbing in the form of off-trail hiking and exploring. I also do kayaking, but there are two kinds of that, like the two kinds of skiing – downhill skiing and cross-country skiing. With kayaking, one kind is white-water kayaking in fast-flowing rivers. That I don't do. What I do is the kind where you paddle in the sea from island to island or down a nice quiet river or around a lake, more or less to explore the shore. You can sometimes stop and camp. On a major island near Seattle, reachable by ferry, I have a second house which is right on a tidal beach, and my kayaks are there, ready to be put in the water whenever adventure calls.

I: No wonder you never left Seattle.

R: That is the real answer to your question. Fill your life with a lot of enthusiasms, and it will help you to be creative and keep a balance. I am often asked by people about hobbies. I always have my outdoor interests for that and have never pushed them aside. I have always kept them going, and they have helped my productivity despite the time they take. In fact, I have always done things I enjoy and think are important. I suppose I'm as competitive as anybody else and would like imagine being out in front in at least one form of recreation, but I don't even like to play a game of chess. I don't like one-on-one competition of that kind. I was never much into sports as a child, yet I have some physical endurance. I do well in the mountains and I feel I can excel in that scene. Another advantage offered by mountains is that you can escape from too much bother, too much noise, by going out into quiet, inspiring nature. You then get new ideas. In the past, before we depended on computers for writing, I could sit high on a peak for an entire day with a pile of papers, putting together an article.

I: Other than algorithmic or computational approaches, have new ideas in algebra, analysis and geometry contributed to fundamental advances in optimization?

R: I like this question because I can turn it completely around. I think it suggests that optimization is an area which takes existing mathematics and applies it. Actually it's quite the opposite. Optimization grew out of new demands in mathematical thinking. After all, a lot of mathematics was inspired by the challenges of astronomy, building pyramids, commerce, and the like. In physics everything is modeled by equations, but now in economics and systems management, for example, there are different needs. What I believe is that optimization theory has contributed to a new kind of mathematics, a new kind of analysis. I only wish that people in the pure mathematics departments had more access to knowing about this. A lot of mathematics tends to go on in a closed little world and in-group. You know, all areas of science and mathematics have a social component – revolving around who knows who. People just aren't aware of optimization-inspired developments. A good example of such developments is in my most recent book *Implicit Functions and Solution Mappings*, which came out in 2009, written with a colleague [Asen L. Dontchev] who is a Bulgarian-American, now in Michigan. In the mathematics of the past, the main model was solving a system of equations. Then if you wanted to know how the solution depends on parameters, you were led to the implicit function theorem. But now there are different models, such as problems of optimization or game structures in which several agents compete in optimizing from their own perspectives. How does the solution to the model depend on the model's parameters in such cases? You can't use the classical implicit function theorem. You need a new version for broader kinds of "solution mappings," which assign to a parameter vector the corresponding solution or set of solutions. What can be said about the derivatives of such a mapping? They have to be one-sided generalized derivatives. What does that mean? The book presents the kind of analysis that can serve in such a situation.

I: So it seems that optimization actually created new mathematics.

R: That is exactly what I believe very strongly. Nowadays there's convexity in statistics and set-based probability theory, along with many other areas. More recently I got into the theory of risk. I started out with a colleague [Stanislav Uryasev] a dozen years ago. He's 20 years younger than I am and he's at the University of Florida.

I: Your work is mainly in the theoretical aspects of optimization. Has the computer come up with results or scenarios that are counter-intuitive and not mathematically proved or understood?

R: That's not the way computers influence optimization, because we aren't working with classical conjectures

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or anything that resembles that. However, there's an important aspect which, as they say, boggles the mind. It is that optimization deals with incredibly large problems in which you can have millions of variables and millions of inequality constraints. How are you going to solve them? You can only hope to do it with careful attention to structure. That may involve discretization in space or time, or stochastic representation, or a more novel kind of approximation, and we get into territory beyond what people can usually imagine. With advances in computers, what is very important is the feedback from computational methods – what can be done and what can't be done. After all, optimization was originally inspired by broader capabilities in computing. As more computational ability comes up, it's not just that you compute bigger problems. It's that you have new ways of thinking about them, requiring extensive mathematical development.

I: What do you think about quantum computers?

R: We have problems in optimization which are still far beyond our ability to solve. They are problems with enormous numbers of variables and constraints, which arise for example through, discretization as already mentioned, or in stochastic representation. In Monte Carlo methods, say, you can generate approximations through statistical sampling, and an explosion in the dimensionality can occur. But if you have a system evolving in a lot of time periods and each period has such an explosion of stochastic branches through sampling, it's easy to see how fantastically huge a problem of optimizing or managing that system can become. How can you best model it? How can you effectively work with the huge model? So, the level of computing will have a big influence. Quantum computers could really help.

I: What about parallel processing?

R: That helps too. It's also important, but with parallel processing it's not just the ability of the computers to perform many actions simultaneously. How do they communicate with each other? Results have to be combined, and how do influences go back and forth? It's not just a matter of machine technology. It's a matter of understanding the design of the communications that take place. Parallel processing doesn't end the story by itself.

I: I believe that you work on several projects with different people at the same time. What are the areas of application that you are working on now?

R: My current work is in three separate areas basically. One is on the pure mathematical side in the sense of theoretical development such as in the book I told you about *[Implicit Functions and Solution Mappings]*, which is sort of outgrowth of convex analysis and variational analysis. By the way, the book *Variational Analysis* by Roger Wets [and me], written about 12 years ago, was already number 50 in that list of top 100 cited. This was only several years after it came out. So that's one side. Another side of my current work is economic modelling, specifically economic equilibrium in markets. That's mostly what I do in my collaboration in Chile and also now with Wets. The third side is the theory of risk which is focused at the University of Florida, and that is what is propelling a lot of the speaking invitations I get. I got interested in risk because I like to work on topics which have practical applications and at the same time exhibit the beauty of thought that I demand as a mathematician. You don't want to waste time on something that is ad hoc and temporary. You want to think you have found the real essence of a topic so you can put it together usefully for the next generation. The theory of risk is moreover deeply involved with convex analysis. That started in finance. I have long been working in optimization problems where there is uncertainty – you have make decisions in advance of fully knowing the future circumstances they will have to confront - but I discovered that people in finance had some fresh ideas. It became clear to me that these ideas could also be good in engineering, for example in reliability of design. Engineering design has many constraints – like requiring the probability that the bridge may collapse should be less than 1 percent, to give you some kind of indication. It turns out that such probabilistic conditions are very ill behaved mathematically. There are some better ways to look at the issue, and one of my passions at the moment is to try to convey this new way of looking at risk to the engineering community. I was involved with a conference in October in Minneapolis, Minnesota, where all kinds of engineers were talking about risk and uncertainty. This interest has also led to another collaboration, with a civil engineer and optimization specialist [Johannes Royset] who is at the Naval Postgraduate School in Monterey, California.

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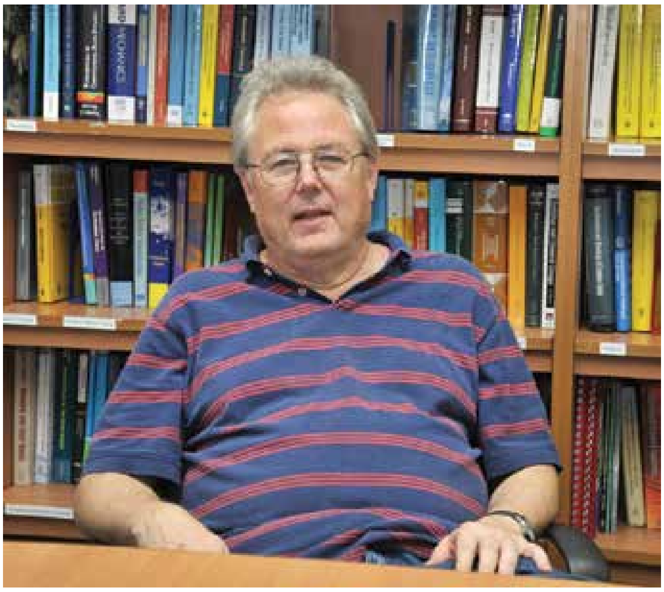
I: You are also involved in stochastic analysis, isn't it?

R: That's right. This is another underpinning of work in risk, because optimization with uncertainty gets very much involved with stochastic analysis in the sense that, in making decisions that have consequences in the future, you have to use the available data to construct a good representation of what might happen in the future. Also, on a different plane, there are many, many situations in engineering where you have a statistical database from experimental tests, say, of the properties and behavior of some material according to the mix of ingredients you put into it, but no underlying theory. There is no law, even in physics, that explains the test results fully and can be extrapolated beyond the particular instances tested. What you get into is a lot of interplay between statistical estimation and optimization, and I was talking about that here at the National University [of Singapore]. It leads to new developments in statistics. For example, there is a classical form of regression, least-squares regression. But we found that if we are going to work with some kinds of risk we should do regression with a different measure of error other than a sum of squares. That is what I'm very much involved with. I also gave lectures on it to the Department of Statistics at Heidelberg University in July. I know it's at a late stage in my career, but the pleasure of being in that stage is that you have a broader view. If you are still very interested in a field, you may be able to bring ideas together and see them in a way that other people have not yet noticed. You can try to bring this to people's attention, and it can be a lot of fun when your later career gives you platforms for that.

I: Have you done any consultation work with industry?

R: There were various opportunities, but somehow they didn't really come to fruition until recently. I'm now doing some consulting with Codelco which is a gigantic copper company in Chile, and that's why I'm going back there in March. This has to do with risk, and reliable engineering is behind it as well. To see how all these things fit together, I'll describe briefly. Codelco has copper mines which can involve making a cavity in the mountain as large as 200 meters high from floor to ceiling – imagine, such an enormous cavity. They do this by blasting over many years. Once they have exhausted that cavity they start again at a lower level of the mountain. As they do this, there are micro-earthquakes caused by the blasting and there are cracks, shears, slips and all that. My colleagues in Chile are modelling the geophysics of it, but then risk comes in. First of all, where should they place sensors to monitor all the potentially dangerous activity? What would be the best locations? That is an optimization problem. Another is how to protect the tunnels that have the workers in them at the various levels of the mine. Nothing is perfect; you can never build a tunnel so heavily reinforced that nothing bad would ever happen. Or if you could, maybe you wouldn't have a budget big enough. What is the trade-off? That's where the theory of risk comes in, and that's what my consultation is about.

Richard T. Durrett: Mathematical Modelling in Ecology, Genetics and Cancer Research >>>



Richard T. Durrett

Interview of Richard T. Durrett by Y.K. Leong

Richard Timothy Durrett made important contributions to probability theory and is world-renown for his multidisciplinary work involving probabilistic methods and mathematical modelling in ecology, genetics and cancer research.

Durrett obtained his BS and MS degrees in mathematics from Emory University (in Atlanta, Georgia) and PhD in operations research from Stanford University. He then taught at the University of California at Los Angeles for a decade before joining the mathematics department of Cornell University in 1985. After moving to Cornell, his interest quickly shifted to problems in ecology and genetics and he began a long and distinguished collaboration with biological scientists. In 2002, he became a joint member of the newly formed Department of Biological Statistics and Computational Biology (formerly Biometrics) in the College of Agriculture and Life Sciences. He founded the Cornell Probability Summer Schools of which he is still the scientific advisor even after he moved to Duke University at Durham in 2010 as the James B. Duke Professor of Mathematics.

As director of the National Science Foundation-funded program VIGRE (Vertically InteGrated Research and Education) in Cornell University's mathematics department, he has been involved in a wide range of activities that reach out to students from the high school level to graduate level. At the professional level, he is active in the organization of international conferences and meetings on probability and biology.

He has received numerous honors for his wide-ranging contributions to mathematics and the biological sciences, notably elected fellowship of the Institute of Mathematical Statistics, American Academy of Arts and Sciences, National Academy of Sciences (US) and American Association for the Advancement of Science.

He has written 7 books and contributed to more than 170 research papers on probability theory and multidisciplinary research in ecology and the biological sciences. He has been on the editorial board of a number of international journals, in particular, the *Annals of Applied Probability*.

Durrett's early work centred around theoretical questions on Brownian motion and stochastic processes. Since 1985 he has directed his interest and energy to the use of probabilistic ideas and methods in mathematical modelling in ecology and the biological sciences. His work in population genetics has been used to study the evolution of DNA repeat sequences and to look for footprints of adaptive evolution. The methods that he developed for the study of genome rearrangement have been used in the comparative genetic study of tomatoes, eggplant and peppers. His work on cancer models is relevant to regulatory sequence evolution and multi-stage carcinogenesis. A recent addition to his research interests is the study of stochastic processes on random graphs.

Among the many invited lectures he has given around the world are the 45-minute lecture at the International Congress of Mathematicians 1990 in Kyoto, Medallion Lecture 2006 and Wald Lecture 2008. His Wald Lecture *Probability problems arising from genetics and ecology: philosophy and anecdotes* was given at the (7th) World Congress in Probability and Statistics, which was jointly sponsored by the Bernoulli Society and the Institute of Mathematical Statistics (US) and held in the Asia-Pacific region for the first time, at the National University of Singapore from 14 to 19 July 2008.

During the Institute's program on *Probability and Discrete Mathematics in Mathematical Biology* (14 March - 10

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June 2011), he gave tutorials on cancer modelling and invited talks on networks in the subprogram on *Discrete Mathematics and Probability in Networks and Population Biology*. It was during this event that Y.K. Leong interviewed him on 10 May 2011 on behalf of *Imprints*. The following is an edited and vetted version of this interview which gives us an insight into the path he took from mathematics to nominally operations research and eventually to what the layman may dramatically and excusably perceive as "life-threatening" issues in ecology and cancer research. Short of pursuing a medical career, he has travelled a path that crosses medical pathways and seems to fulfil, in an indirect and unexpected way, his mother's dream for him years before. This interview also leaves you in no doubt that the rarefied atmosphere of mathematics does have something to offer that is beneficial to this planet and its inhabitants in the future to come.

Imprints: I believe that you had originally wanted to become a medical doctor. How did you end up doing mathematics at Emory University first and then a doctorate in operations research instead at Stanford University?

Richard T. Durrett: Becoming a doctor was more of my mother's dream than my dream. You also have to see this in the context of the 1970s in the United States. There was the Vietnam War and one crap reason for being a doctor is that you don't have to fight in the war. Along came a system of draft lottery and I drew number 343 in the lottery, which meant that I certainly did not have to go to fight. I realized this leaves me free to do mathematics. Another reason for not becoming a doctor was that in science I was very good with theory but I was never very good with the lab. I realized being a doctor was all lab rather than theory.

I: Subsequently you went to do operations research instead.

D: Well, again this was in 1973. When I graduated from college the job market was very bad. There was no temporary position, so most people got into side jobs effectively for six years and then they either got tenure or not. There was a lot of unemployment in math. So I decided that if I was going to disappoint my mother and do mathematics, I should do something where I could get a job afterwards. I thought that employment was good for operations research and also I was interested in game theory which is something studied in operations research.

I: Any reason why Stanford University?

D: It seems to be one of the best places for operations research. I applied to Stanford and Cornell and several other places, but I think Stanford was my first choice.

I: What was the topic of your PhD thesis and who was your advisor?

D: My advisor was Dr Donald Iglehart. He was interested in queuing theory and in one of his classes he posed a question that he was interested in. We know that random walks converge to Brownian motion but suppose I condition the random walk not to hit 0, does it converge to Brownian motion conditioned not to hit 0? This is a technical question because Brownian motion avoids the axes with probability zero. I was able to solve this problem and this became my thesis.

I: Was this in the mathematics department?

D: No, this was in the operations research department. I took a lot of courses from mathematics and from statistics. In particular, I took several courses from Kai Lai Chung [(1917-2009)] who was in mathematics at that time and also from Sid Resnik who was a professor in statistics.

I: Subsequently you went from there to UCLA. Any specific reason why UCLA?

D: Well, it was a school with a very good probability group… I can't really exactly say why I chose that from my many options but I did.

I: Your research interests seem to be centered in applications of probability to solve concrete problems in other fields like biology and ecology. Do you think that this could be traced back to your original ambition to become a doctor?

D: No *[Laughs]*. I think I explained that it was more my mother's desire than mine. I certainly spent about 10 years after I got out of graduate school being a very pure mathematician. After I came to Cornell I got interested in doing applied problems from Simon Levin, a famous ecologist. What I learned from working on applied problems is that when you work on a question that comes from another field it quite often has much more interesting answers than the ones that you made up yourself.

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I: How do you choose the problems that you want to work on?

D: I wish I knew. I read a lot of articles, I talk to people, I hear talks. I was serious when I said I wished I knew.

I: Problems in biological sciences like genetics seem to require multidisciplinary team work. What is the key to this kind of collaboration?

D: It takes a long time to figure how to do this. I mentioned that I met Simon Levin but it was two or three years before I could figure out how to do things mathematically that was interesting to him from a biological perspective. It takes some time learning the other field and listening to other people to find out what they are interested in and then trying to figure out how to do things. I mean, in doing applied science probably 80 percent of the work is to figure out what the right question is. And then after that it's often relatively straightforward to solve.

I: Is Levin a biologist?

D: He's originally a mathematician. He's in the Department of Ecology [and Evolutionary Biology] at Princeton [University] but right now he's probably better known for his work in ecology, most of which is very mathematical. He won the Kyoto Prize about 5 years ago.

I: Not many mathematicians go into ecology.

D: Well, not many mathematicians are interested in the real world, in ecology, in particular, but at this conference there were many mathematicians who were interested in questions from genetics and there are several here who are interested in problems from ecology. Both are very good sources of mathematical questions. But you have to take some time to find out what the biologists are interested in.

I: Are problems in ecology real life problems?

D: Yes, they are real life problems but they are not necessarily the ones I work on. I do a lot of work on spatial models and that's again one of Simon Levin's expertise. Many models I can mimic assume things that are homogeneously mixing, and we would like to know if things are distributed in space how do they change the ecological competition? So I don't work on very practical problems. It's more sort of theoretical.

I: Are there any mathematical models in ecology that have provided solutions to practical problems?

D: Well, by me, no, but let me change the subject a little bit. There are things that I have done in genetics that turn out to be practical and useful. I mean, Steve Tanksley at Cornell asked me the question about how many plants he needed to grow to localize a certain gene within 100,000 spaces. I was able to give him an answer based on mathematics that he and his and co-worker have found useful since then. In ecology, no, I haven't contributed anything, but there are several things I have done that are useful in genetics.

I: In ecology, are the mathematical problems mostly of a statistical nature?

D: Certainly there is a lot of room for statistical work but there's also a lot of room for mathematical modelling. Certainly if you want to apply models you have to estimate parameters. But there is also the general effort that if I have a model with specified parameters how does that perform? That is the type of work that I'm more interested in.

I: Some years ago, there was a claim that by studying the mitochondrial DNA that is passed down along the female line only, all human beings can be traced to one common female ancestor in the distant past. Is this a scientific fact or a probabilistic assertion?

D: Probably a little bit of both. I mean, we are not going to be able to look back into the fossil record and identify this woman, but by making certain assumptions about how the population interact, we can get an idea about when this common ancestor would have lived. There is a great amount of uncertainty about the time, I think, maybe a confidence interval of between 80 to 400 thousand years ago. But there is not much doubt that of all the humans on the planet now, their mitochondria DNA have a common ancestor. It's just a question of when that occurred.

I: Common ancestor doesn't really mean one single person, does it?

D: Well, that's what the phrase means. Well, mitochondria DNA is passed from the mother to all the children but only the daughters matter. As you work backwards in time, you have one mitochondria DNA here. So as you work

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backwards further and further, the lineages collapse to one person and after that they continue to be within one family line. So yes, there is one most recent common ancestor of all the women on the planet in terms of their mitochondria DNA.

I: It's quite amazing, isn't it?

D: I guess so. For me, it is mathematically a fact. Also, another thing that has happened is that the population is growing exponentially forward in time, so if you work backwards in time it decreases. So it makes it easier to have a common ancestor and some of the complications make it hard it to compute when this time approaches.

I: I wish this fact is more well-known. It will make us feel closer to each other.

D: Well, I guess so. That will be a good thing if people come closer to each other.

I: I believe your team has done a project trying to compare the genetic makeup of tomatoes, eggplant and peppers. Has your team completed this project?

D: I did this work in consultation with Steve Tanksley. He studied these three species and he wanted to understand the relationship between these things because it allows you to take discoveries in tomatoes in one species and translate them into another species. So if you find a gene in tomatoes that help you to grow bigger tomatoes then you have a better understanding how to grow bigger eggplants. We studied how the chromosomes between the various species are related. There's still much work to do to understand the relationship between these species. So in some sense the work is never really completely completed.

I: Any particular reason for choosing these three fruits?

D: Steve Tanksley has the data on it. I mean, they are chosen because they are closely related. If you think about a tomato and an eggplant they don't really look a lot alike, but if you think about it a little bit more, the tomato has a lot of loose seeds embedded in it and so does the eggplant. The fact that one is red and the other is purple is not such a huge genetic difference.

I: How different are they genetically?

D: I couldn't really tell you, seriously.

I: For example, the genetic difference between humans and chimps is less than two percent, isn't it?

D: Yes, it's some small amount. We're related by something like six million years but you have to remember that humans have a rather long generation time period. So that divides it by 20. You know, eggplants and tomatoes are annual species. So the same number of years may be a larger number of generations. Also the genetics of plants are complicated by the fact that about 10,000 years ago, plants were domesticated and that caused large genetic changes when farmers selected species that grew well. This really complicated the genetics of plants.

I: Are cancer models disease-specific?

D: Yes, very much so. Different cancers have different mechanisms. In leukemia you have stem cells in your bone marrow that produce your white blood cells and cancer comes from having mutation in those stem cells. In colon cancer you have many little indentations in your colon called polyps and genetic mutations happening in one of these polyps start the process of cancer. This is one of those things we are looking for when you have a colonoscopy.

So the geometry and the consequences of the cancer in different organs are very different. At Duke I am talking to some people who study ovarian cancer and one of the problems there is that it is hard to detect and quite often there is a short window between when the cancer starts and when it spreads to other organs. And after it spreads the cancer is very hard to treat, so we want to know if there are screening strategies to diagnose it before it spreads.

I: Is there any model that provides a mechanism for different types of cancer?

D: Well, we do a lot of work with branching process models. The network is a first approximation in many systems. Now in my more recent work with Franziska Michor who is with the Dana-Farber Cancer Institute, we realize that spatial models may be needed for some solid tumors and in some other situations like the spread of breast cancer through a

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duct in the breast. So there is small repertoire of models that we are relying on. But one needs different models.

I: What are spatial models?

D: If you have your pancreas you have a lot of cells sitting next to each other, and if I want to understand how the growth of a solid tumor proceeds then I need to look at the geometry of the object. So the model takes that into account: it's a spatial model. In branching processes models all I care about is the number of cancer cells and not how they are arranged. That feature makes spatial models more complicated because you need to know the arrangement of the cells as well as the number.

I: How much geometry do you need to know?

D: By geometry, I am saying it in terms of Euclidean geometry. Maybe I should say the shapes of the growing tumors are important. High school geometry is not going to help very much.

I: What about topology?

D: I personally don't think you need to know topology.

I: Those things curl around in a complicated way in the body. Is it possible to look at their geometry?

D: Well, it is possible. The body has features like blood vessels and connective tissues that abstract geometry does not have. If you want to talk about string theory and relativity, you can talk about the geometry. I'm not so sure if many geometries give the answer.

I: Maybe it involves some new kind of geometry, is it possible?

D: Well, in mathematics, anything is possible but some things are more likely than others.

I: You have recently moved from Cornell University to Duke University. Is there any particular reason for this move?

D: Well, that's a pretty long and complicated question. Let me give you the short version. I felt that there were many more opportunities for me to do multidisciplinary work at Duke. Of course, the decision is not made in the abstract. The fact that my two children just recently graduated from college made it much easier to move. So it's a combination of various things: the opportunities at Duke and the fact that I was no longer tied down to Ithaca due to my children. Also it's nice to be away from the snow at Ithaca and have relatively warm weather at Durham.

I: I remember that in one of your websites there is a picture with a lot of snow.

D: Well, that's my house in Ithaca. It was soon after we sold our house; we had 17 inches of snow in one day. That's what the picture was about, it's my front yard. I look at that I can remember it in July or August when it is very hot in Durham, much like it is hot here, hot and humid.

I: Is it in the mountains?

D: No, it's not in the mountains, it's just further north. The mountains are not very high but it is just so far north in the United States. Singapore is on the equator. Ithaca is at a latitude like 40 degrees north, halfway between the equator and the North Pole.

I: I believe you have a lot of doctoral students during your career. Do you have any now?

D: Yes. I have three students at Cornell, two of whom are finishing up this year, and two students at Duke who have just started to work with me. I very much enjoy working with students.

I: What kind of advice would you give to students who are interested in this kind of multidisciplinary work?

D: That's a good question. You have to find somebody you can work with and it's very tricky because you have to learn about the subject matter that you want to work on and you have to learn the mathematics at the same time. So it is a very tricky game of, you know, to be a mathematician you have to prove mathematical results but you also want to do things that relate to the subject matter. So it's very hard to balance the two: to find results that illuminate the biology and are interesting to mathematicians. It's a very hard thing but I think it is also very worthwhile.

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Publications >>>

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I: Would it also depend on the kind of personality of the person in doing multidisciplinary work?

D: Well, yes, you certainly have to be interested in the other subject matter. You have to be good in talking to other people and interacting with things. I think many people can do this type of work. It does not necessarily require one special type of individual.

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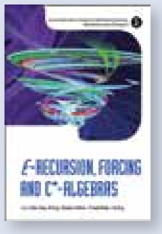

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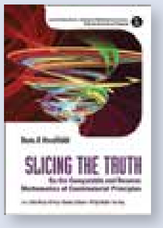

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